

Interferometers

We will make several passes on each topic - hopefully we have full coverage by the end.

Pass #1. Introduction

Interferometers are devices employed in the study of interference patterns produced by various light sources.

They are conveniently divided into two main classes: those based on *division of wavefront*, and those based on *division of amplitude*.

1.1. Michelson interferometer: theory

The Michelson interferometer employs a division of amplitude scheme.

It can be used to carry out the following principal measurements:

- Width and fine structure of spectral lines.
- Lengths or displacements in terms of wavelengths of light.
- Refractive indices of transparent solids.
- Differences in the velocity of light along 2 different directions.

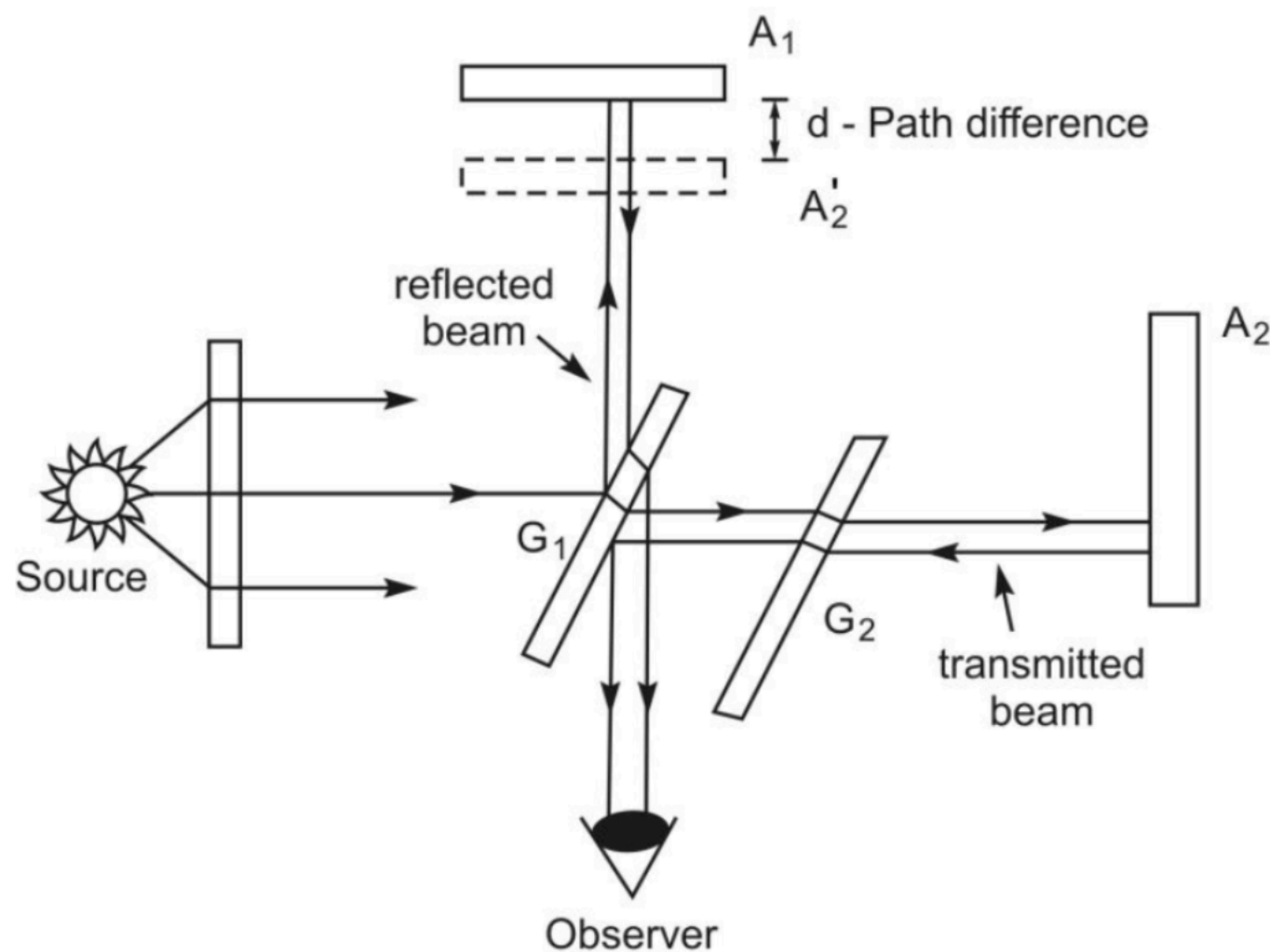


Figure 1-1: A schematic diagram of the Michelson interferometer.

It operates as follows: we “divide” the wave amplitude by partial reflection using a beamsplitter G_1 , with the two resulting wave fronts maintaining the original width by having reduced amplitudes.

A beam splitter is nothing more than a plate of glass, which is made partially reflective: as such, the splitting occurs because part of the light is reflected off of the surface, and part is transmitted through.

The two beams obtained by amplitude division are sent in different directions against plane mirrors, then reflected back along their same respected paths to the beam splitter to form an interference pattern.

The core optical setup, which is labelled in Figure 1-1, consists of two highly polished plates, A_1 and A_2 , acting as the above-mentioned mirrors, and two parallel plates of glass G_1 and G_2 - - one is the beam splitter, and the other is a compensating plate, whose purpose will be described later.

The light reflected normally from mirror A_1 passes through G_1 and reaches the eye.

The light reflected from the mirror A_2 passes back through G_2 for a second time, is reflected from the surface of G_1 and into the eye.

The purpose of the compensating plate G_2 is to render the path in glass of the two rays equal.

This is not essential for producing effective, sharp, and clear fringes in monochromatic light, but it is crucial for producing such fringes in white light (a reason will be given in the “White Light Fringes” section).

The mirror A_1 is mounted on a carriage, whose position can be adjusted with a micrometer.

To obtain fringes, the mirrors A_1 and A_2 are made exactly perpendicular to each other by means of the calibration screws (Fig. 1), controlling the tilt of A_2 .

There are two very important requirements that need to be satisfied along with the above set up in order for interference fringes to appear:

Use an *extended light source*.

The point here is purely one of illumination: if the source is a point, there is not much space for you to see the fringes on.

You can convince yourself of the usefulness of using an extended source by positioning a variable size aperture in front of an extended source and shrinking its radius to the minimum possible (thus effectively converting it to a point source).

As you would see, the field of view over which you can see the fringes shrinks right with it.

Hence, it is in your best interest to use as big of a source as possible (a different screen is of further great aid here).

The light must be *monochromatic*, or nearly so.

This is especially important if the distances of A_1 and A_2 from G_1 are appreciably different.

The spacing of fringes for a given color of light is linearly proportional to the wavelength of that light: hence the fringes will only coincide near the region where the path difference is zero.

The solid line here corresponds to the intensity of interference pattern of green light, and the dashed curve — to that of red light.

We can see that only around zero path difference will the colors remain relatively pure: as we move farther away from that region, colors will start to mix and become impure and unsaturated - already about 8-10 fringes away the colors mix back into white light, making fringes indistinguishable.

Hence the region where fringes are visible is very narrow and hard to find with non-monochromatic light.

Some of the light sources suitable for the Michelson interferometer are a sodium flame or a mercury arc.

If you use a small source bulb instead, a ground-glass diffusing screen in front of the source will do the job; looking at the mirror A_1 through the plate G_1 , you then see the whole field of view filled with light.

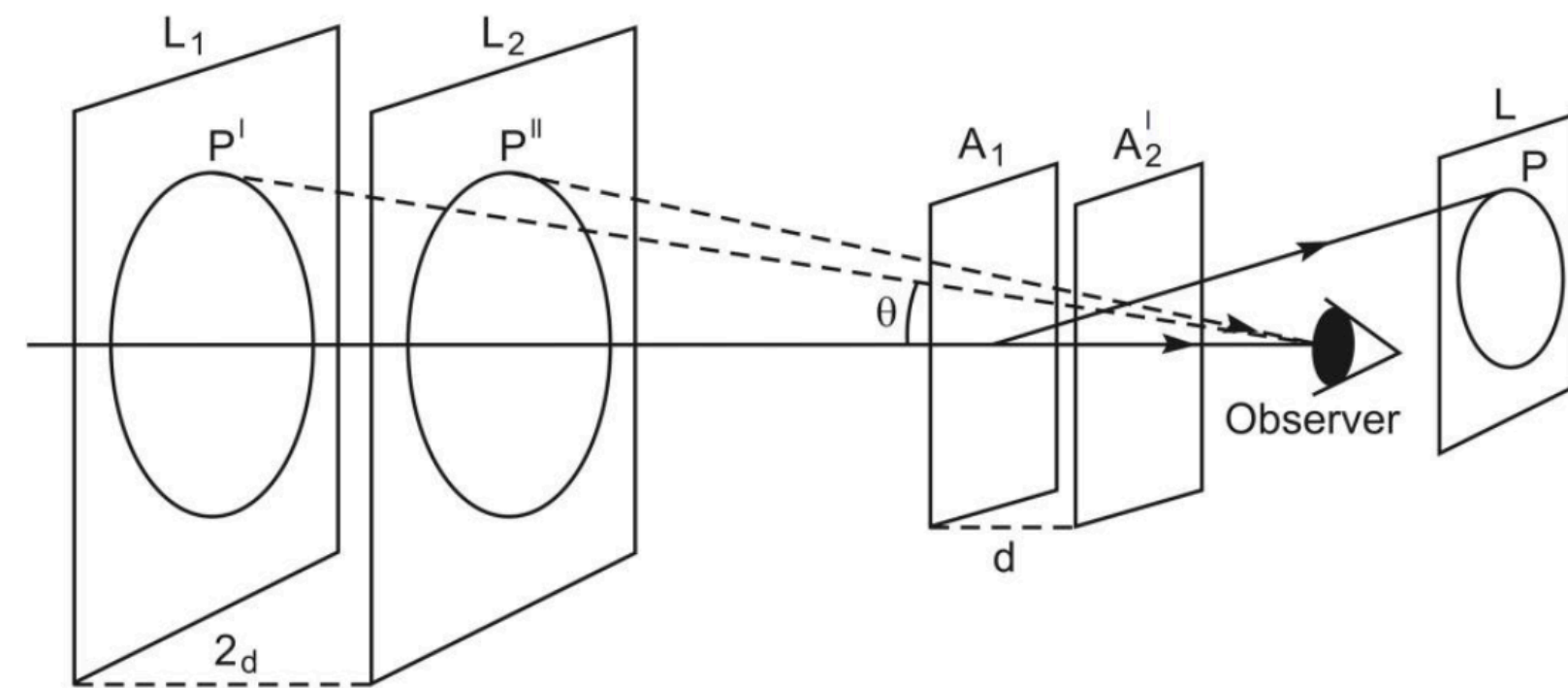


Figure 1-2: Virtual images from the two mirrors created by the light source and the beamsplitter in the Michelson interferometer.

Since light in the interferometer gets reflected many times, we can think of the extended source as being at L , where L is behind the observer as seen in Fig. 1-2; L forms 2 virtual images, L_1 and L_2 , in mirrors A_1 and A'_2 , respectively.

The virtual sources in L_1 and L_2 are said to be in phase with each other (such sources are called *coherent sources*), in that the phases of corresponding points in the two are exactly the same at all times.

The real mirror A_2 has been replaced by its virtual image A'_2 formed by the reflection in G_1 : hence A'_2 is parallel to A_1 .

If d is the separation of A_1 and A'_2 , the virtual sources are then separated by $2d$, as can be seen in the diagram (Fig. 1-2).

When d is exactly an integer number of half wavelengths, every ray that is reflected normal to the mirrors A_1 and A'_2 will always be in phase.

Rays of light that are reflected at other angles will not, in general, be in phase.

This means that the path difference between two incoming rays from points P' and P'' will be $2d \cos \theta$, where θ is the angle between the viewing axis and the incoming ray.

We can say that θ is the same for the two rays when A_1 and A'_2 are parallel, which implies that the themselves are parallel.

The parallel rays will interfere with each other, creating a fringe pattern of maxima and minima for which the following relation is satisfied:

$$2d \cos \theta = m\lambda \quad (1)$$

where d is the separation of A_1 and A'_2 , m is the fringe order, λ is the wavelength source of light used, θ is as above (if the two are nearly collinear, we, of course, have $\theta \approx 0$ — this is the case for the fringes in the very center of the field of view).

Since, for a given m , λ is constant the maxima and minima lie on a circular plane about the foot of the perpendicular axis stretching from the eye to the mirrors.

As was mentioned before, the Michelson interferometer uses division by amplitude scheme: hence the resultant amplitudes of the waves, a_1 and a_2 , are fractions of the original amplitude A , with respective phases a_1 and a_2 .

We can calculate the phase difference between the two beams based on the respective mirror separation.

If the path difference is $2d \cos \theta$, then the difference in phase simply

$$\delta = 2\pi \frac{2d \cos \theta}{\lambda} \quad (2)$$

Here the ratio of the path difference to the wavelength tells you what fraction of a wavelength have you passed, and multiplication by 2π makes it a fraction of the full period of a sinusoid, thus giving you exactly the sought phase difference.

By starting with A_1 a few centimeters beyond A'_2 , the fringe system will have the general appearance which is shown in fig. 1-3, where the rings of the system are very closely spaced.

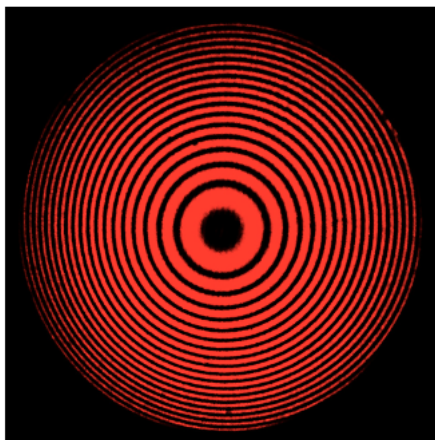


Figure 1-3: The circular fringe interference pattern produced by a Michelson interferometer.

As the distance between A_1 and A'_2 decreases, the fringe pattern evolves, growing at first until the point of zero path difference is reached, and then shrinking again, as that point is passed.

This implies that a given ring, characterized by a given value of the fringe order m , must have a decreasing radius in order for (2) to remain true.

The rings therefore shrink and vanish at the center, where a ring will disappear each time $2d$ decreases by λ .

This is because at the center, $\cos \theta = 1$, and so we have the simplified version of equation (2),

$$2d = m\lambda \quad (3)$$

From here we see that the fringe order changes by 1 precisely when $2d$ changes by λ , hence for a fringe to disappear we need to decrease $2d$ by λ .

Localized fringes

In case when the mirrors are not exactly parallel, fringes can still be observed for path differences not much greater than a few millimeters (using monochromatic light, of course).

The space between the mirrors is wedge-shaped, as can be seen in fig. 1-4: thus the two rays reaching the from eye the mirrors are no longer parallel and appear to diverge.

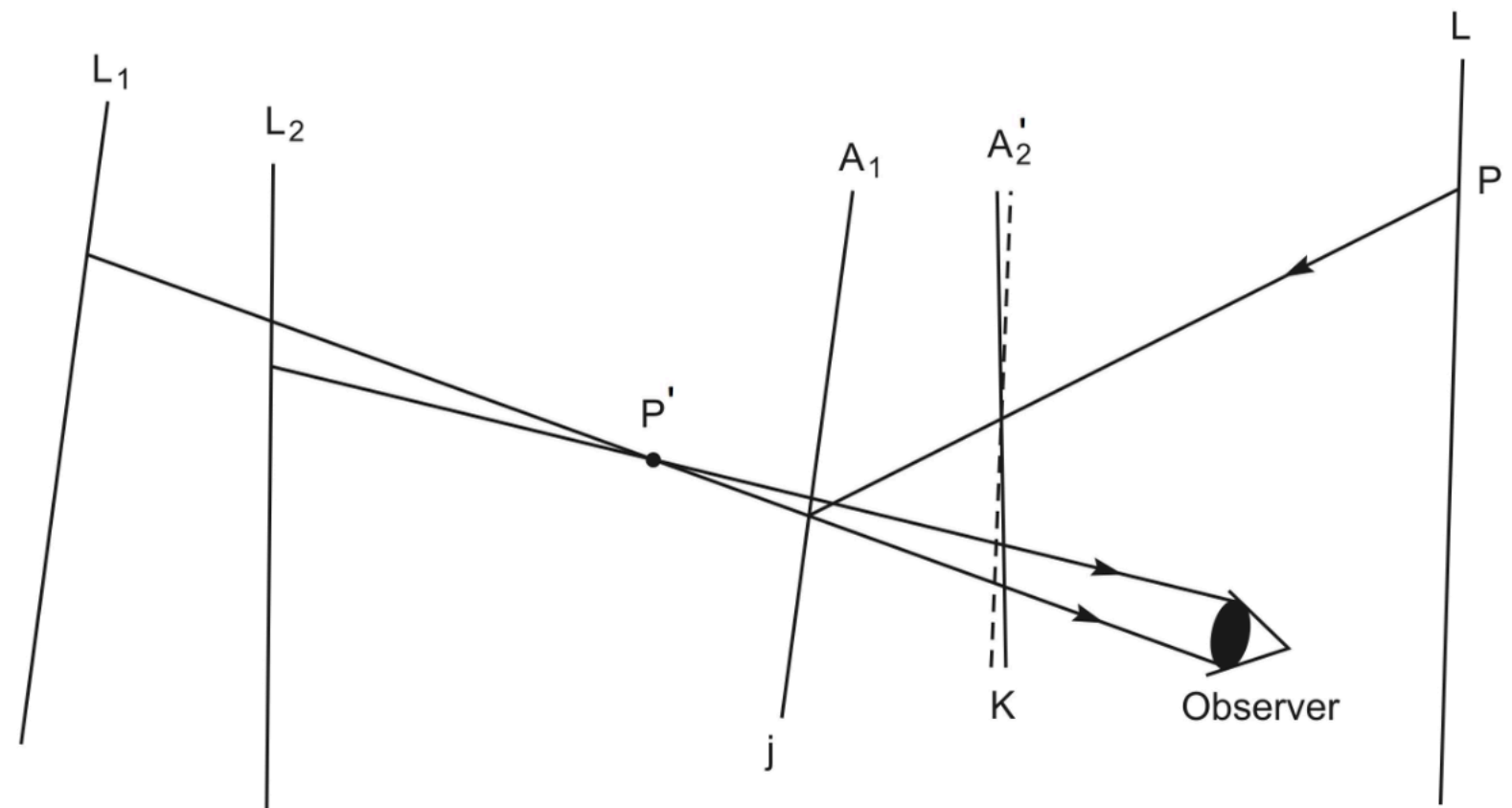


Figure 1-4: Formation of localized fringes with non-perpendicular mirrors — the air wedge is clearly seen.

Hence, the interference picture will be more like that of fig. 1-5: the fringes are now semi-circles, with the centre lying outside the field of view — such fringes are often called localized fringes.

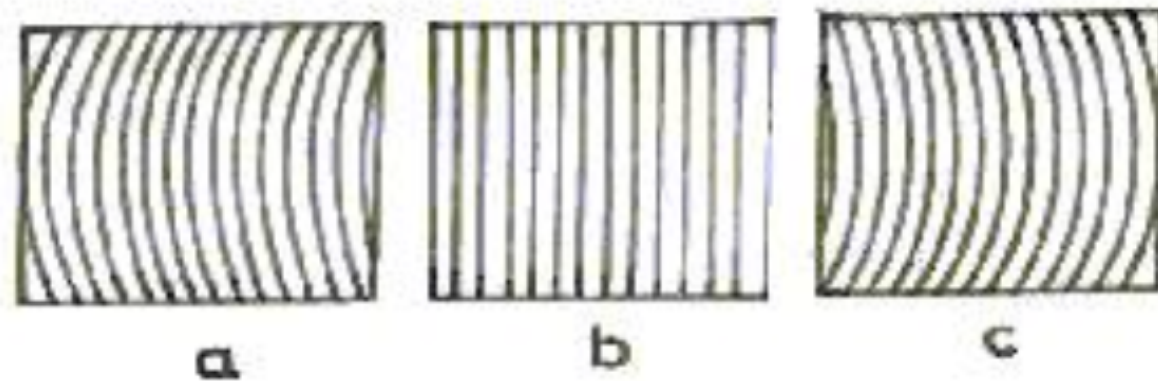


Figure 1-5: The localized fringe interference patterns produced by a Michelson interferometer: (a) and (c) are depictions of curved fringes, implying the mirror is far from the region of zero path difference; and (b) shows straight, parallel fringes — this must be at or very near the point of zero path difference.

The reason these fringes are almost straight is primarily because of the variation of the thickness of air in the wedge, as that is now the main reason for the variation of the path difference between the two beams across the field of view.

One would expect all fringes to be perfectly straight, parallel to the edge of the wedge: however, that is not the case, as the path difference still does vary somewhat with the angle θ , especially if d is large.

Depending on the magnitude of d , we can observe different interference patterns: as we change the path difference, the fringes become straighter, until we hit point of zero path difference.

At that point, if we were looking at circular fringes, they would fill the whole field of view, become very large circles — that means that localized fringes would become parallel lines, as if there were small sections of the circumferences of very large circles.

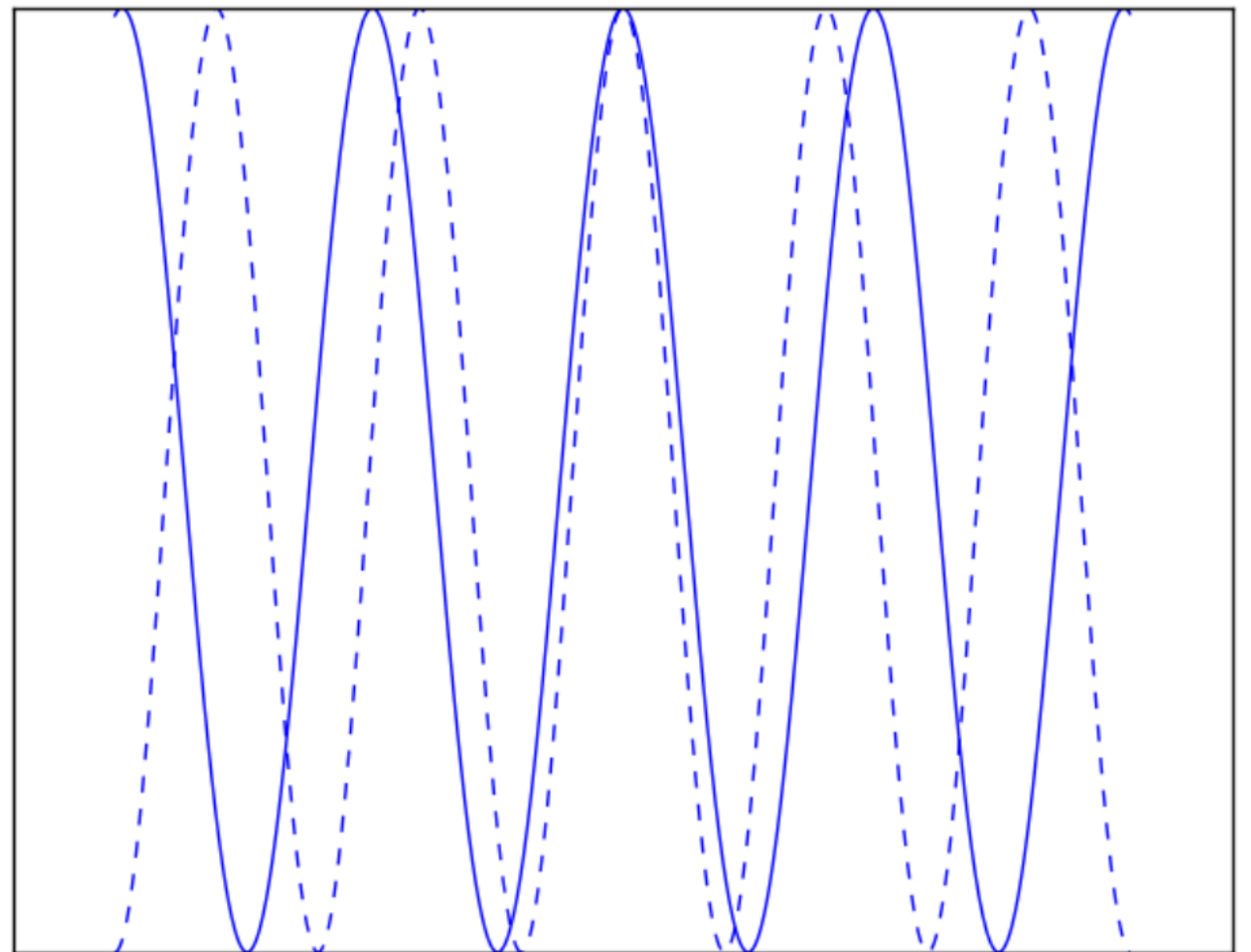
The association “large circular fringes — parallel localized fringes” will be important in the next section, when we use it to locate white light fringes.

White Light Fringes

If instead of using monochromatic light, we wish to study the fringes created by white light, no fringes will be seen at all except for when the path difference between A_1 and A'_2 does not exceed a couple of wavelengths.

This is well-demonstrated in fig. 1-6: the dashed line corresponds to the intensity of the interference pattern of green light, while the solid line — to that of yellow light.

Figure 1-6: The intensity curves of interference of green light (dashed line) and yellow light (solid line). The dispersion of the interference patterns away from the region of zero path difference is readily observed.



As you can see from the diagram, the patterns only overlap over the narrow range of zero path difference between the two incoming beams: now if there are many different wavelengths involved in an interference process, as is the case for white light, one can conclude that anywhere too far away from the region of zero path difference the colors will mix back up into white light and no fringes will be visible.

With white light, there will be a central dark fringe, bordered on either side by 8 or 10 colored fringes.

Since the region over which the white light fringes are visible is so narrow, trying to search for it with white light alone is too time-consuming.

Instead you can first approximate its location by using monochromatic light and finding the region of zero path difference.

It will correspond to one of two regions: (a) if the mirrors are perfectly parallel and we are observing circular fringes, the region with the largest circular fringes is the region of zero path difference; (b) if the mirrors are almost parallel and we are observing localized fringes, then the region with straight, parallel fringes will be the region of zero path difference.

Once we have approximately found the right region, we switch back to white light, and move VERY slowly through the region: the bright fringes should come into view.

These fringes will only occur over a very narrow range of path difference values, corresponding to about a 20 degree turn on the micrometer — hence the need to move slowly, otherwise you can miss them.

We advise that, upon finding the fringes, you mark the approximate position of the micrometer, to simplify future search (as the position of white light fringes will be needed for other experiments).

1.2 Fabry-Perot interferometer: theory

Another commonplace division-of-amplitude interferometer is the Fabry-Perot interferometer, which uses a principle similar to that of the Michelson interferometer to produce interference fringes.

The core of such a device consists of two parallel flat glass plates, one movable, one fixed, the inner surfaces of which are coated with a partially reflective metallic layer (see fig. 1-7).

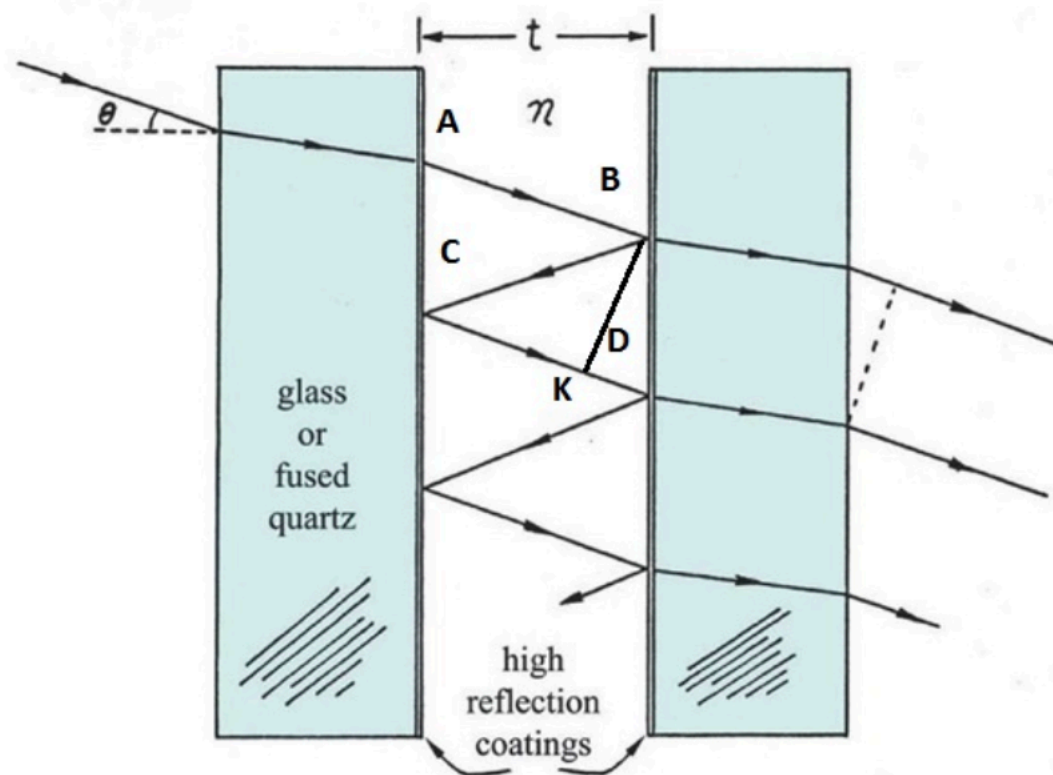


Figure 1-7: The reflected and transmitted beams of light going through the two glass surfaces of a Fabry-Perot interferometer (letters indicate points of reflection/refraction).

Due to the coating, a beam of light incident on the first plate at an angle θ to the horizontal produces a series of beams passing through to the other side, as each continuously gets either transmitted through the second plate to go on to the observer, or bounces back and forth between the inner surfaces until it does (it could also potentially come back out from the side the original beam entered the arrangement, but those rays are of no consequence to us).

Each of the beams arrives at the point of observation with a path difference of δ produce an interference pattern. Let the distance between the plates be t .

We see from fig. 7 that the path difference δ between the rays exiting at B and D is exactly

$$\delta = BC + CK$$

In the diagram, the line BK is normal to CD. The angle between BC and CK is 2θ , and the triangle BCK is a right angle one.

Hence we may write

$$CK = BC \cos 2\theta$$

Moreover, we can relate the hypotenuse BC to the distance between the plates via

$$BC \cos \theta = t$$

Thus we have, for the path difference

$$\delta = BC + CK = BC(1 + \cos 2\theta) = 2BC \cos^2 \theta = 2t \cos \theta \quad (4)$$

The condition for constructive interference is, as always

$$n\lambda = 2t \cos \theta \quad (5)$$

where n is the fringe order, and between the glass plates and watch the fringes disappear in the center of the field of view, thus allowing us to do almost exactly the same measurements as we could with a Michelson interferometer.

The advantage of the Fabry-Perot is its high resolving power: it makes it a valuable tool in the study of the Zeeman effect and the hyperfine structure of certain spectral lines.

One point has to be made concerning this device: since the interference only occurs for light incident on the plate at an angle θ , a perfectly parallel beam of light may not produce fringes: hence we must once again use an extended light source to remedy this problem.

Pass #2: The Michelson Interferometer

The **Michelson interferometer** (invented by the American physicist Albert A. Michelson) is a precision instrument that produces interference fringes by splitting a light beam into two parts and then recombining them after they have traveled different optical paths.

Figure 2-1 depicts the interferometer and the path of a light beam from a single point on the extended source S, which is a ground-glass plate that diffuses the light from a monochromatic lamp of wavelength λ_0 .

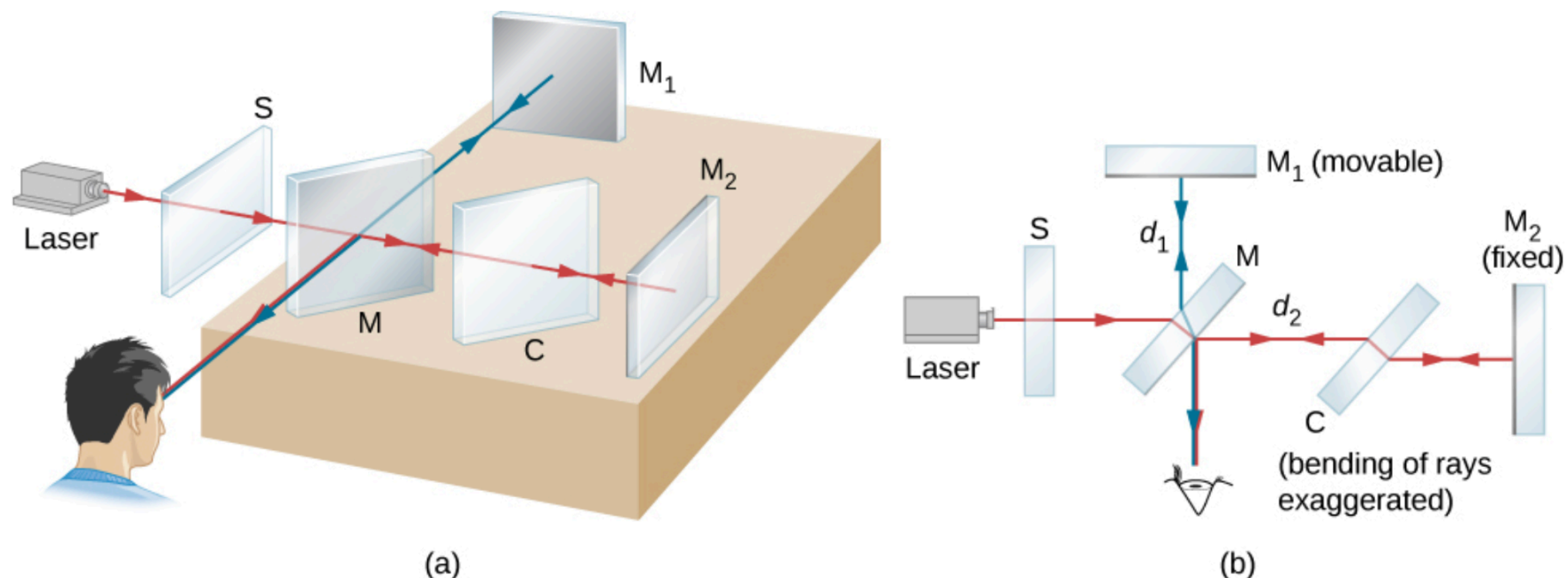


Figure 2-1 (a) The Michelson interferometer. The extended light source is a ground-glass plate that diffuses the light from a laser. (b) A planar view of the interferometer.

The beam strikes the half-silvered mirror M, where half of it is reflected to the side and half passes through the mirror.

The reflected light travels to the movable plane mirror M_1 , where it is reflected back through M to the observer.

The transmitted half of the original beam is reflected back by the stationary mirror M_2 and then toward the observer by M.

Because both beams originate from the same point on the source, they are coherent and therefore interfere.

Notice from the figure that one beam passes through M three times and the other only once.

To ensure that both beams traverse the same thickness of glass, a compensator plate C of transparent glass is placed in the arm containing M_2 .

This plate is a duplicate of M (without the silvering) and is usually cut from the same piece of glass used to produce M.

With the compensator in place, any phase difference between the two beams is due solely to the difference in the distances they travel.

The path difference of the two beams when they recombine is $2d_1 - 2d_2$, where d_1 is the distance between M and M_1 , and d_2 is the distance between M and M_2 .

Suppose this path difference is an integer number of wavelengths $m\lambda_0$.

Then, constructive interference occurs and a bright image of the point on the source is seen at the observer.

Now the light from any other point on the source whose two beams have this same path difference also undergoes constructive interference and produces a bright image.

The collection of these point images is a bright fringe corresponding to a path difference of $m\lambda_0$ (Figure 2-2)

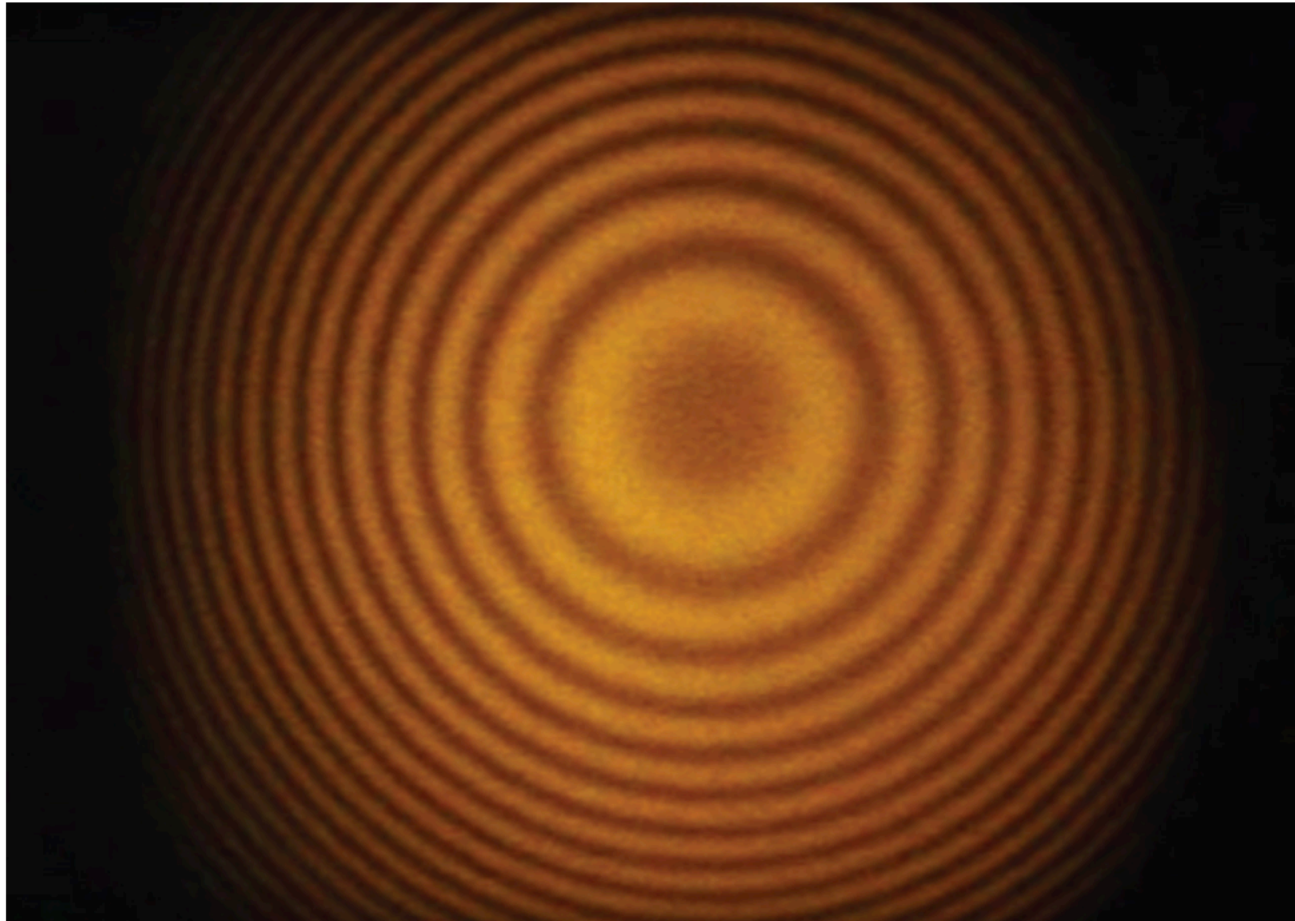


Figure 2-2 Fringes produced with a Michelson interferometer.

When M_1 is moved a distance $\Delta d = \lambda_0/2$, this path difference changes by λ_0 , and each fringe moves to the position previously occupied by an adjacent fringe.

Consequently, by counting the number of fringes m passing a given point as M_1 is moved, an observer can measure minute displacements that are accurate to a fraction of a wavelength, as shown by the relation

$$\Delta d = m \frac{\lambda_0}{2}.$$

Michelson interferometer interference condition

Example: Precise Distance Measurements by Michelson

A red laser light of wavelength 630 nm is used in a Michelson interferometer.

While keeping the mirror M_1 fixed, mirror M_2 is moved.

The fringes are found to move past a fixed cross-hair in the viewer.

Find the distance the mirror M_2 is moved for a single fringe to move past the reference line.

We use the result of the Michelson interferometer interference condition to find the distance moved Δd .

For a 630-nm red laser light, and for each fringe crossing ($m=1$), the distance traveled by M_2 if you keep M_1 fixed is

$$\Delta d = m \frac{\lambda_0}{2} = 1 \times \frac{630 \text{ nm}}{2} = 315 \text{ nm} = 0.315 \mu\text{m}.$$

Significance

An important application of this measurement is the definition of the standard meter.

The length of the standard meter was once defined as the mirror displacement in a Michelson interferometer corresponding to 1,650,763.73 wavelengths of the particular fringe of krypton-86 in a gas discharge tube.

Measuring the Refractive Index of a Gas

In one arm of a Michelson interferometer, a glass chamber is placed with attachments for evacuating the inside and putting gases in it.

The space inside the container is 2 cm wide.

Initially, the container is empty.

As gas is slowly let into the chamber, you observe that dark fringes move past a reference line in the field of observation.

By the time the chamber is filled to the desired pressure, you have counted 122 fringes move past the reference line.

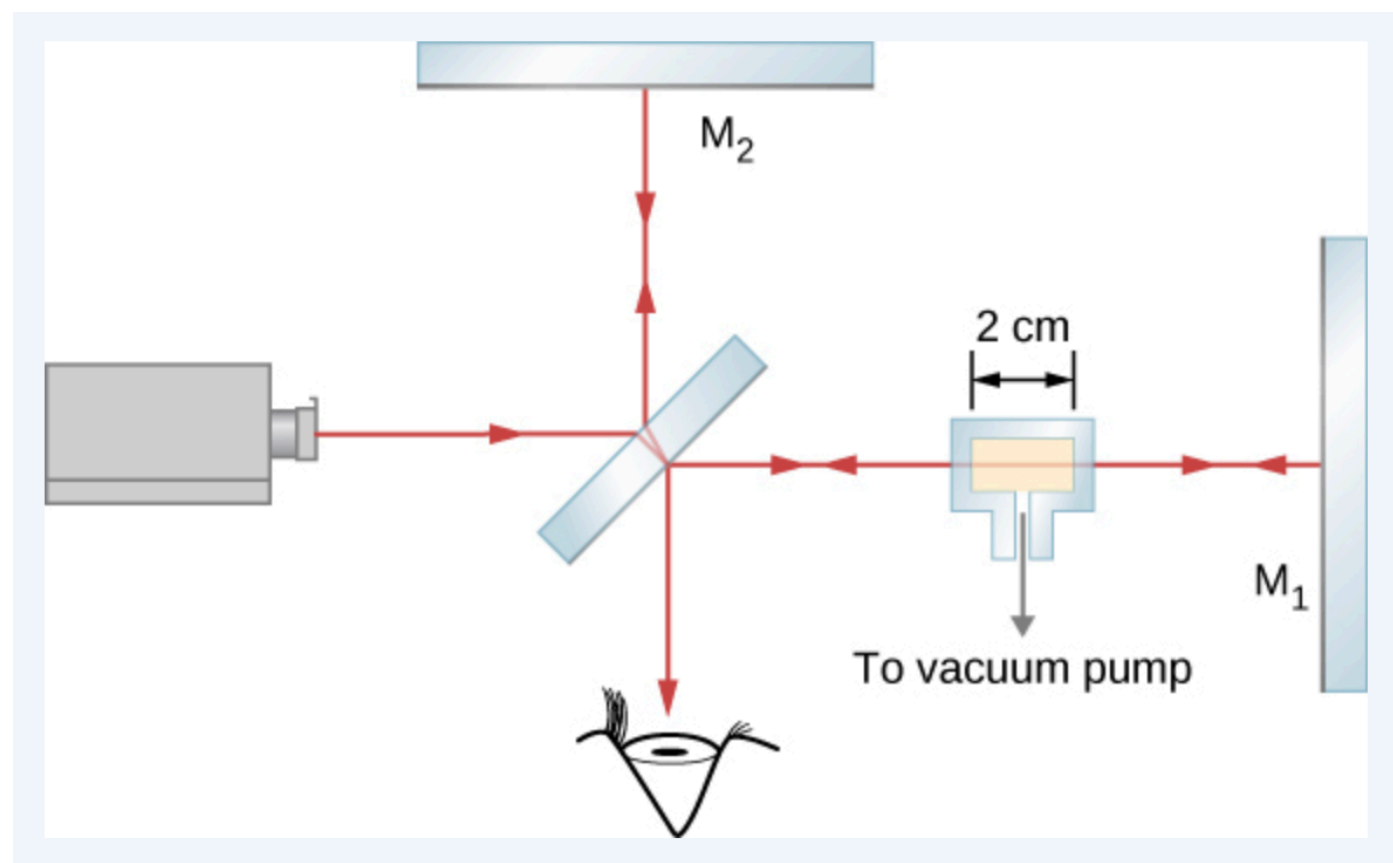
The wavelength of the light used is 632.8 nm.

What is the refractive index of this gas?

Strategy

The $m = 122$ fringes observed compose the difference between the number of wavelengths that fit within the empty chamber (vacuum) and the number of wavelengths that fit within the same chamber when it is gas-filled.

The wavelength in the filled chamber is shorter by a factor of n , the index of refraction.



Solution

The ray travels a distance $t = 2$ cm to the right through the glass chamber and another distance t to the left upon reflection.

The total travel is $L = 2t$.

When empty, the number of wavelengths that fit in this chamber is

$$N_0 = \frac{L}{\lambda_0} = \frac{2t}{\lambda_0}$$

where $\lambda_0 = 632.8$ nm is the wavelength in vacuum of the light used.

In any other medium, the wavelength is $\lambda = \lambda_0/n$ and the number of wavelengths that fit in the gas-filled chamber is

$$N = \frac{L}{\lambda} = \frac{2t}{\lambda_0/n}.$$

The number of fringes observed in the transition is

$$\begin{aligned} m &= N - N_0, \\ &= \frac{2t}{\lambda_0/n} - \frac{2t}{\lambda_0}, \\ &= \frac{2t}{\lambda_0}(n - 1). \end{aligned}$$

Solving for $(n-1)$ gives

$$n - 1 = m \left(\frac{\lambda_0}{2t} \right) = 122 \left(\frac{632.8 \times 10^{-9} m}{2(2 \times 10^{-2} m)} \right) = 0.0019$$

and $n = 1.0019$

Significance

The indices of refraction for gases are so close to that of vacuum, that we normally consider them equal to 1.

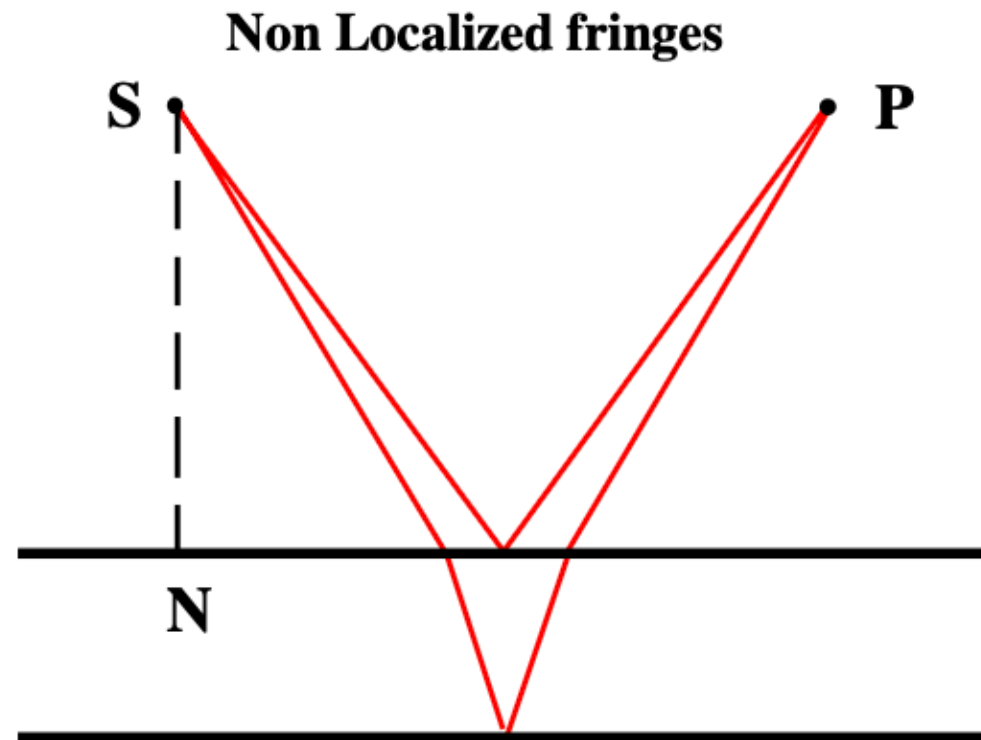
The difference between 1 and 1.0019 is so small that measuring it requires a correspondingly sensitive technique such as interferometry.

We cannot, for example, hope to measure this value using techniques based simply on Snell's law.

Basic Classical Interferometers

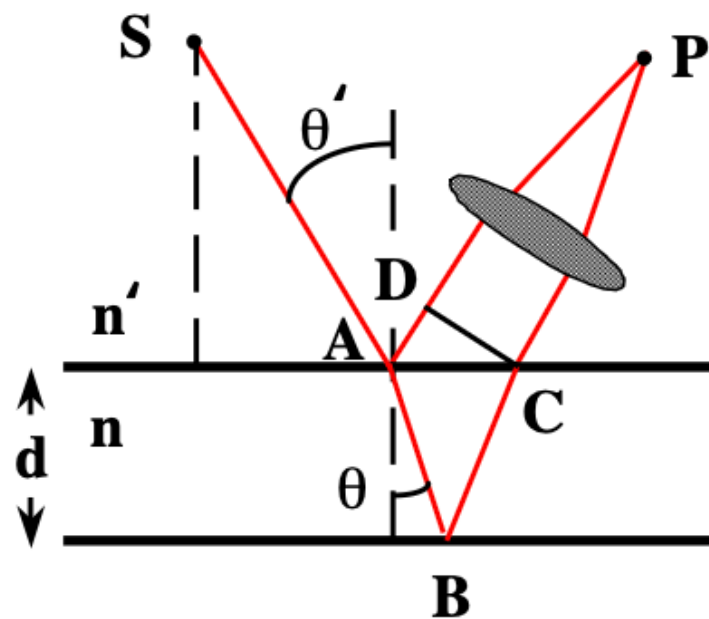
- **Plane Parallel Plate**
- **Fizeau**
- **Michelson**
- **Twyman-Green**
- **Mach-Zehnder**
- **Lateral Shear**
- **Radial Shear**

Plane Parallel Plate - Point Source



By symmetry, fringes in plane parallel to plate are circular about normal SN

Plane Parallel Plate - Extended Source (Path Difference)



$$\Delta l = 2nd \cos \theta$$

Optical path difference

$$\Delta l = (AB + CB)n - n'AD$$

$$AB = CB = \frac{d}{\cos \theta}$$

$$AD = AC \sin \theta'; AC = 2d \tan \theta$$

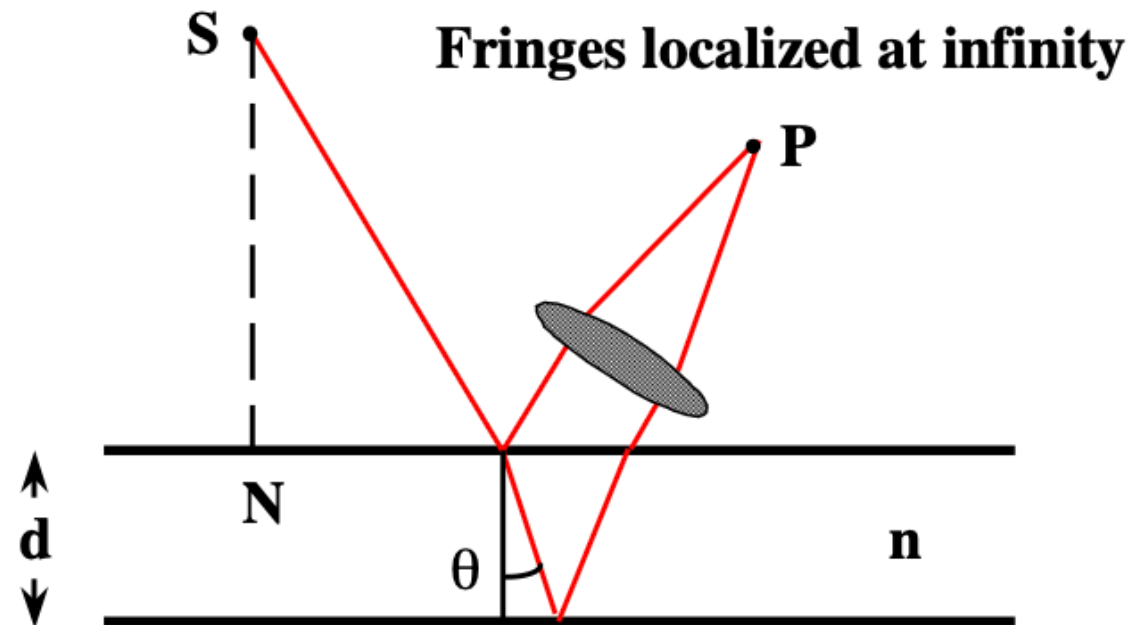
$$= 2d \tan \theta \sin \theta'$$

$$n' \sin \theta' = n \sin \theta$$

$$\Delta l = \frac{2nd}{\cos \theta} - 2n'd \frac{\sin \theta}{\cos \theta} \frac{n \sin \theta}{n'}$$

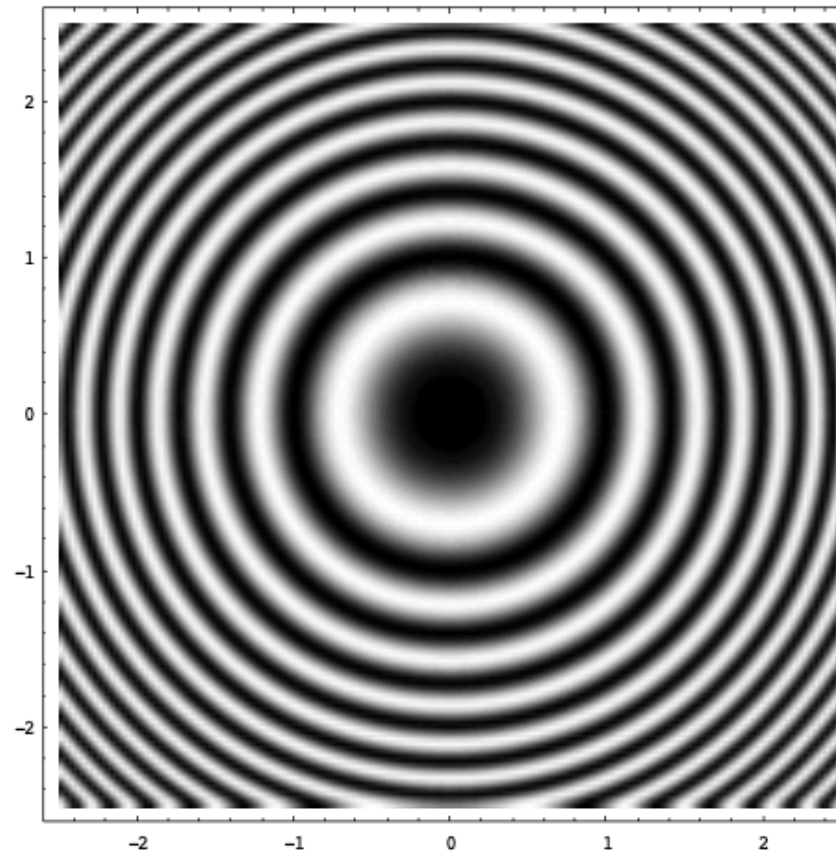
$$= \frac{2nd}{\cos \theta} [1 - \sin^2 \theta]$$

Plane Parallel Plate - Extended Source (Haidinger Fringes)

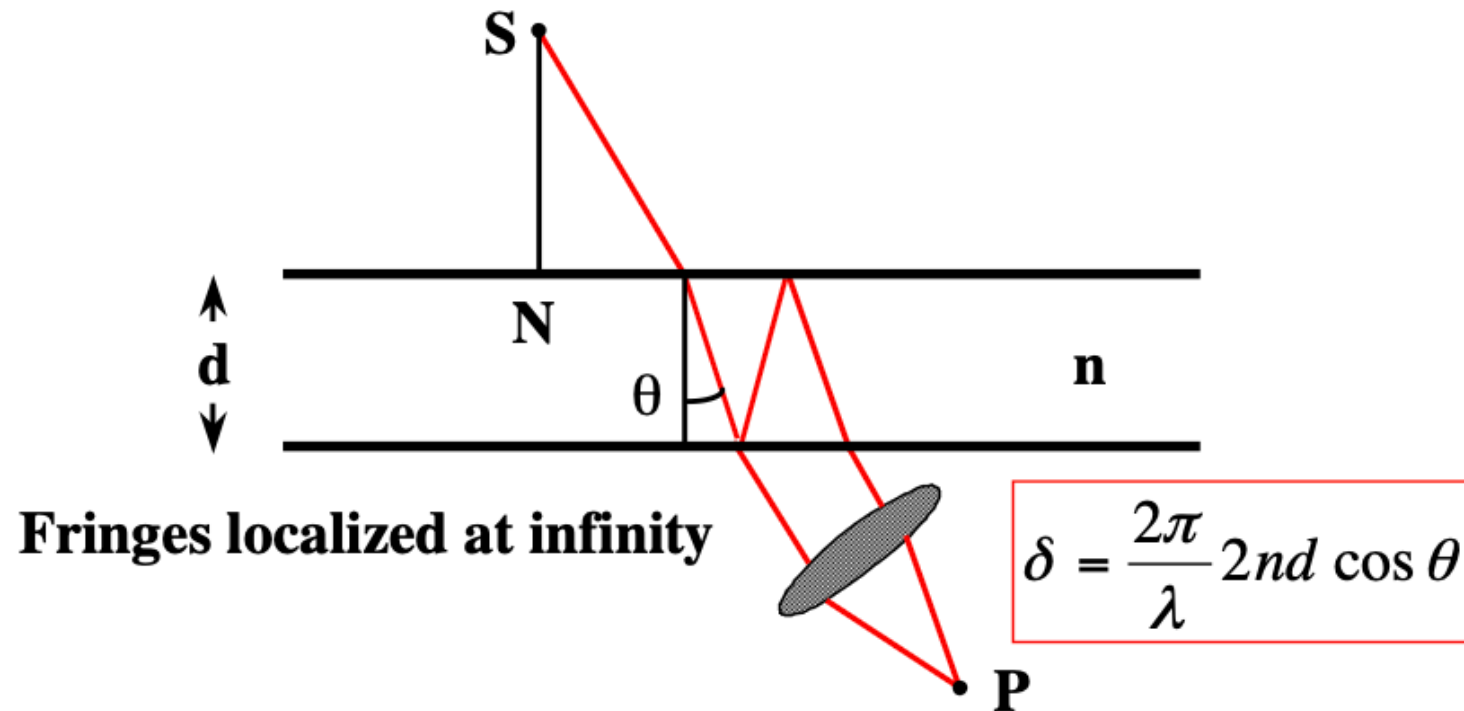


$$\delta = \frac{2\pi}{\lambda} 2nd \cos \theta \pm \pi$$

Haidinger Fringes

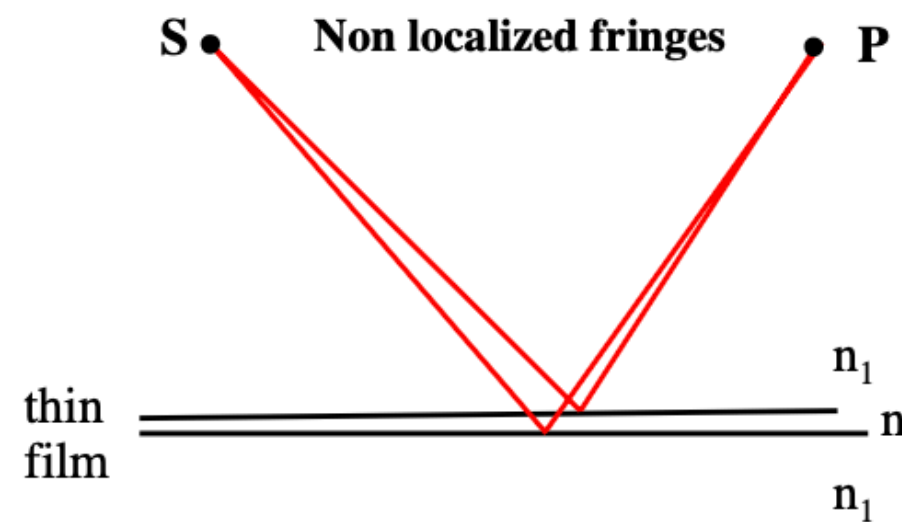


Plane Parallel Plate - Extended Source (Transmitted Light)



Low reflectance surfaces give low visibility fringes.
Transmitted and reflected fringe patterns are complimentary.

Fizeau Fringes - Point Source (1862)

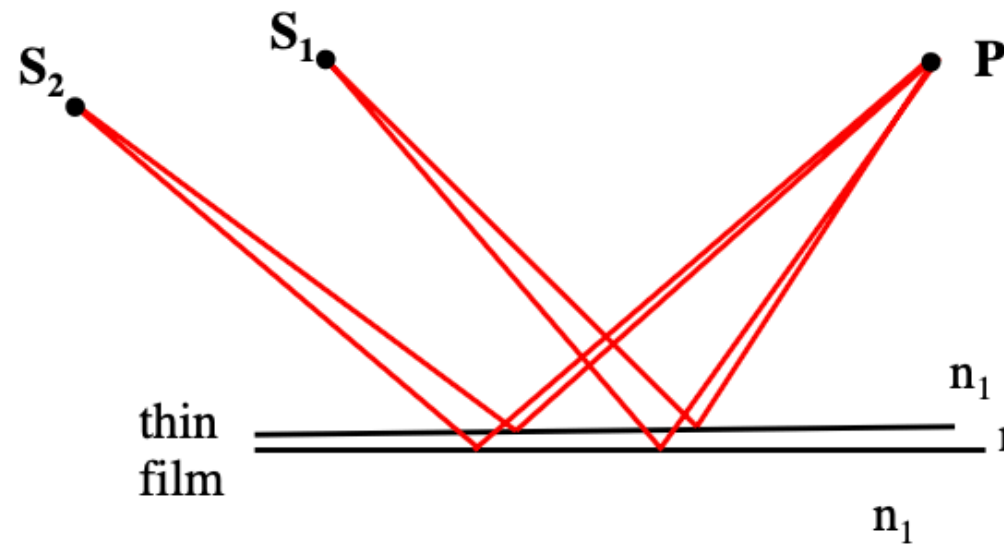


$$\delta = \frac{2\pi}{\lambda} 2nd \cos \theta \pm \pi$$

d is film thickness (function of position)

θ is angle within film (function of position)

Fizeau Fringes - Broad Source (1862)



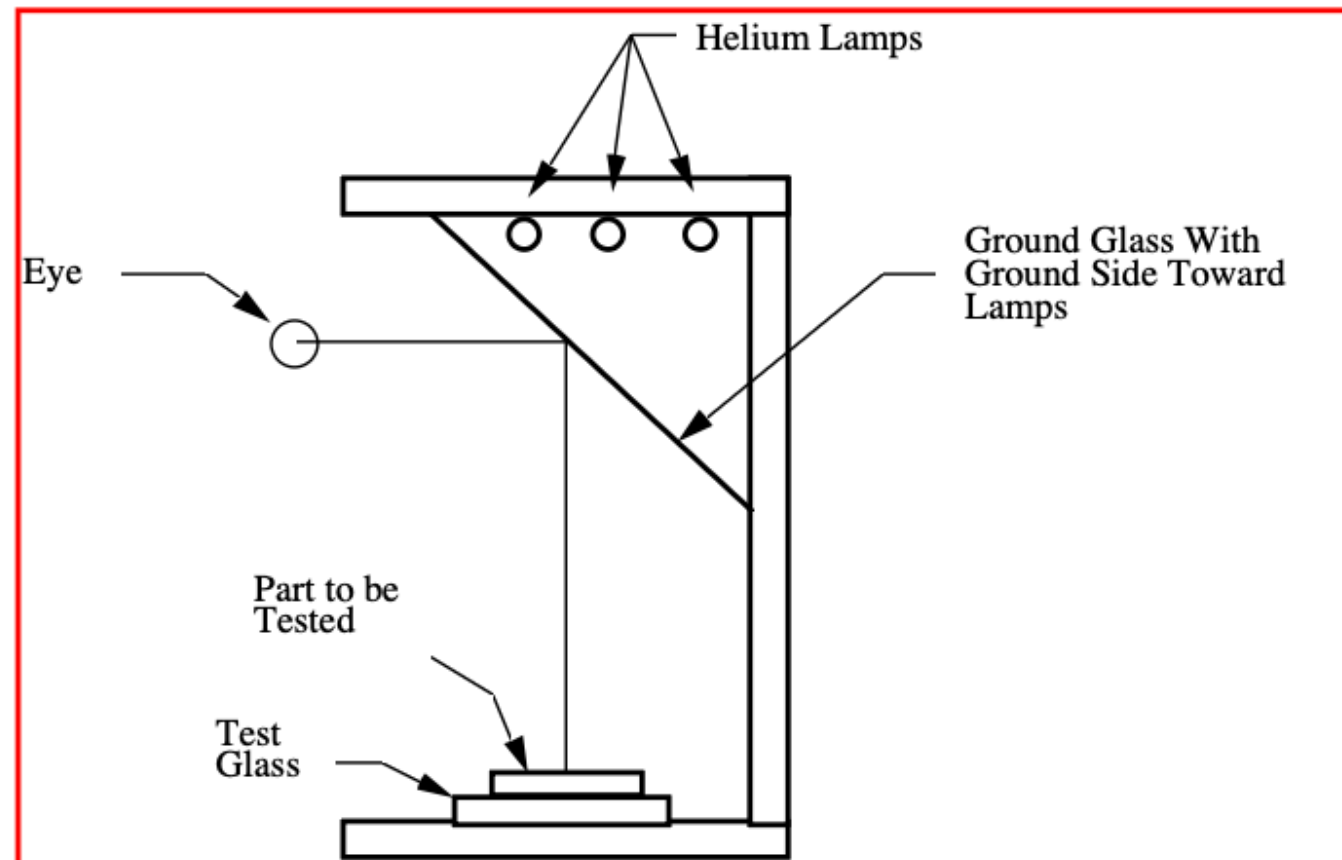
Fringes localized near film

Near the film rays from source points see approximately same d

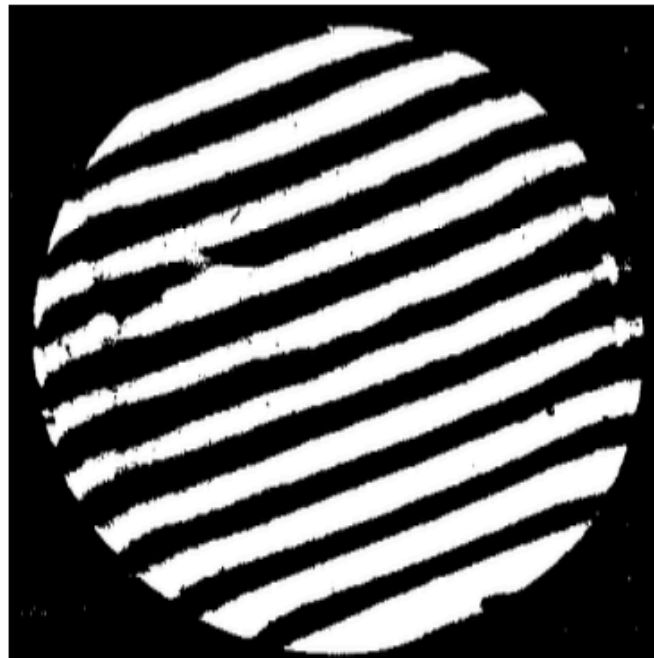
Variations in $\cos \theta$ reduced if

- a) camera has small aperture focused on film
- b) if $\theta \approx 0$, $\cos \theta \approx 1$ for moderate spread in θ

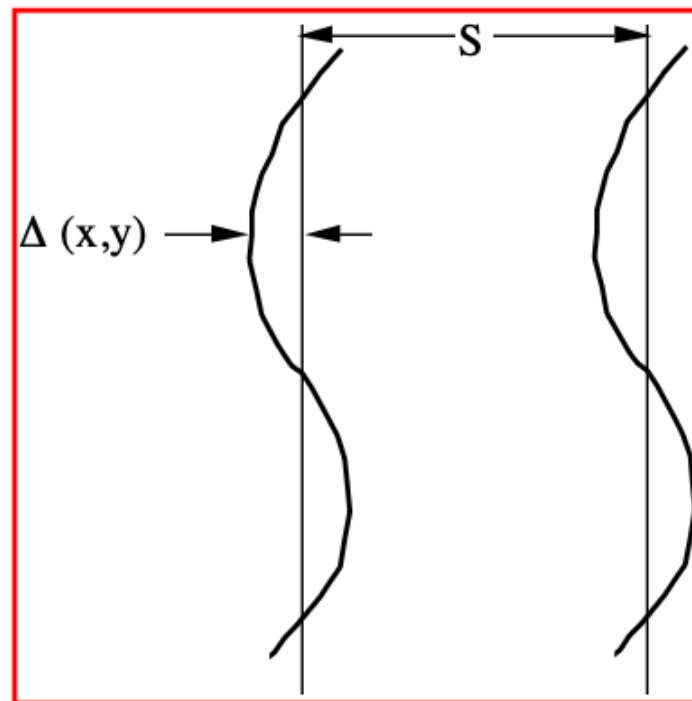
Classical Fizeau Interferometer



Typical Interferogram obtained using Fizeau Interferometer

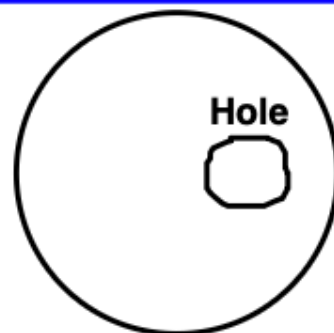


Relationship between Surface Height Error and Fringe Deviation

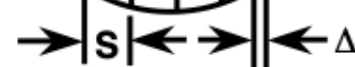
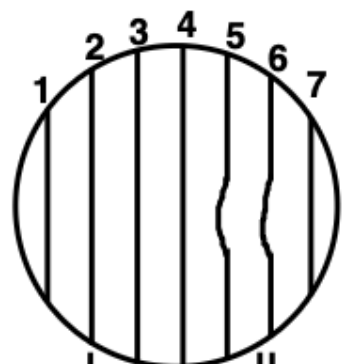
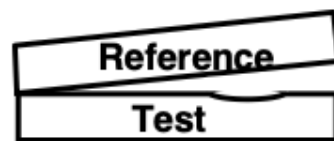


$$\text{Surface height error} = \left(\frac{\lambda}{2}\right)\left(\frac{\Delta}{S}\right)$$

Fizeau Fringes



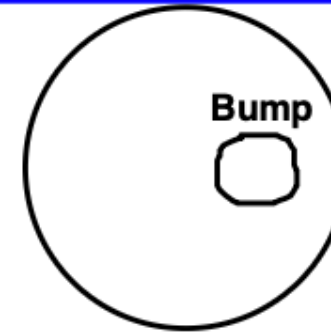
Top View



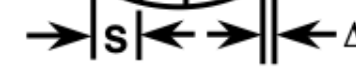
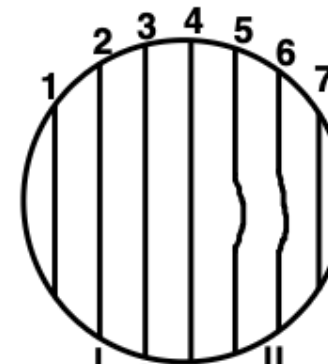
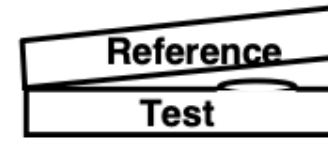
Interferogram

**For a given fringe
the separation
between the two
surfaces is a
constant.**

$$\text{Height error} = (\lambda/2)(\Delta/S)$$

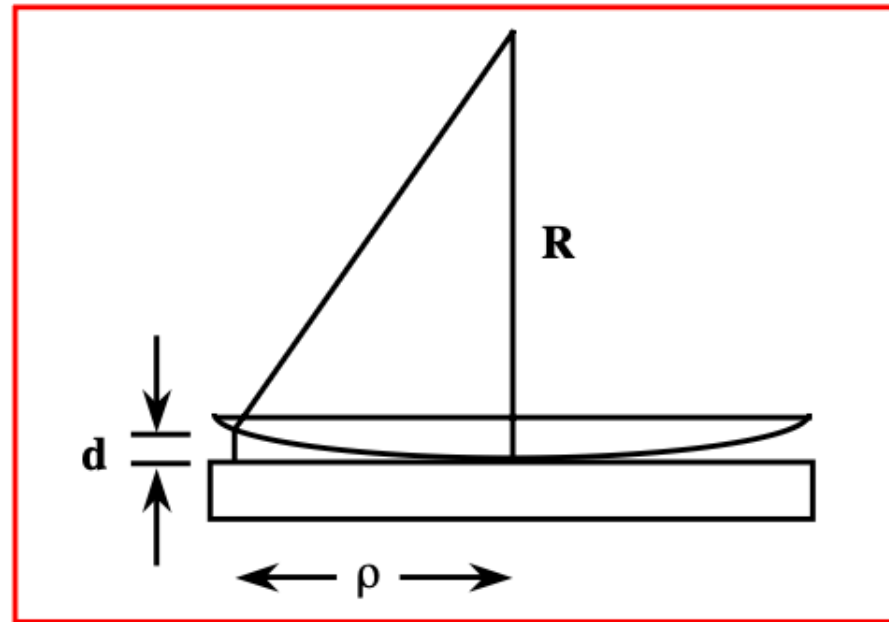


Top View



Interferogram

Newton's Rings



For m th dark fringe from center

$$d_m = m \frac{\lambda}{2}$$

$$\rho_m \approx \sqrt{m\lambda R}$$

Soap Bubbles and Oil Films

For bright fringe

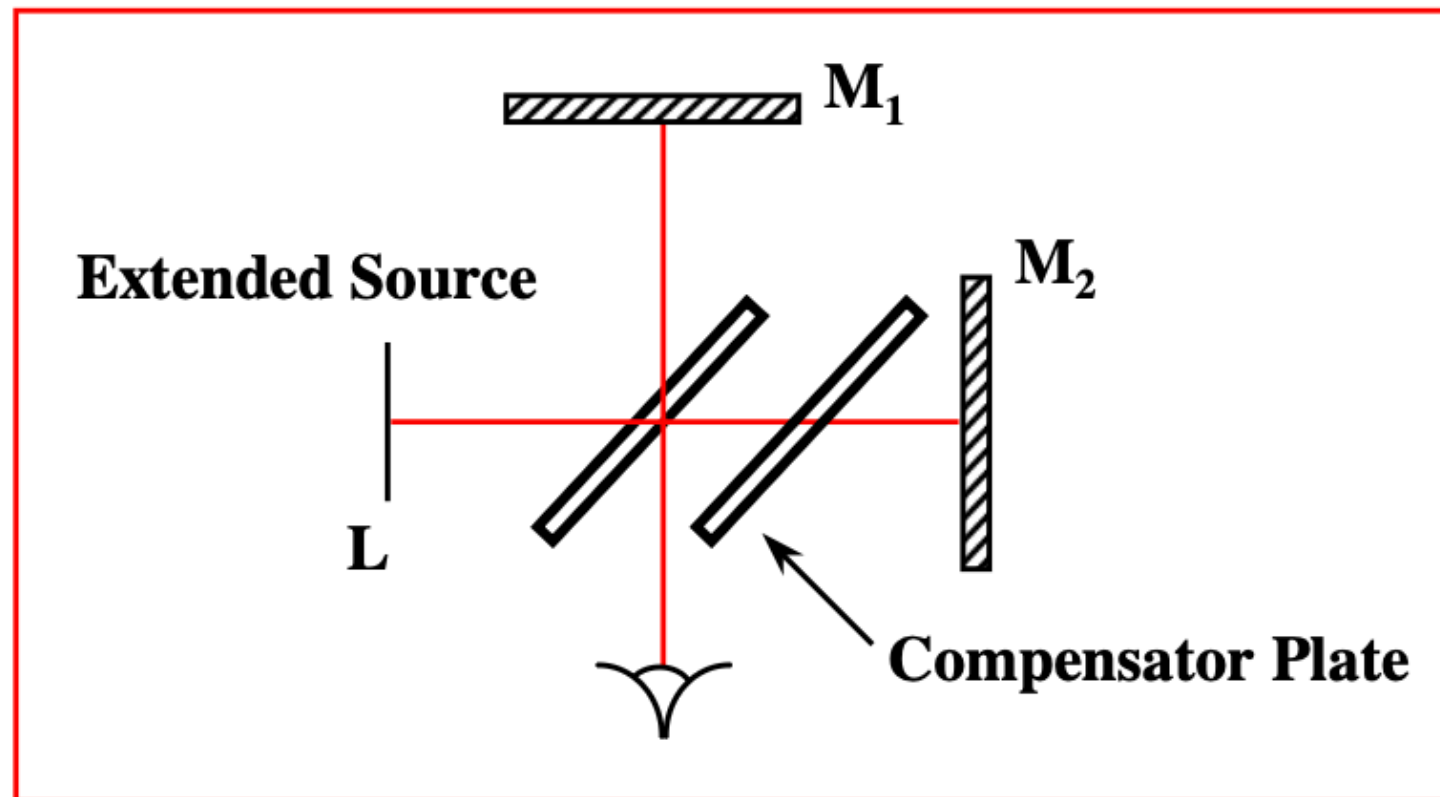
$$\frac{2\pi}{\lambda} 2nd \cos \theta + \pi = m2\pi$$

If $d \gg \lambda$, m varies greatly for change in λ .

If $d = \text{few } \lambda$, m varies slowly with λ .

**Therefore, with thin films see color fringes.
Color changes with variations in thickness and θ .**

Michelson Interferometer (1881)



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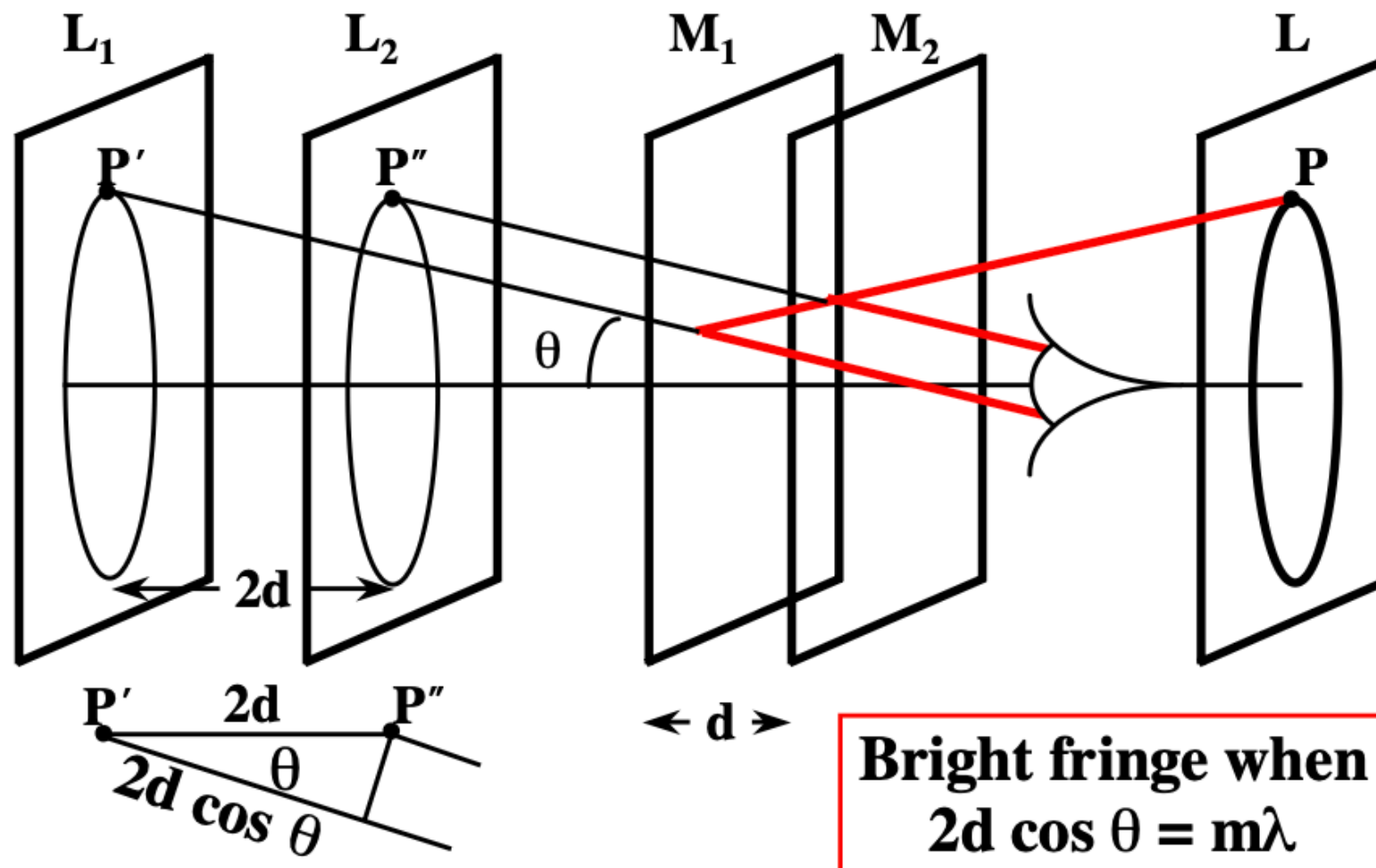
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Michelson Interferometer

(Fringes of Equal Inclination - Haidinger)



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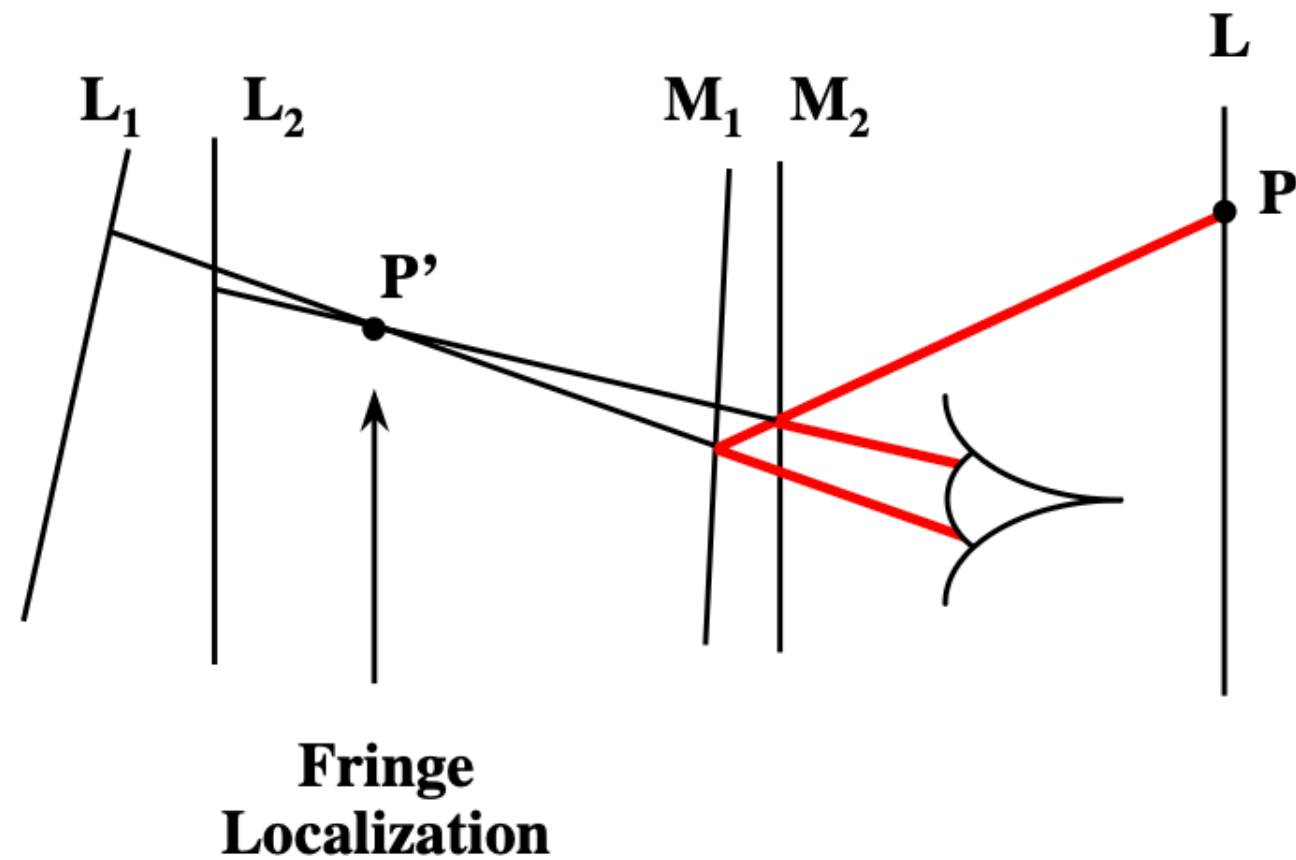
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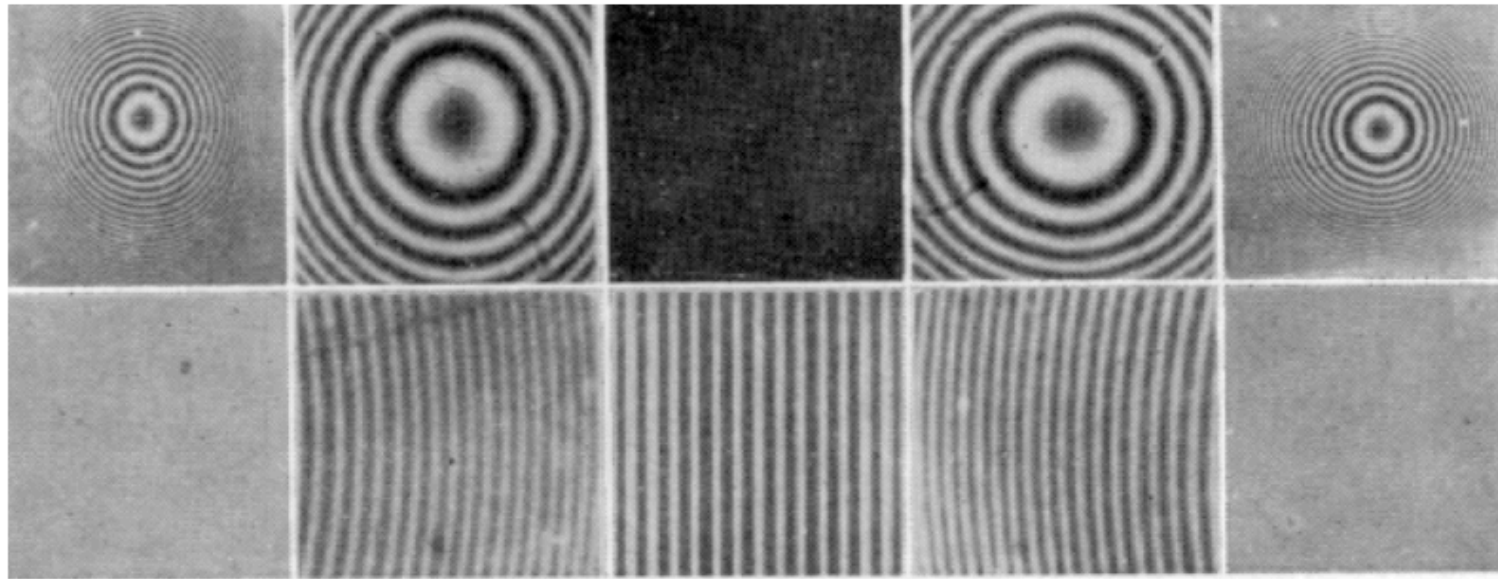
the

Michelson Interferometer

(Fringes of Equal Thickness)



Michelson Interferometer Fringes

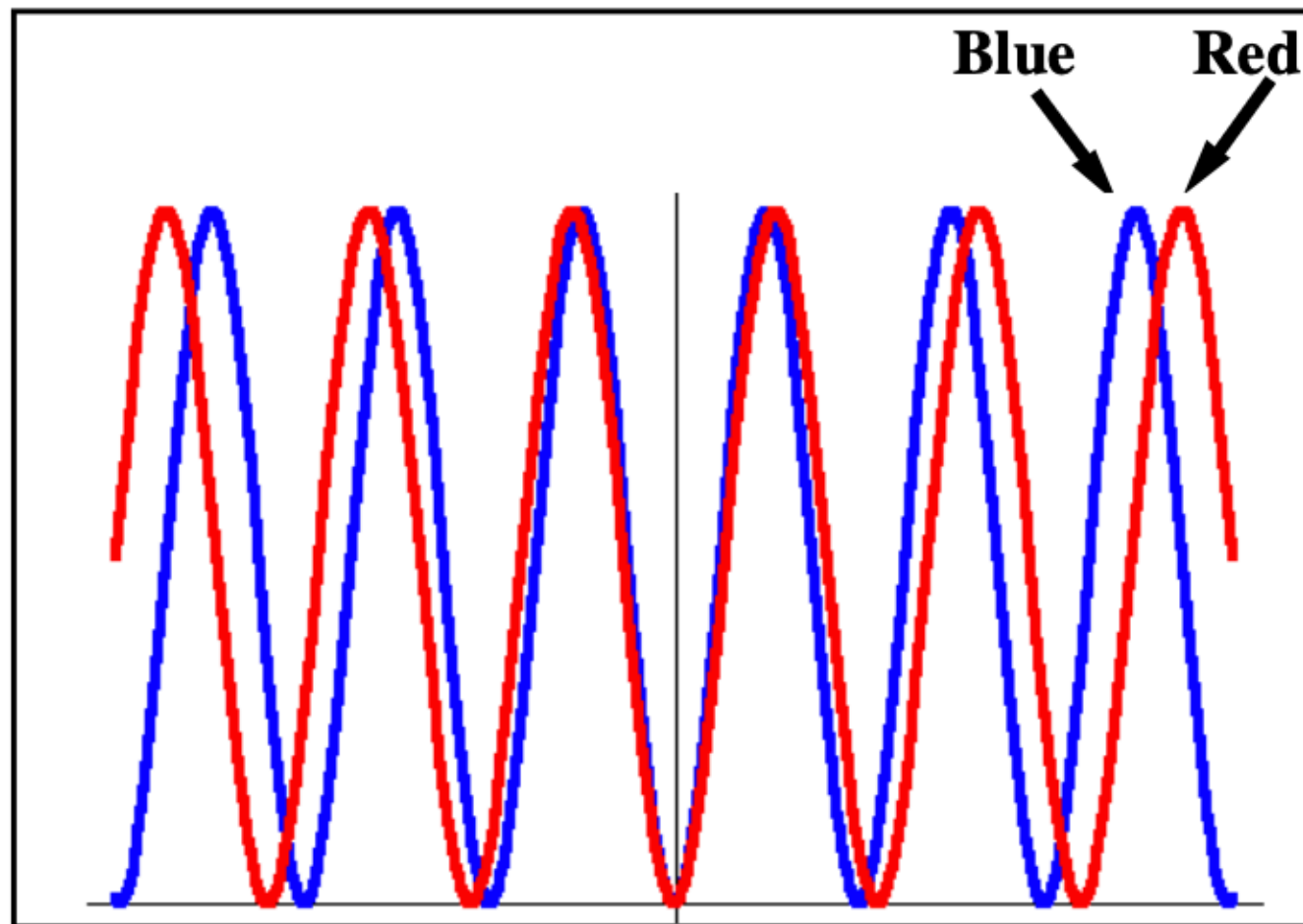


Upper row - Fringes of Equal Inclination

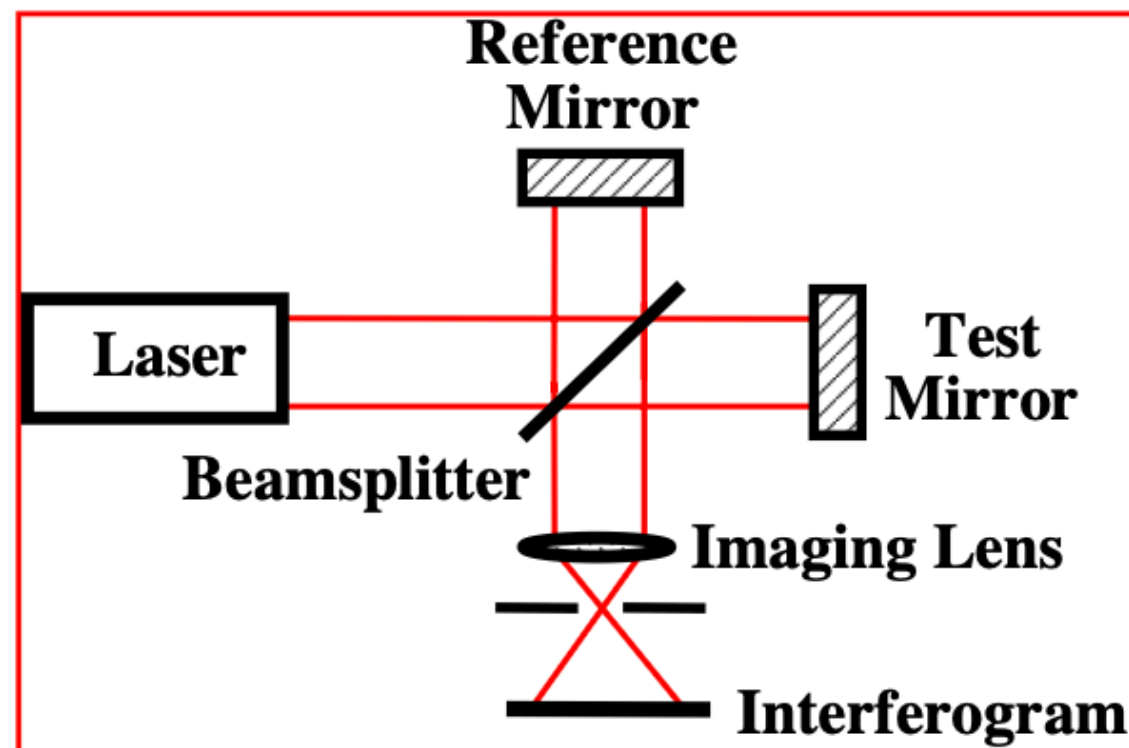
Lower row - Fringes of Equal Thickness

Path differences increases outward from the center

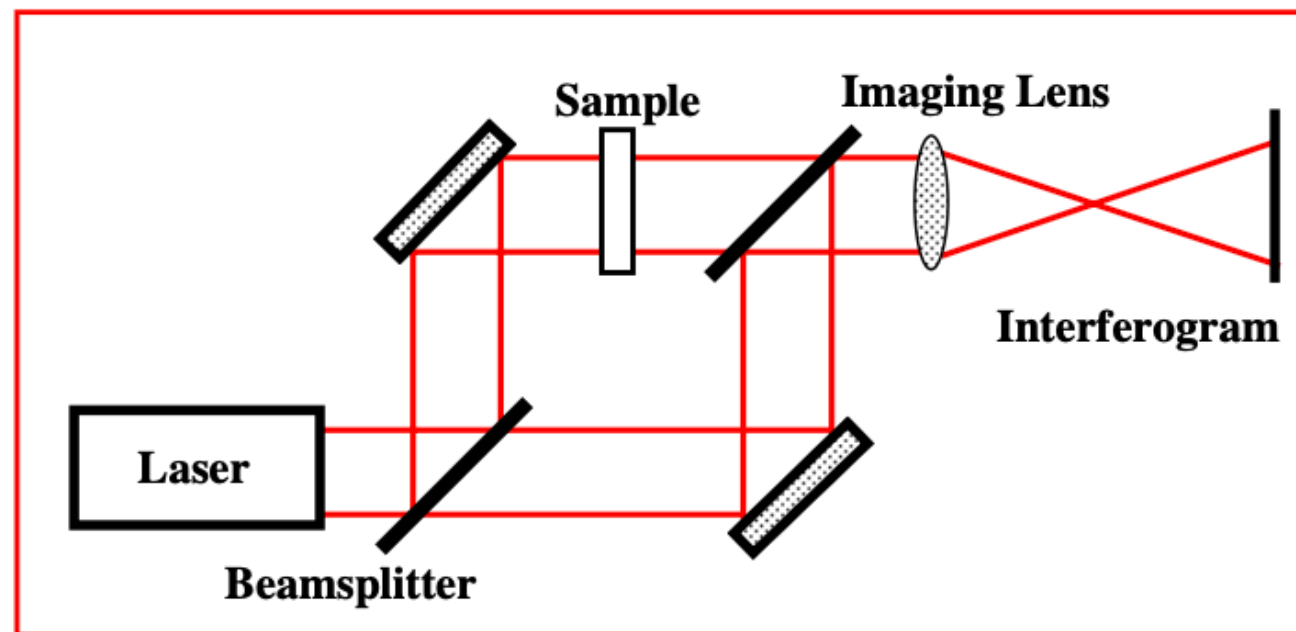
White Light Fringes



Twyman-Green Interferometer (Flat Surfaces)



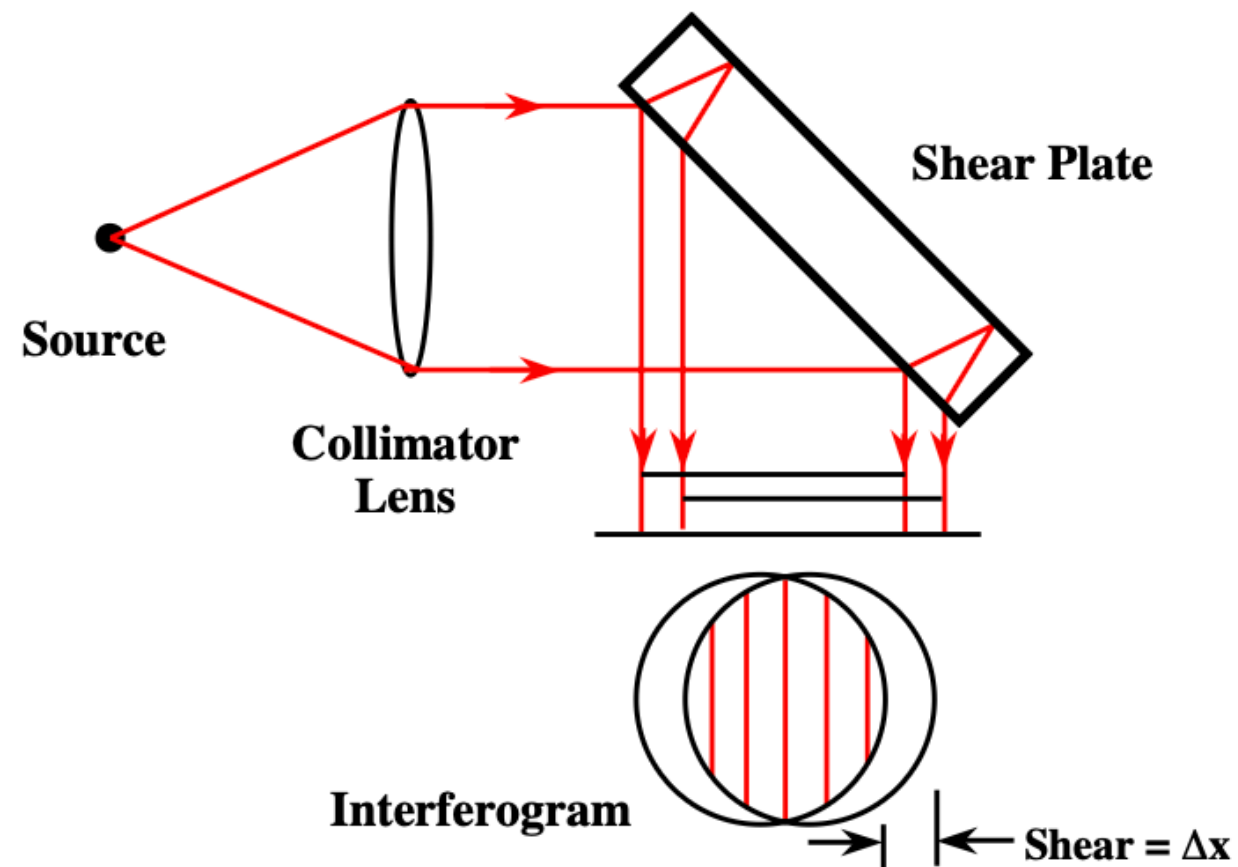
Mach-Zehnder Interferometer



Testing samples in transmission

Lateral Shear Interferometry

Measures wavefront slope



Lateral Shear Fringes

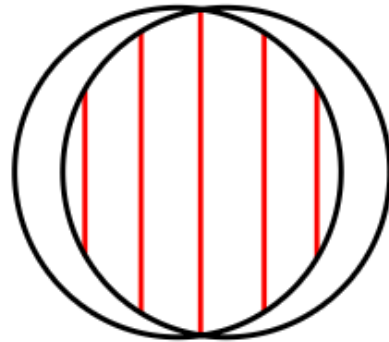
$\Delta W(x,y)$ is wavefront being measured

Bright fringe obtained when
 $\Delta W(x + \Delta x, y) - \Delta W(x, y) = m\lambda$

$$\left(\frac{\partial \Delta W(x,y)}{\partial x} \right)_{\text{Average over shear distance}} (\Delta x) = m\lambda$$

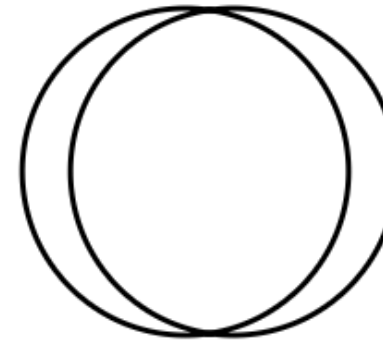
**Measures average value of slope
over shear distance**

Collimation Measurement

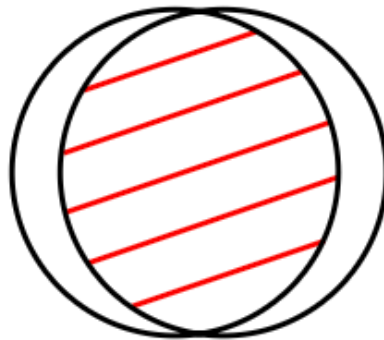


Not collimated

No wedge in shear plate

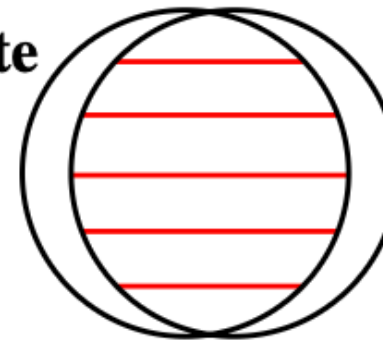


Collimated (one fringe)



Not collimated

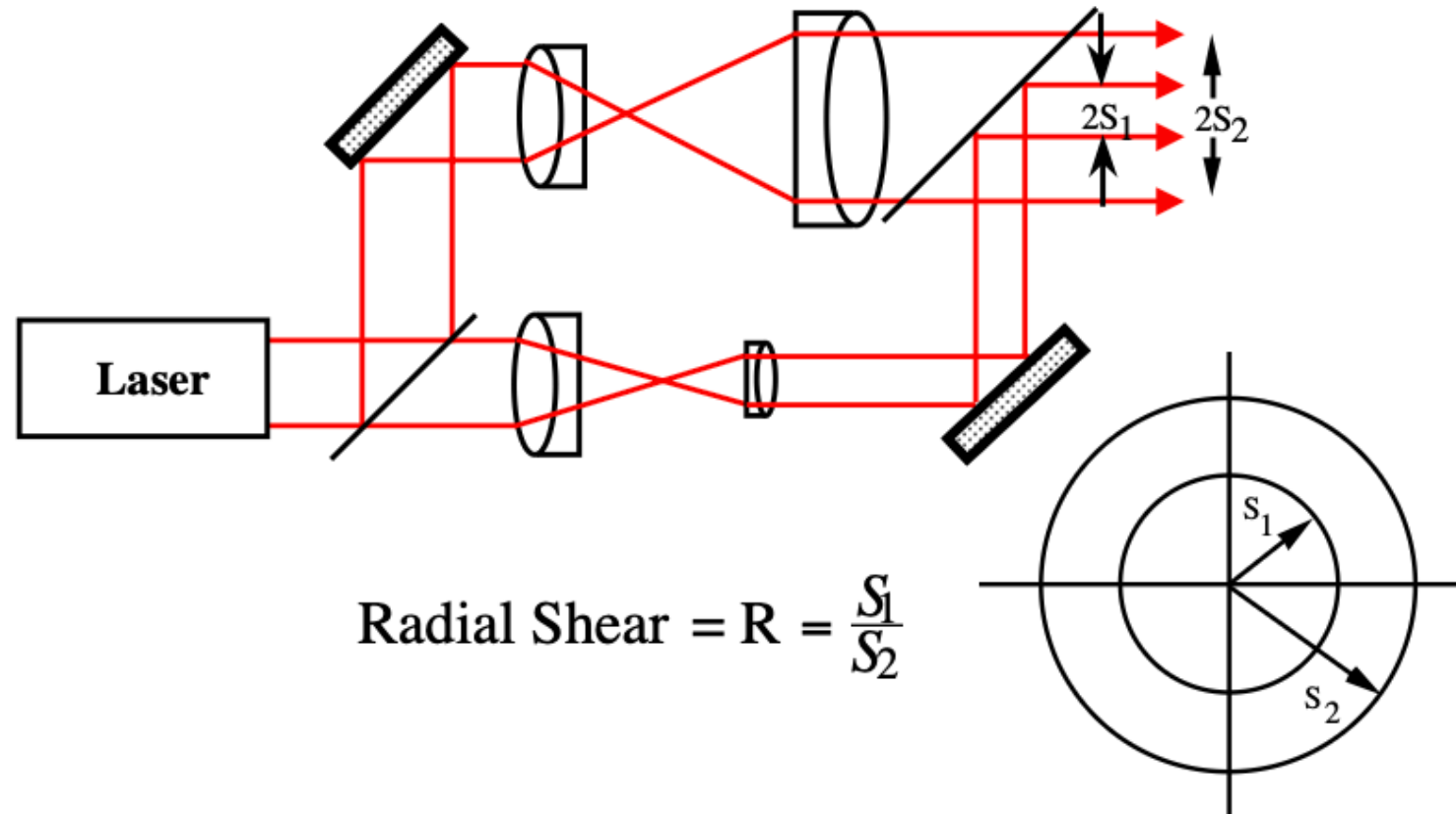
Vertical wedge in shear plate



Collimated

Radial Shear Interferometry

Wavefront is interfered with expanded version of itself



Analysis of Radial Shear Interferograms

Wavefront being measured

$$\Delta W(\rho, \theta) = W_{020}\rho^2 + W_{040}\rho^4 + W_{131}\rho^3 \cos \theta + W_{222}\rho^2 \cos^2 \theta$$

Expanded beam can be written

$$\Delta W(R\rho, \theta) = W_{020}(R\rho)^2 + W_{040}(R\rho)^4 + W_{131}(R\rho)^3 \cos \theta + W_{222}(R\rho)^2 \cos^2 \theta$$

Hence, a bright fringe is obtained whenever

$$\Delta W(\rho, \theta) - \Delta W(R\rho, \theta) = W_{020}\rho^2(1 - R^2) + W_{040}\rho^4(1 - R^4) + W_{131}\rho^3(1 - R^3) \cos \theta + W_{222}\rho^2(1 - R^2) \cos^2 \theta$$

Same as Twyman-Green if divide each coefficient by $(1 - R^n)$

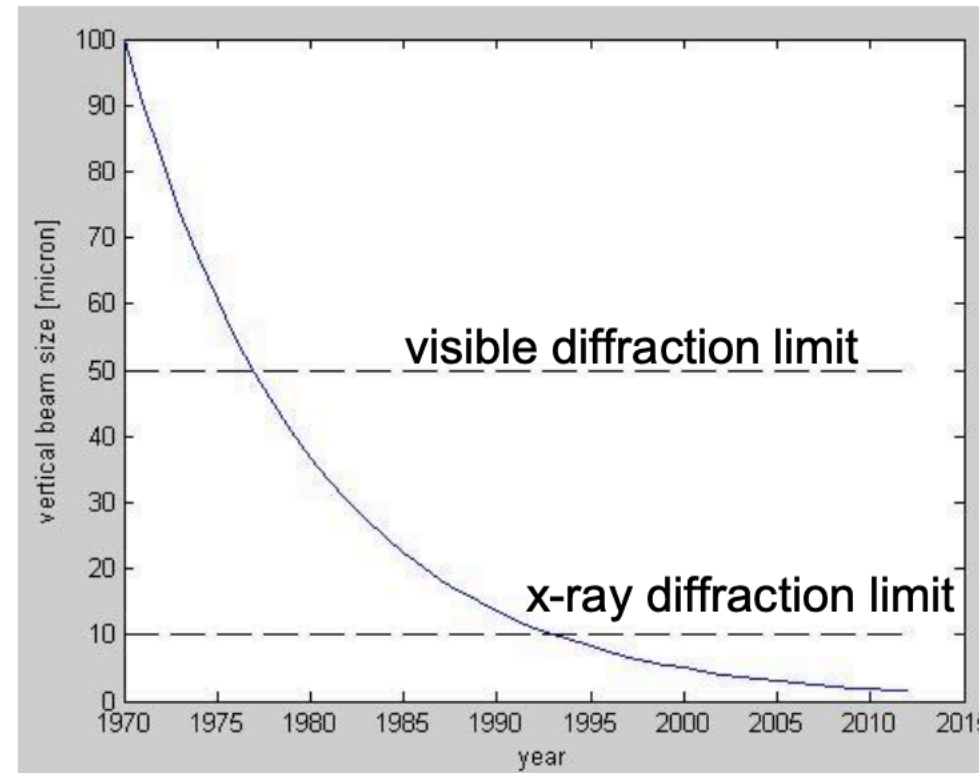
Radial Shear Interferogram

- **Variable Sensitivity Test**
- **Large shear - results same as for Twyman-Green**
- **Small shear - Low sensitivity test**

Pass #5: Michelson's Interferometer - Theory and Practice

- Motivation
- Two-slit Interference – Young's experiment
- Diffraction from a single slit - review
- Extended Source – Partial Coherence
- The Mutual Coherence function
- Van-Cittert/Ziernike theorem
- Stellar Interferometers for SR applications

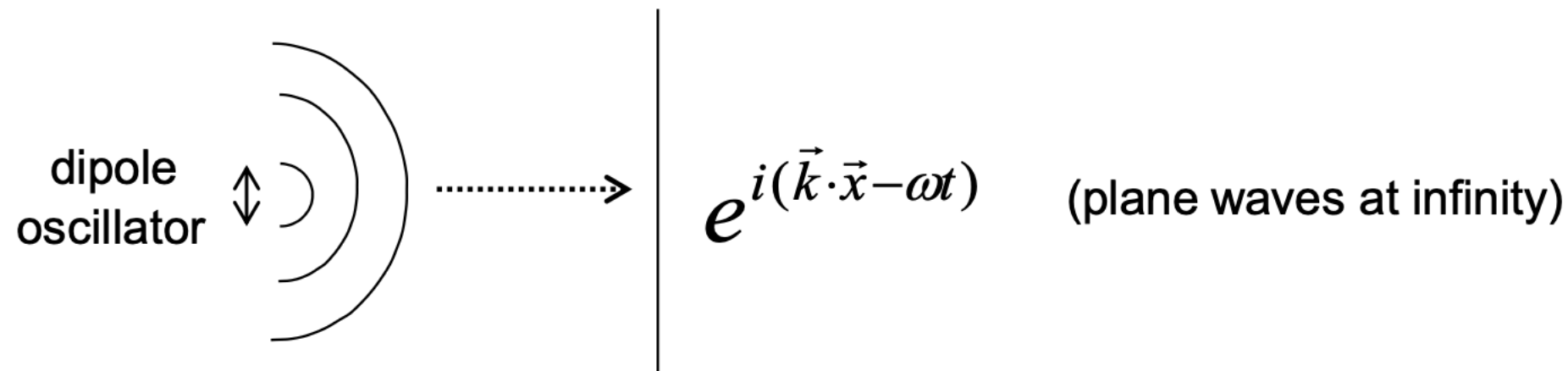
- Electron beam size can be very small



$$\sqrt{\epsilon\beta} = \sqrt{(1)(1 \times 10^{-9} / 1000)} = 1 \mu m!$$

- Need to measure beam size for optics verification, machine monitoring and operation
- Conventional imaging diffraction limited
 - $\sigma_{\text{res}} \sim 50 \mu m$ visible
 - $\sigma_{\text{res}} \sim 10 \mu m$ x-ray pinhole
- What else can be used?

- Take advantage of light *coherence* properties



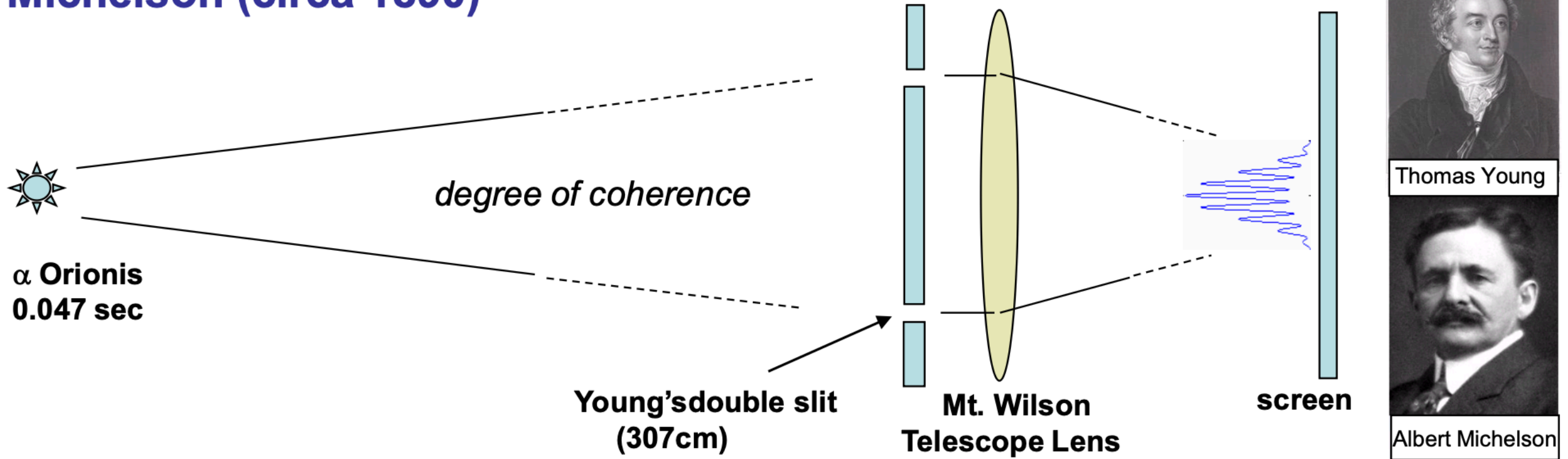
- For a *distributed* source at finite distance the light is only *partially coherent*

A diagram showing a bundle of several lines that converge to a point on the left and then diverge to the right, representing a distributed source. To the right of this diagram is the vector equation $\vec{k} = k_x \hat{x} + k_z \hat{z}$, where the term $k_x \hat{x}$ is enclosed in a dashed oval.

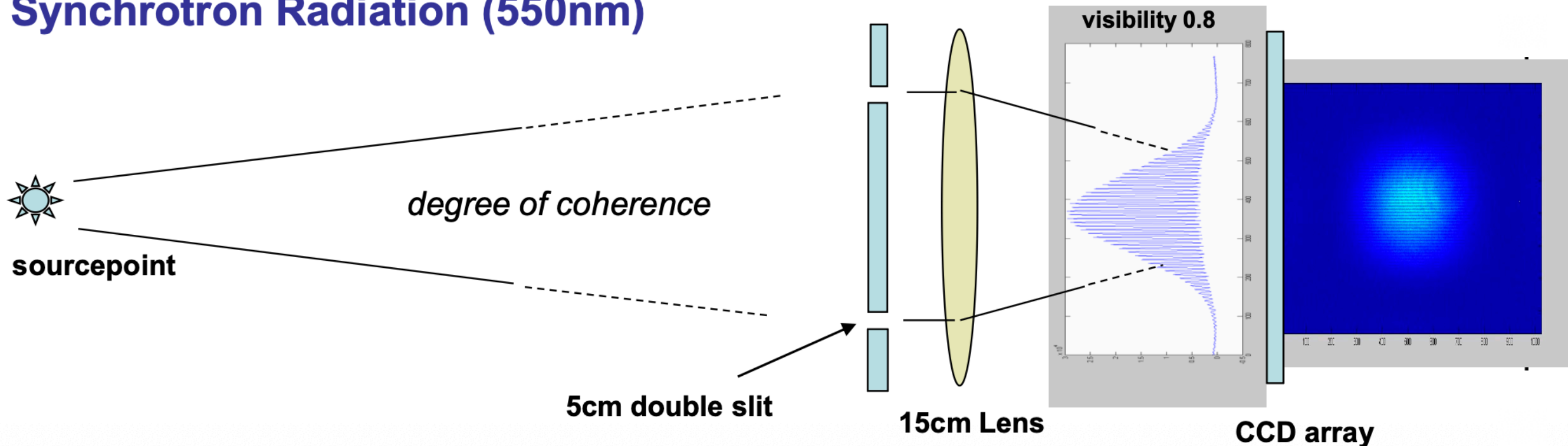
- Interferometry enters world of wavefront physics and statistical optics

Interferometric Beam Size Measurement

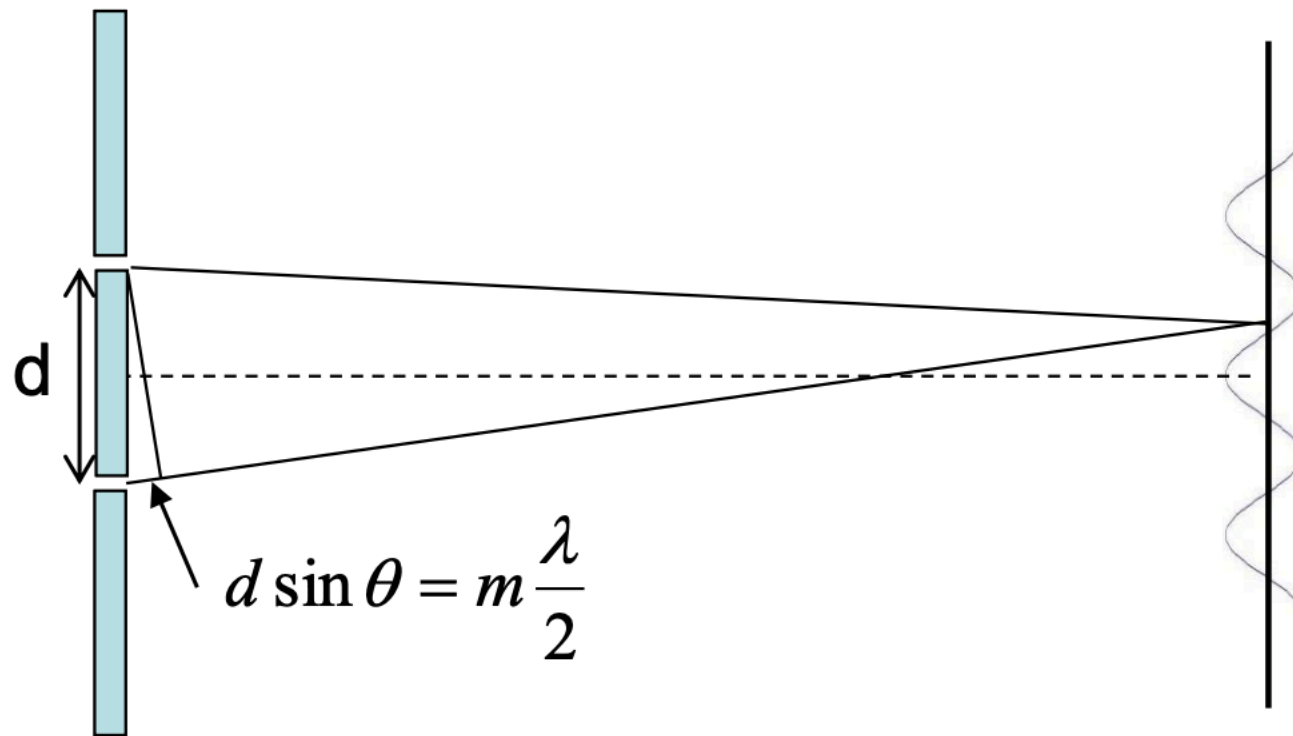
Michelson (circa 1890)



Synchrotron Radiation (550nm)



Two-Slit Interference: Young's Experiment



Field Pattern

$$E = E_o e^{i(\Phi)} + E_o e^{i(-\Phi)} = 2E_o \cos(\Phi)$$

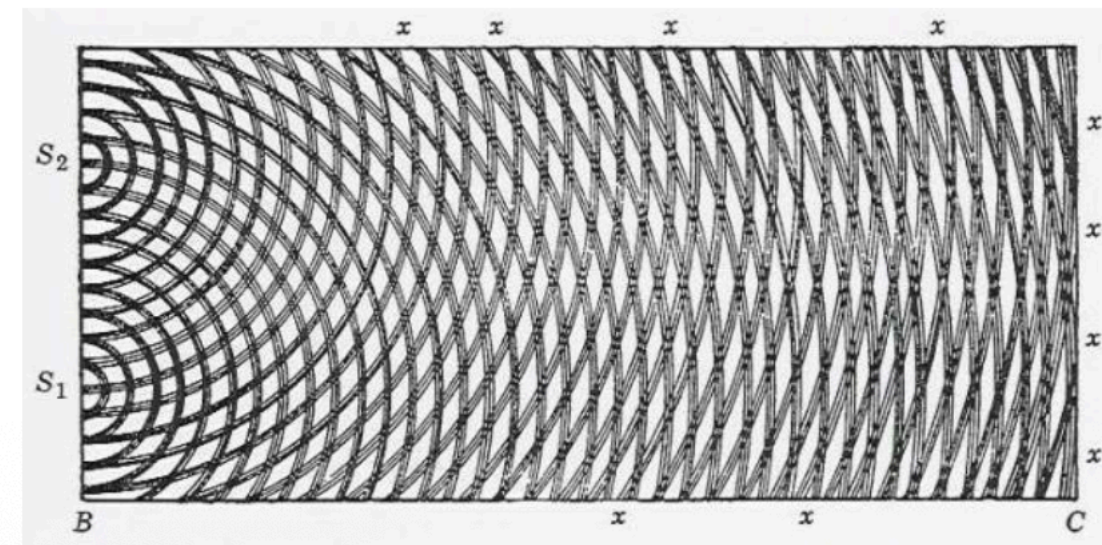
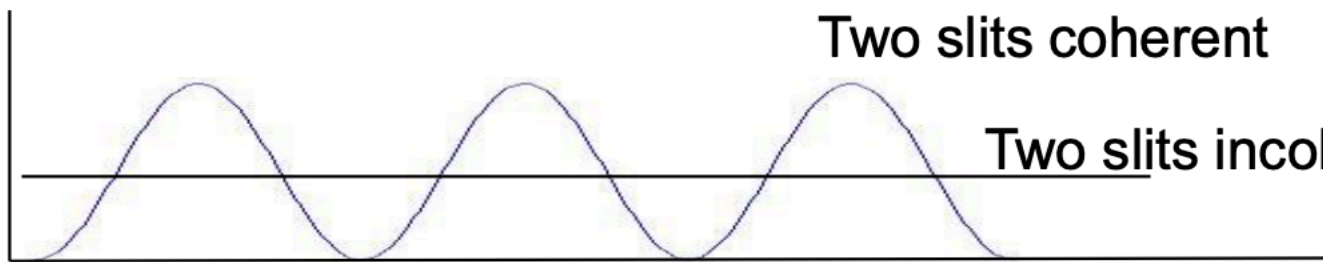
$$\int_{-\infty}^{\infty} (\delta(\Phi) + \delta(-\Phi)) e^{ikx} dx$$

monochromatic light!

Intensity Pattern: $\cos^2(\Phi)$

Two slits coherent

Two slits incoherent



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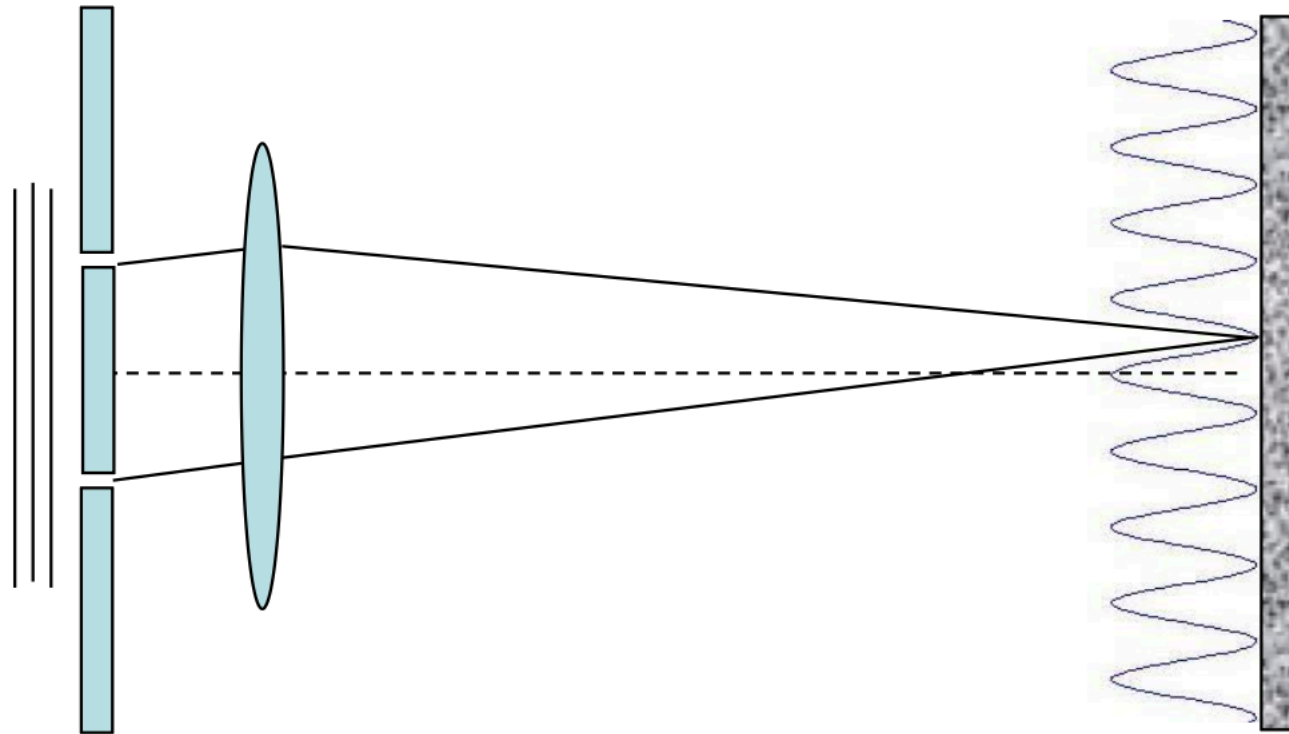
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Two-Slit Interference: Young's Experiment

Use a lens to concentrate image on screen



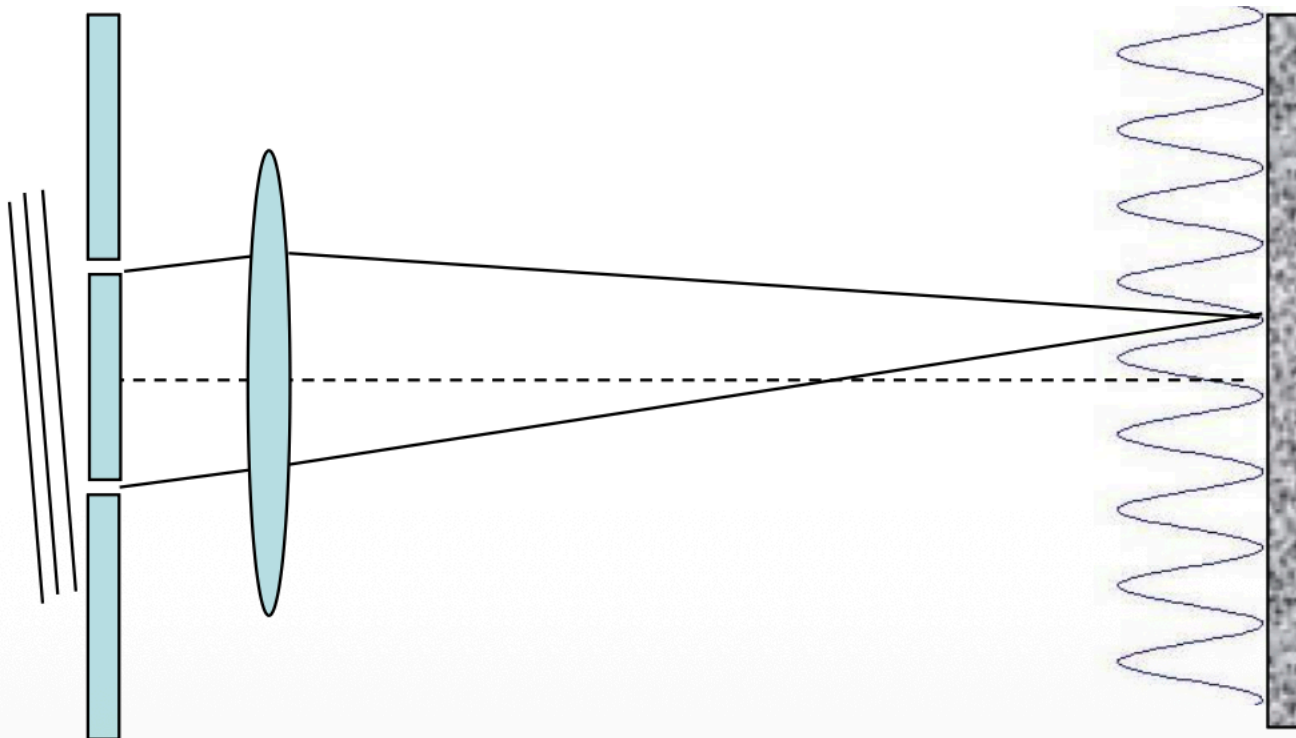
Intensity Pattern

Note Contrast/Visibility

$$V = \frac{I_{Max} - I_{Min}}{I_{Max} + I_{Min}} = 1.0$$

fully coherent!

Change phase/incidence angle, shift pattern phase



$V=1.0$ (no change)

Fundamentals of astronomical optical Interferometry

Interferometry and angular resolution

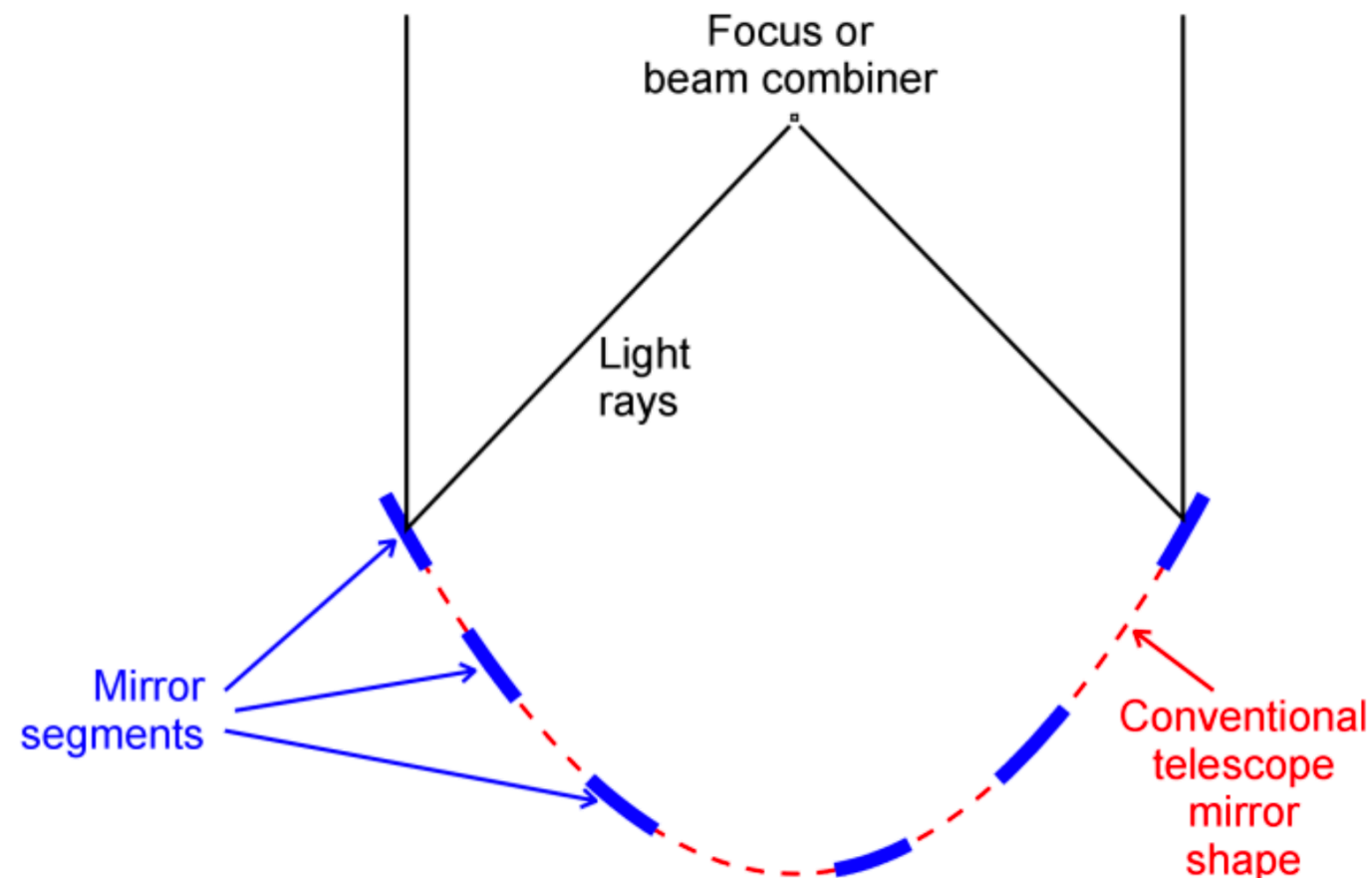
Diffraction limit of a telescope : λ/D (D: telescope diameter)

Diffraction limit of an interferometer: λ/B (B: baseline)

Baseline B can be much larger than telescope diameter D:

largest telescopes : $D \sim 10\text{m}$

largest baselines (optical/nearIR interferometers): $B \sim 100\text{m} \text{ } 500\text{m}$



the
the
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Telescope vs. interferometer diffraction limit

For circular aperture without obstruction : Airy pattern

First dark ring is at $\sim 1.22 \lambda/D$

Full width at half maximum $\sim 1 \lambda/D$

The “Diffraction limit” term = $1 \lambda/D$

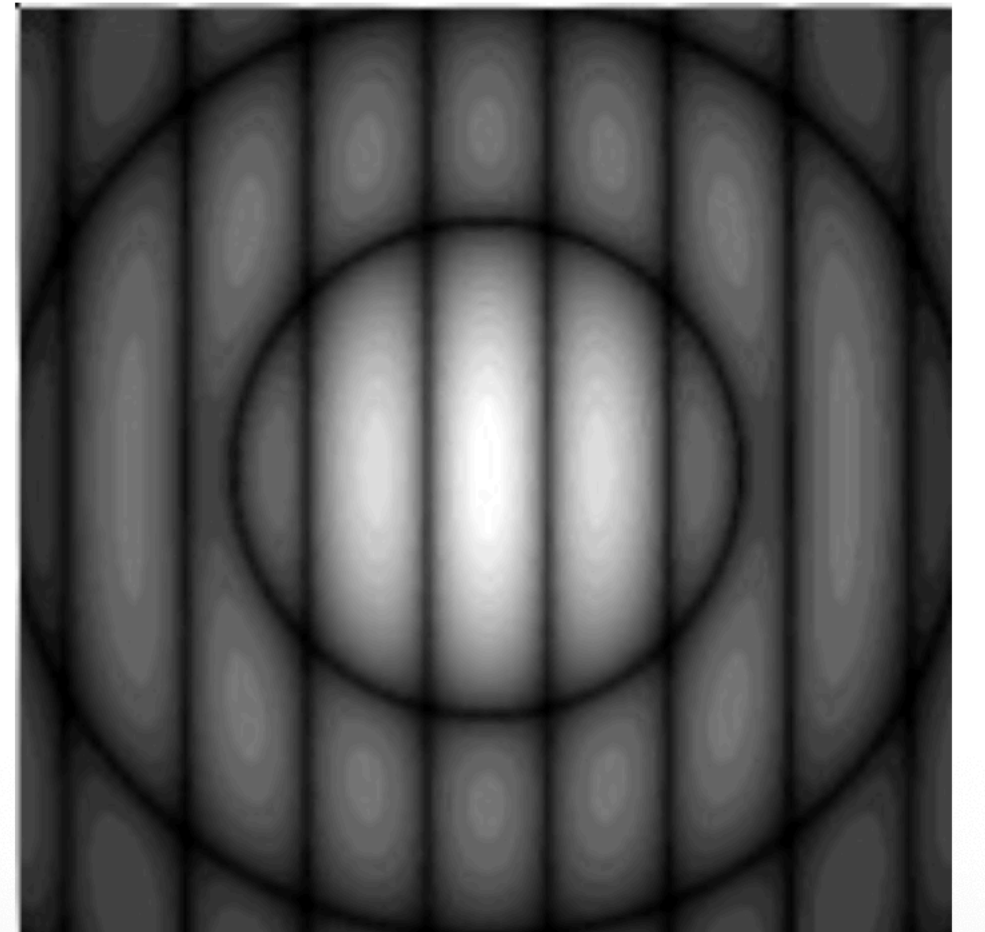
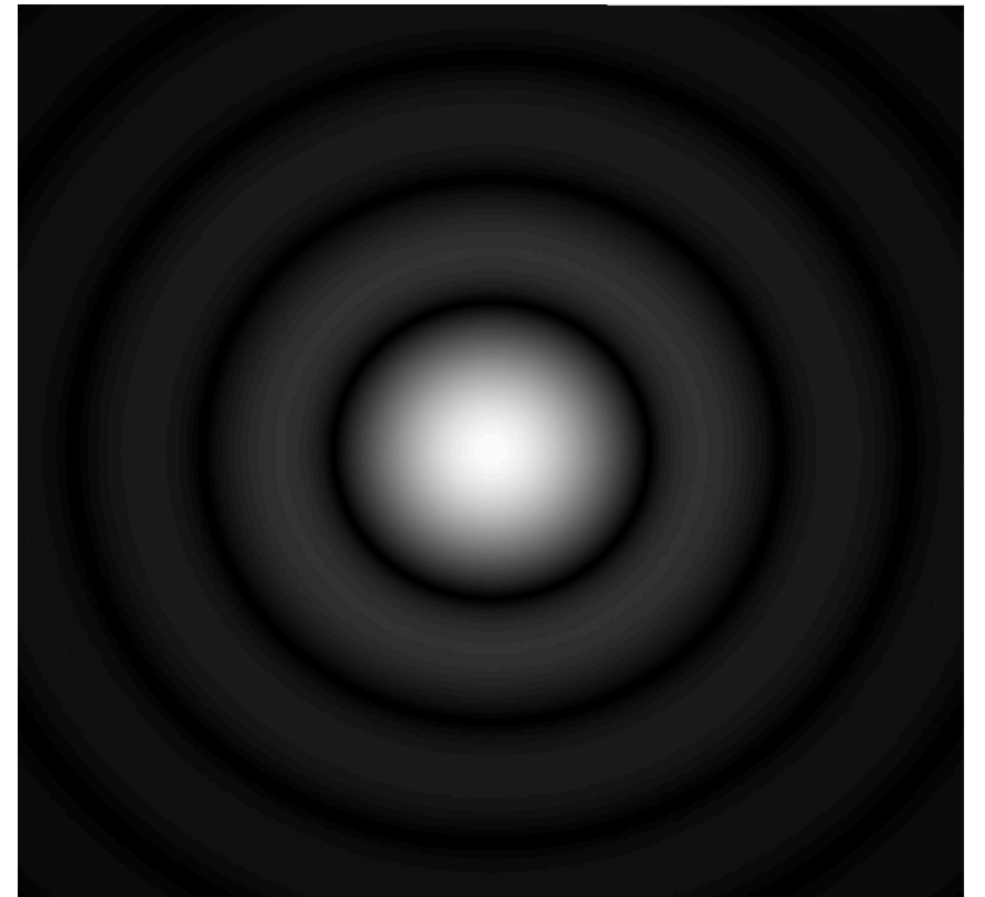
$D=10\text{m}$, $\lambda=2 \mu\text{m} \rightarrow \lambda/D = 0.040 \text{ arcsec}$

This is the size of largest star

With interferometer, $D \sim 400\text{m} \rightarrow \lambda/B = 0.001 \text{ arcsec} = 1 \text{ mas}$

This is diameter of Sun at 10pc

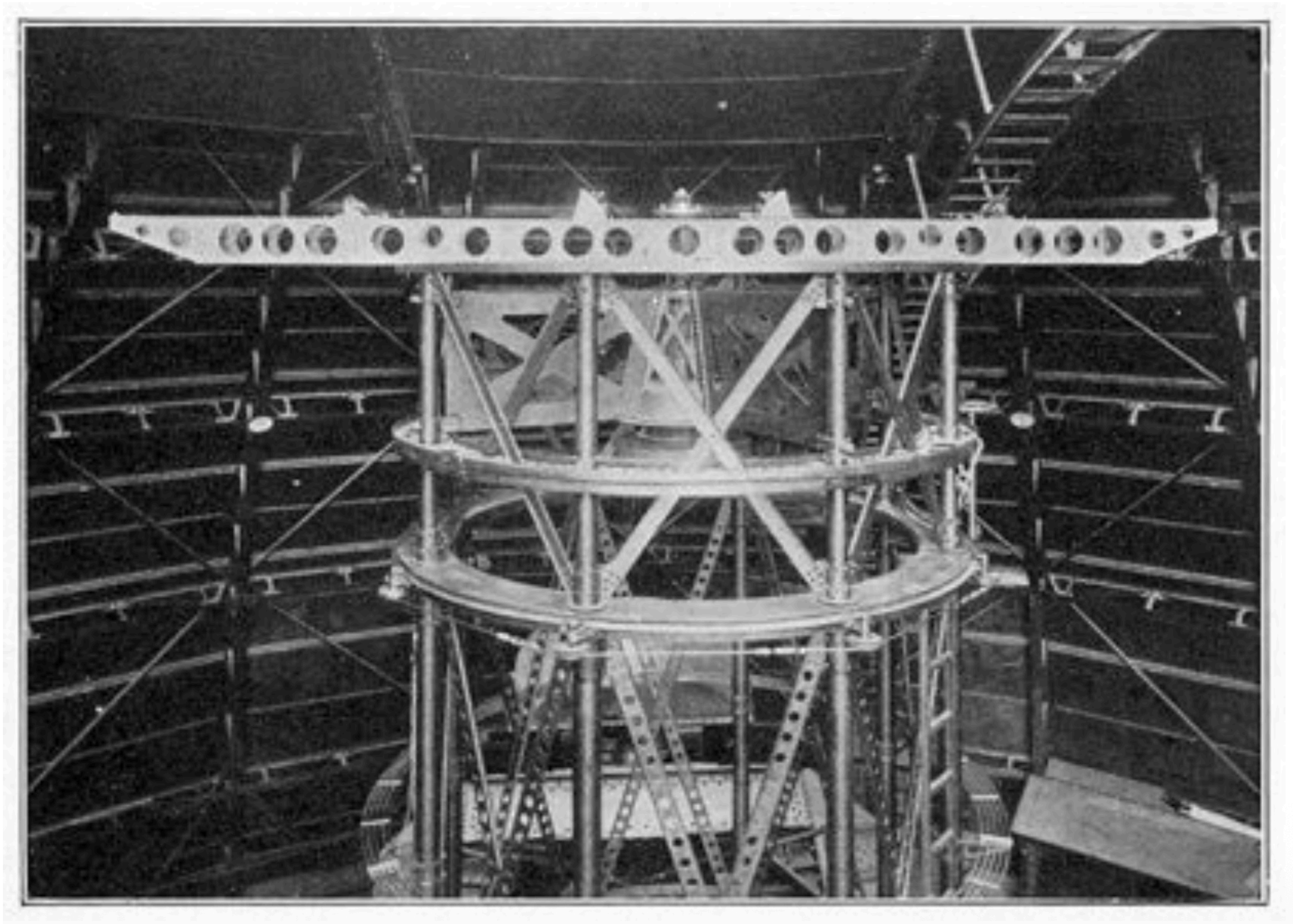
Many stars larger than 1mas



Michelson interferometer (1920)

Diffraction limit of a telescope : λ/D (D: telescope diameter)

Diffraction limit of an interferometer: λ/B (B: baseline)



2-telescope interferometer: Fringe visibility and phase

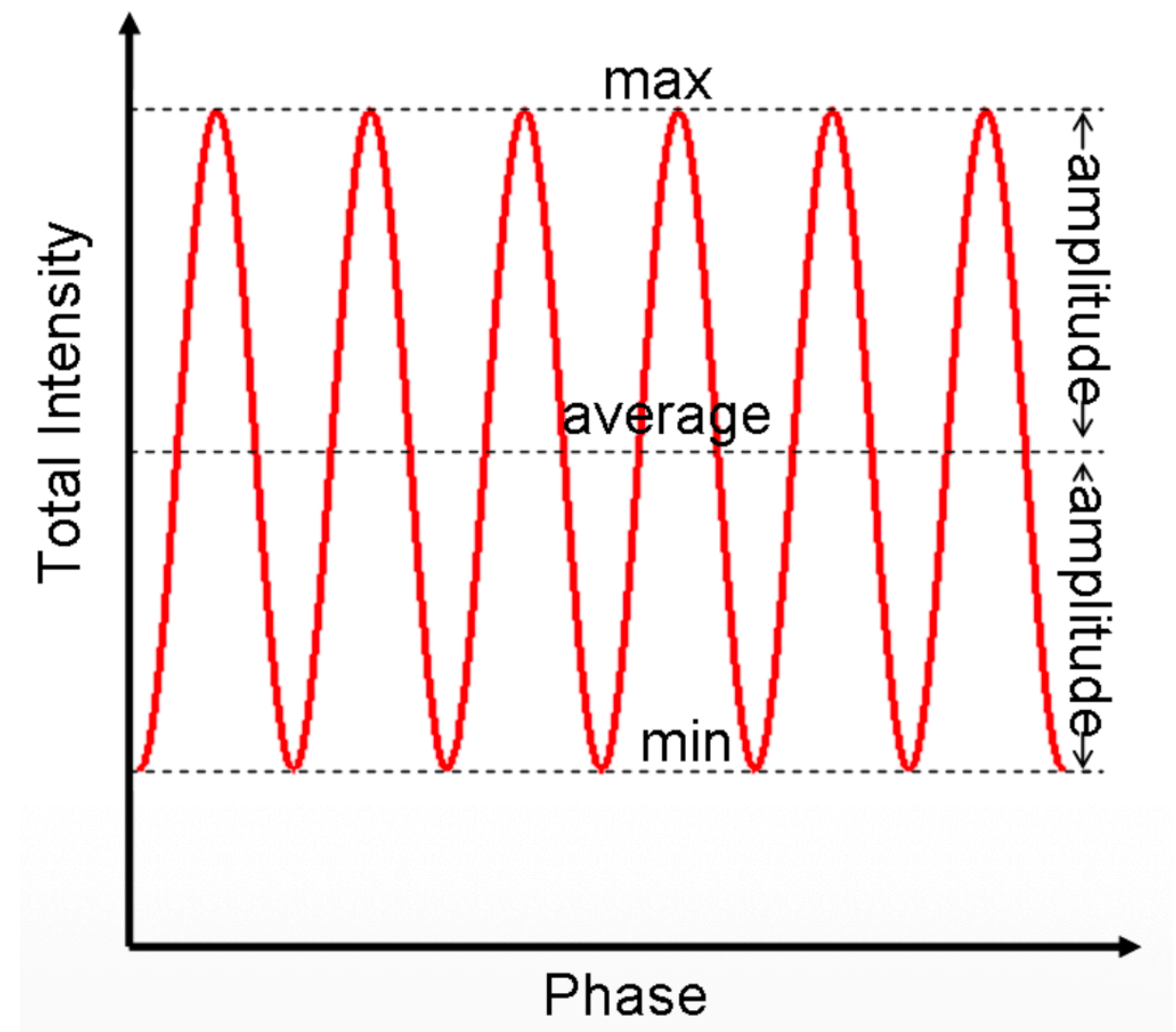
With a single baseline interferometer, the only information measured are:

- fringe average intensity $\rightarrow$ source brightness
- fringe visibility = amplitude / average $\rightarrow$ single constraint on spatial distribution of light, visibility = 1 if source is unresolved (BUT visibility = 1 does not imply that the source is unresolved)
- fringe phase $\rightarrow$ single constraint on spatial distribution of light (BUT needs to be referenced)

Fringe frequency DOES NOT contain information about object, as it is only a function of the baseline

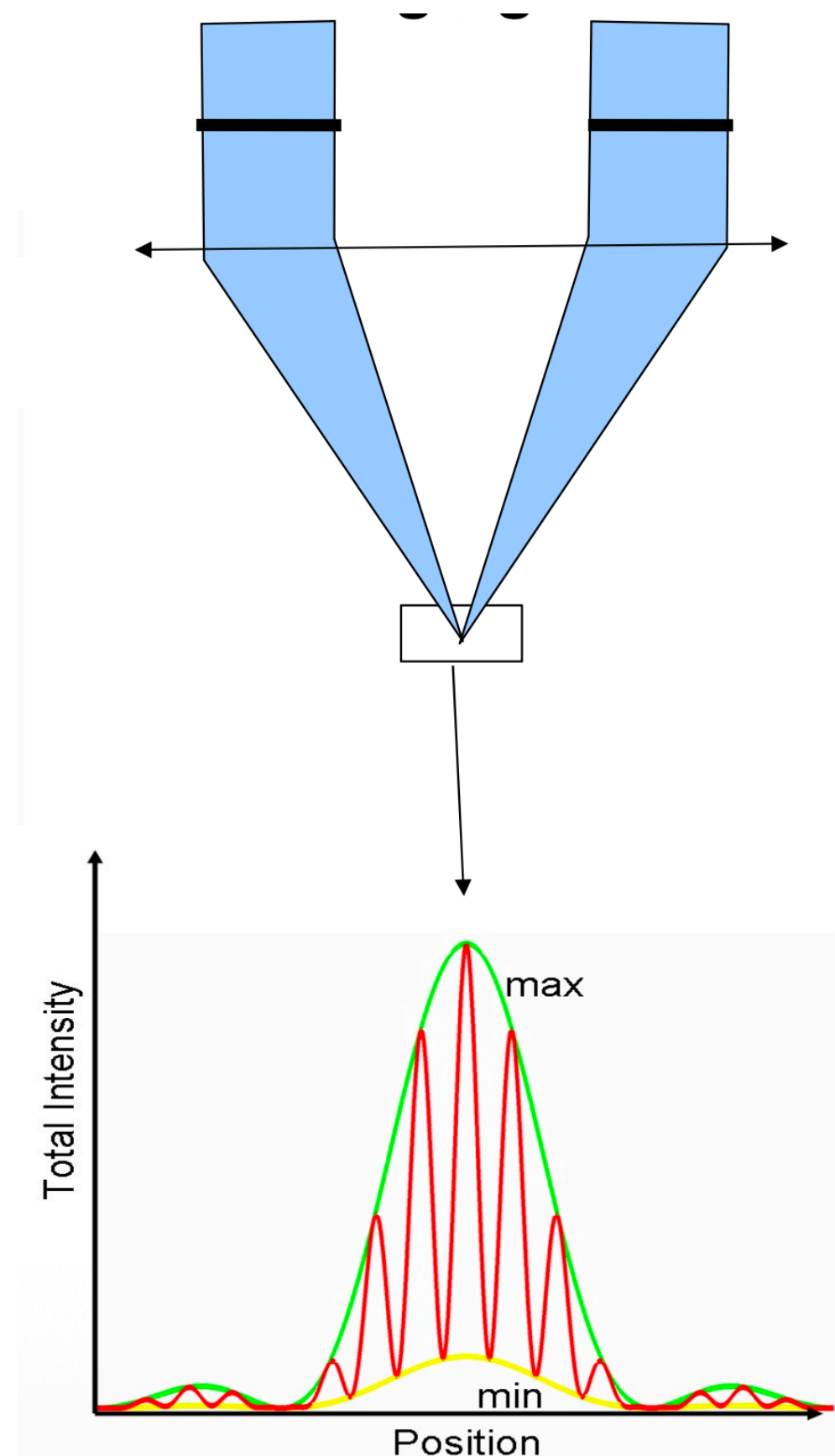
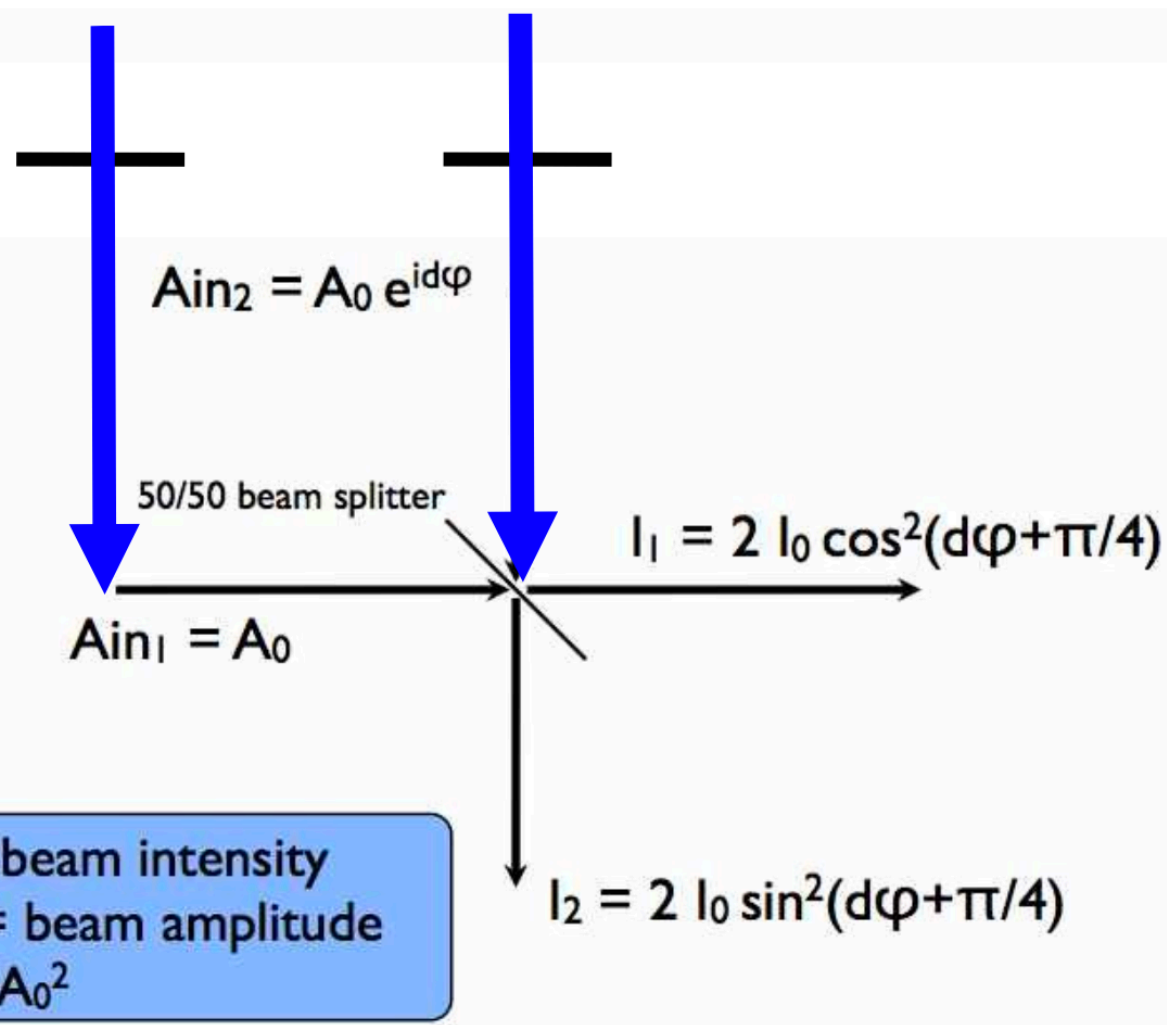
Interferometers measure at most these 3 quantities for a single baseline

This can be done by imaging fringes, or by measuring intensity after combination with a known phase offset



2-telescope interferometer:

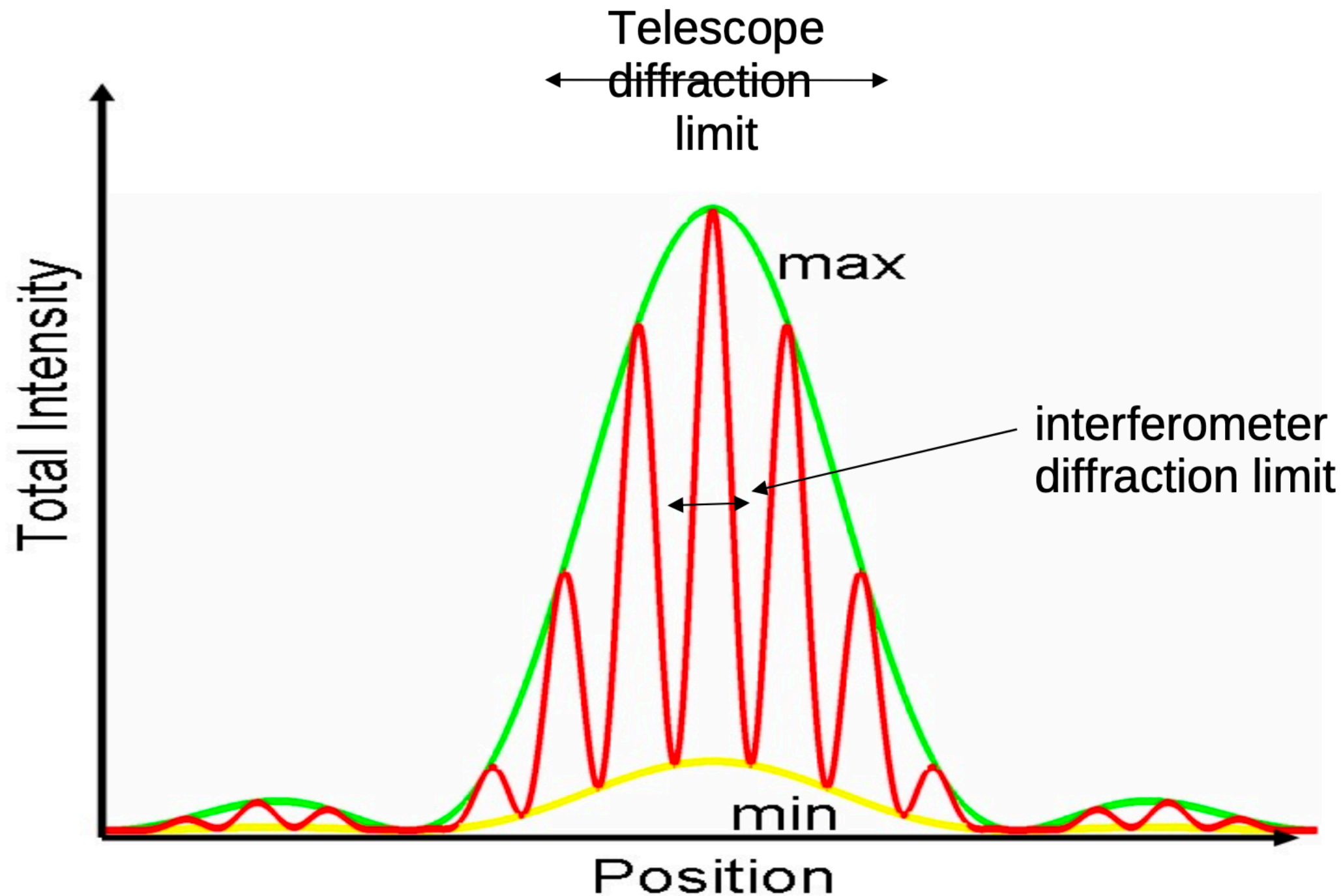
Fringe visibility and phase can be measured either by discrete flux measurements (beam splitter(s)) or direct imaging of fringes



Note:

- at least 3 measurements required to constrain fringe visibility (1), phase (1) and intensity (1)
- fringe scanning can be used to temporally sample fringes

Interferometer field of view



If two sources are separated by more than the diffraction limit of the individual telescopes, they will not interfere in the interferometer:

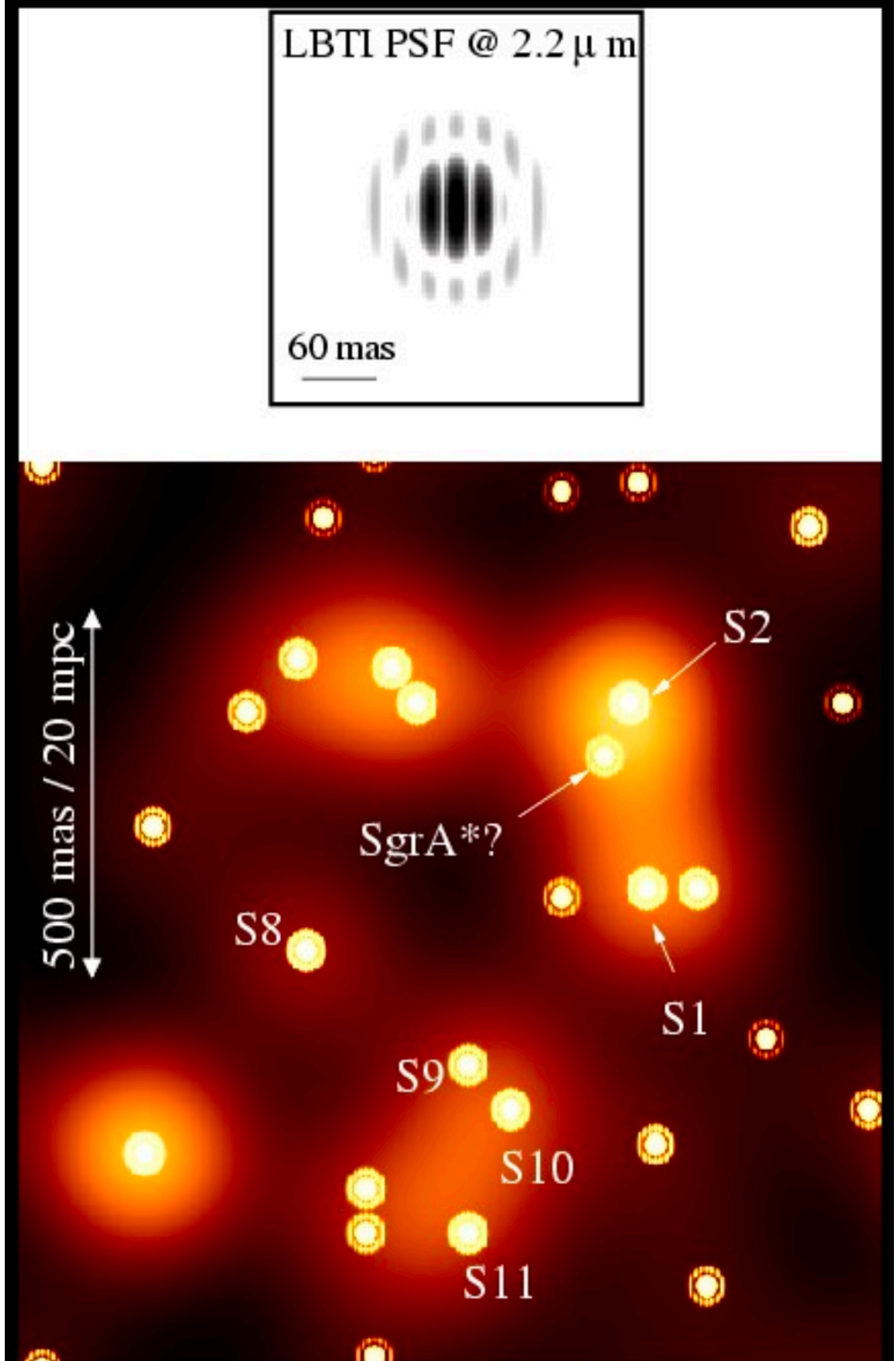
interferometric field of view $\sim$ diffraction limit of a single aperture

Interferometer field of view

BUT: each telescope can have large field of view, and interferometer can create multiple fringe packets (1 per source)

Spatial information at the scale
<
telescope diffraction limit comes from
interferometer

Spatial information at the scale
> telescope diffraction limit comes from
telescopes



Van Cittert-Zernike theorem

Assumes object of intensity $I(l,m)$, spatially incoherent

- l,m : angular coordinates on sky
- spatially incoherent: light different points of the objects cannot interfere
(this is almost always true in astronomical observations)

Theorem expresses mutual coherence (= fringe visibility and phase) between 2 points on a plane perpendicular to line of sight, separated by (u,v)

- u,v : interferometer baseline

$$\Gamma_{12}(u, v, 0) = \iint I(l, m) e^{-2\pi i(ul+vm)} dl dm$$

→ theorem shows that a single baseline of the interferometer measures a single point in the Fourier transform of the object

Intuitively, this makes sense:

Point source (l,m) observed by a single baseline (u,v) :

- Modulo of Fourier transform is = 1 everywhere in (u,v) plane : fringe visibility is always = 1
 - Phase is $(ul+vm)$, proportional to both source position and baseline
- the equation makes sense for a single point source

Spatial incoherence leads to additivity of fringes

Van Cittert-Zernike theorem

$$\Gamma_{12}(u, v, 0) = \iint I(l, m) e^{-2\pi i(ul + vm)} dl dm$$

Physical interpretation:

Each point on the object creates a fringe with visibility = 1, average level = point brightness, and phase = position of the point along line perp. to baseline

Measured fringe is incoherent sum of all individual fringes (double integral in theorem)

Small source, small baseline, large lambda = high visibility

Large source, large baseline, small lambda = low visibility

With a single baseline, single lambda, only one constraint is measured on the object.

Stellar interferometry is very successful because object is simple, and can be parametrized in a few parameters

single measurement can constrain stellar diameter with simple model (perfect disk)

Complicated objects which cannot be described with a few parameters require many baselines

- multiple telescopes (>2)

- use of Earth's rotation to change baseline

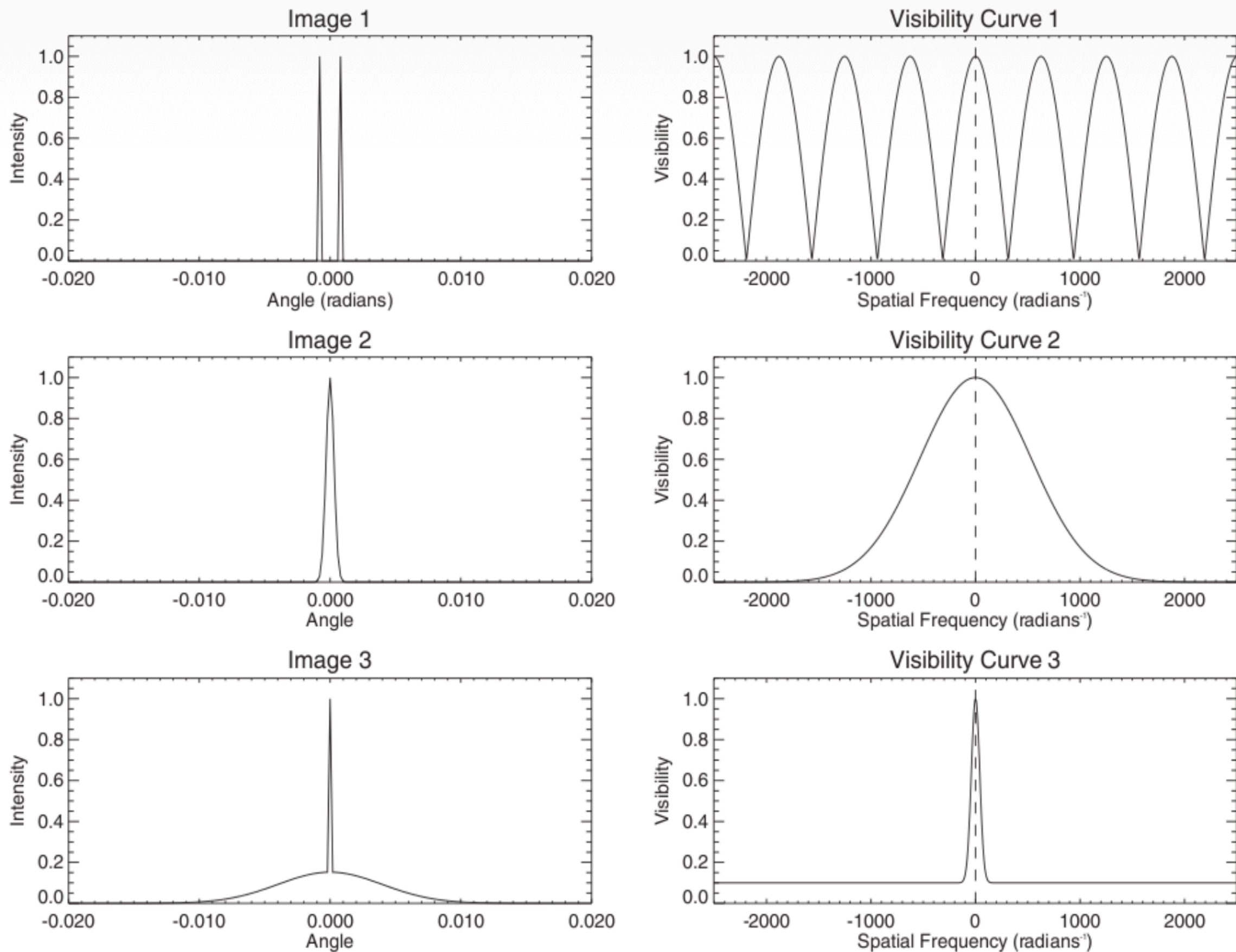


Figure 1 This figure shows simple one-dimensional images and their corresponding visibility curves. The left panels are the images while the right panels correspond to the Fourier amplitudes, i.e. the visibility amplitudes. Note that ‘large’ structure in image-space result in ‘small’ structure in visibility-space.

Stellar Interferometry

Van Cittert-Zernike gives the fringe Visibility as a function of baseline, λ , and stellar apparent size.

$$\Gamma_{12}(u, v, 0) = \iint I(l, m) e^{-2\pi i(ul+vm)} dl dm \longrightarrow V^2(B\theta/\lambda) = \left(2 \frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda} \right)^2$$

(J1 = Bessel function of 1st kind)

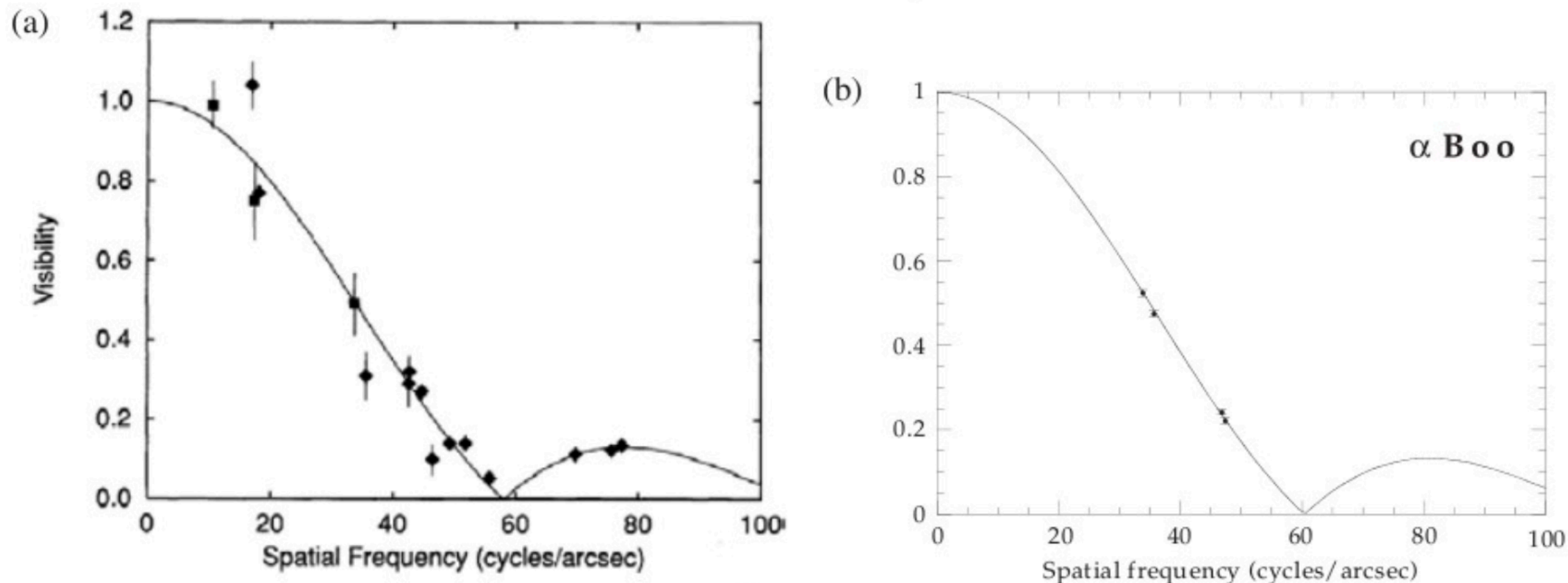


Figure (a) This figure shows visibility data for α Boo by the CERGA interferometer (◆) and the IRMA interferometer (■), and originally appeared in the Publications of the Astronomical Society of the Pacific (Copyright 1993, Astronomical Society of the Pacific; Dyck *et al* (1993), reproduced with permission of the editors). (b) The incredible gain in calibration using spatial filtering and photometric monitoring is evident in this figure reproduced from Perrin *et al* (1998, figure 2(a)) with permission from ESO.

Stellar Interferometry

Measuring distance to Cepheids

$$V^2(B\theta/\lambda) = \left(2 \frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda}\right)^2$$

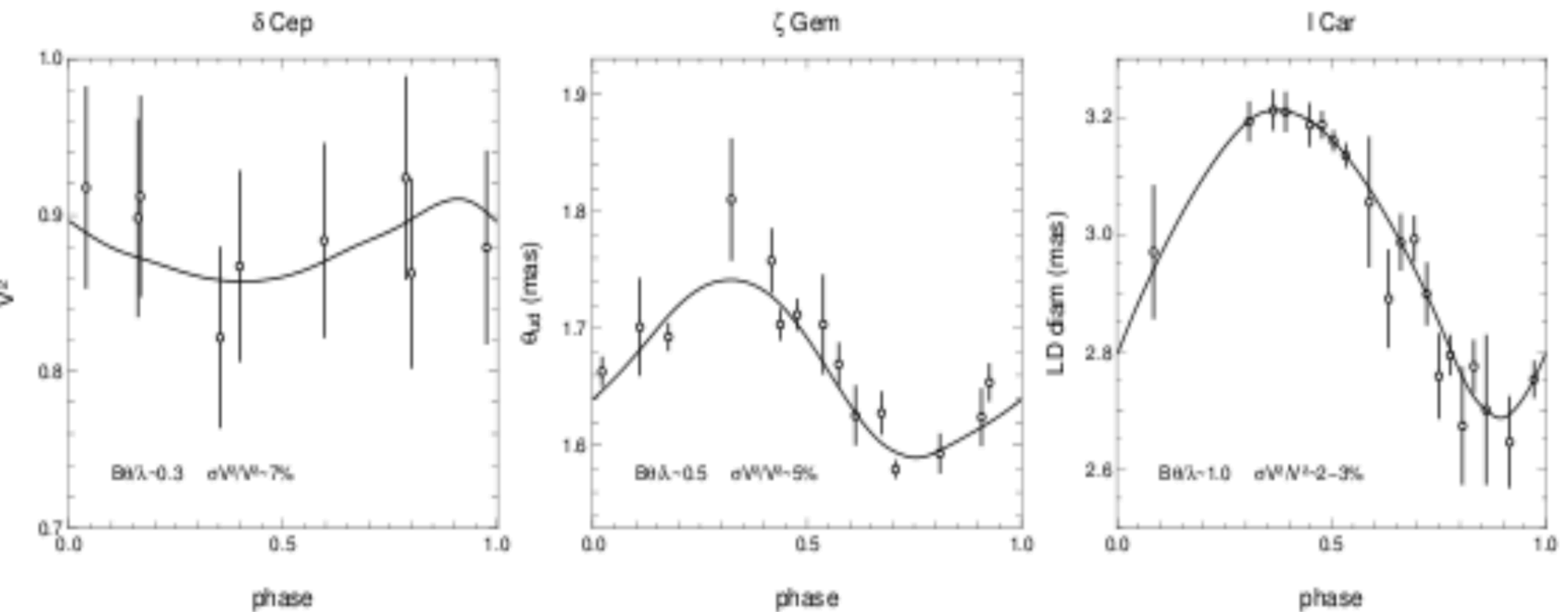


Figure . Different interferometric attempts to measure Cepheid angular diameter variations. From left to right: Mourard et al. (1997⁶), Lane et al. (2000⁷) and Kervella et al. (2004⁸). The left panel is V^2 as a function of phase, while the panels to the right are angular diameters with respect to phase. The thin, continuous line is the integration of the pulsation velocity (distance has been adjusted). From left to right, one can see the effect of increasing resolution ($B\theta/\lambda$) and improving precision ($\sigma V^2/V^2$). In the left panel, the pulsation was not claimed to be detected; the middle panel was the first detection, with a 10% precision on the distance; the right panel displays one of the best: 4% in the distance.

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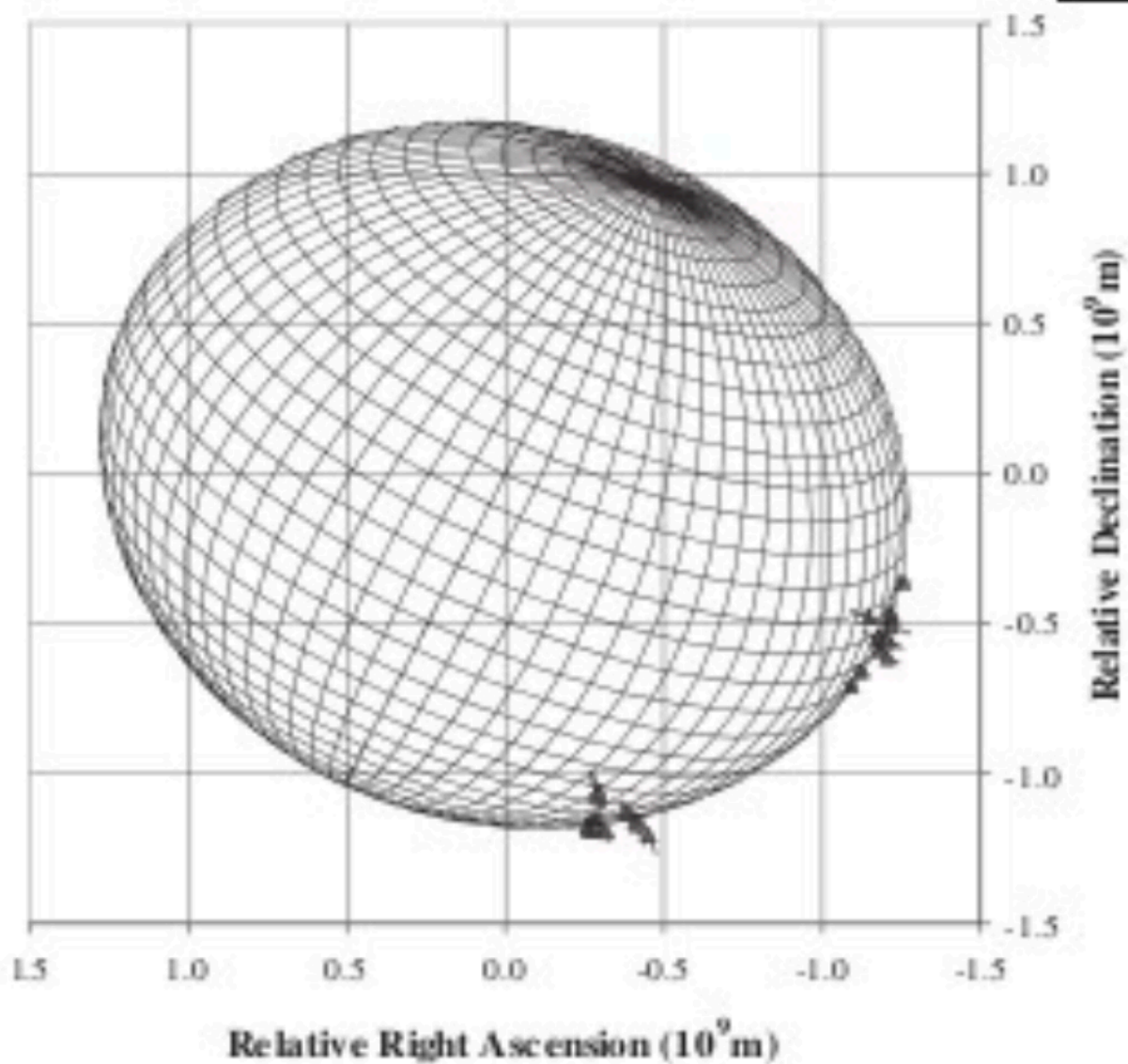
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Stellar Interferometry

Measuring oblateness of Altair

$$V^2(B\theta/\lambda) = \left(2 \frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda}\right)^2$$



Also possible:

Measure limb darkening (deviation from Bessel function above)

Measure diameter as a function of λ :
envelopes around evolved stars

Palomar Testbed Interferometer

the the the

Fundamentals of Radio Interferometry

I: The Basics



Topics

- **Introduction:**
 - **Motivation: Mapping the Sky**
 - **Why Interferometry?**
- **The Basic Interferometer**
 - **Simplifying Assumptions**
 - **‘Fringe’ patterns**
 - **Sine and Cosine Fringes**
 - **Response to Extended Emission**
 - **The Complex Correlator and Complex Visibilities**
 - **Illustrations to aid visualization**
- **Important Note:**
 - **The concepts I’ll be speaking about are not difficult – but they are unfamiliar to beginners.**

Our Goal: Mapping the Sky Brightness

- In astronomy, we wish to know the angular distribution of the emission of distant radiating sources.
 - This can be a function of frequency, polarization, and time.
- ‘Angular Distribution’ means we are interested in the **brightness distribution** of the emission.
- Measuring the brightness means **resolving the emission**.
- Because our targets are so far away, the emission is extremely weak, and of very small angular size.
 - Power from strongest radio source collected over the entire area of Australia in 1 GHz bandwidth = 0.1 watt!
 - 1 arcsecond = angular width of a coin at ~5 kilometres!
- Early (1950s) surveys of the radio sky employed single dishes.
- Nowadays, most (but not all!) observations are done with interferometers.

Why Interferometry?

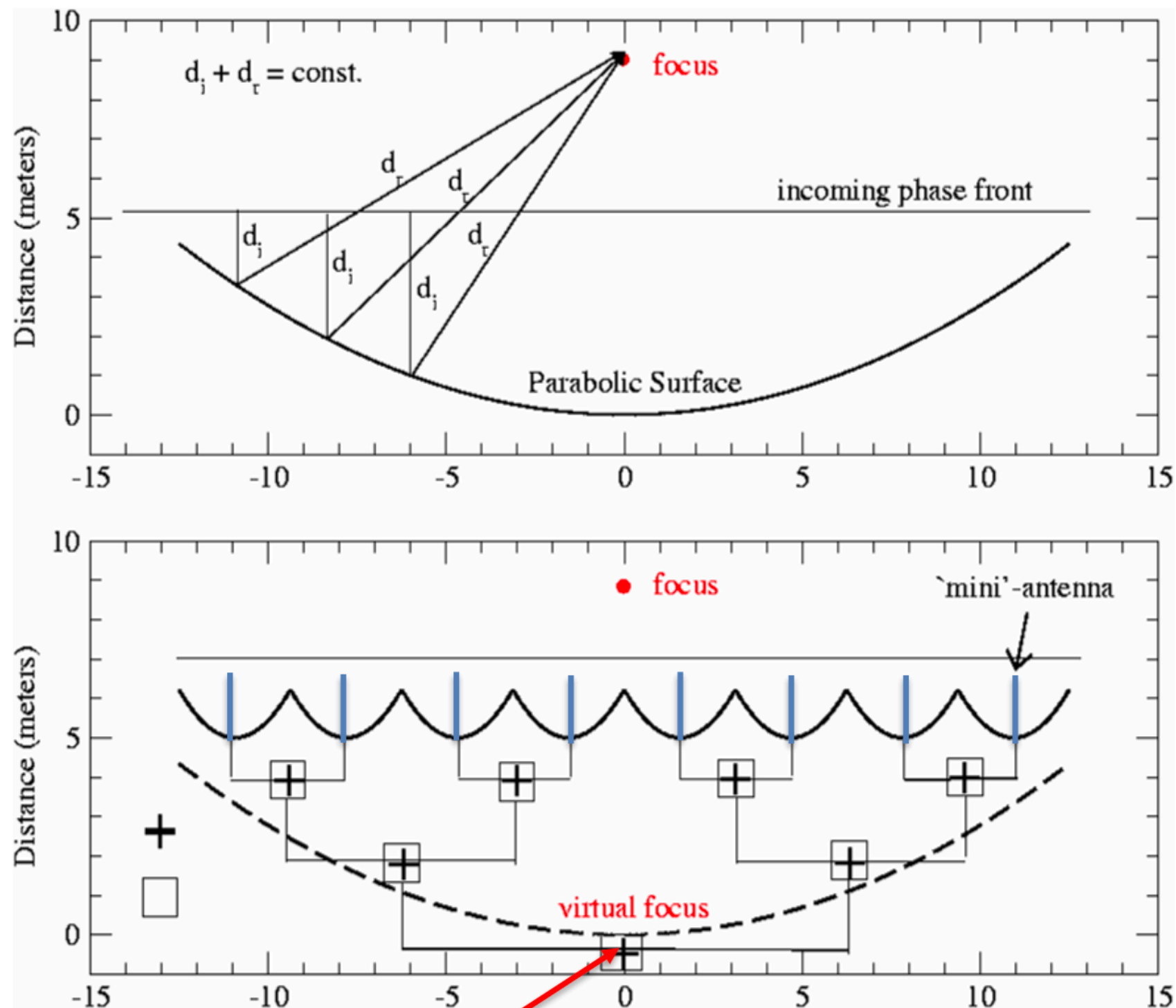
- It's due to **Diffraction** – a consequence of the wave nature of light.
- Radio telescopes coherently sum electric fields over an aperture of size D . For this, diffraction theory applies – the angular resolution is:

$$\theta_{rad} \approx \lambda / D \quad \text{Or, in practical units} \quad \theta_{arcsec} \approx 2 \lambda_{cm} / D_{km}$$

- To obtain 1 arcsecond resolution at a wavelength of 21 cm, we require an aperture of ~42 km – not feasible.
- The (currently) largest single, fully-steerable apertures are the 100-m antennas in Bonn, and Green Bank. Nowhere big enough.
- Can we synthesize a larger aperture with separated antennas?
- The technique of synthesizing a larger aperture through combinations of separated pairs of antennas is called 'aperture synthesis'.

Interferometry – Basic Concept

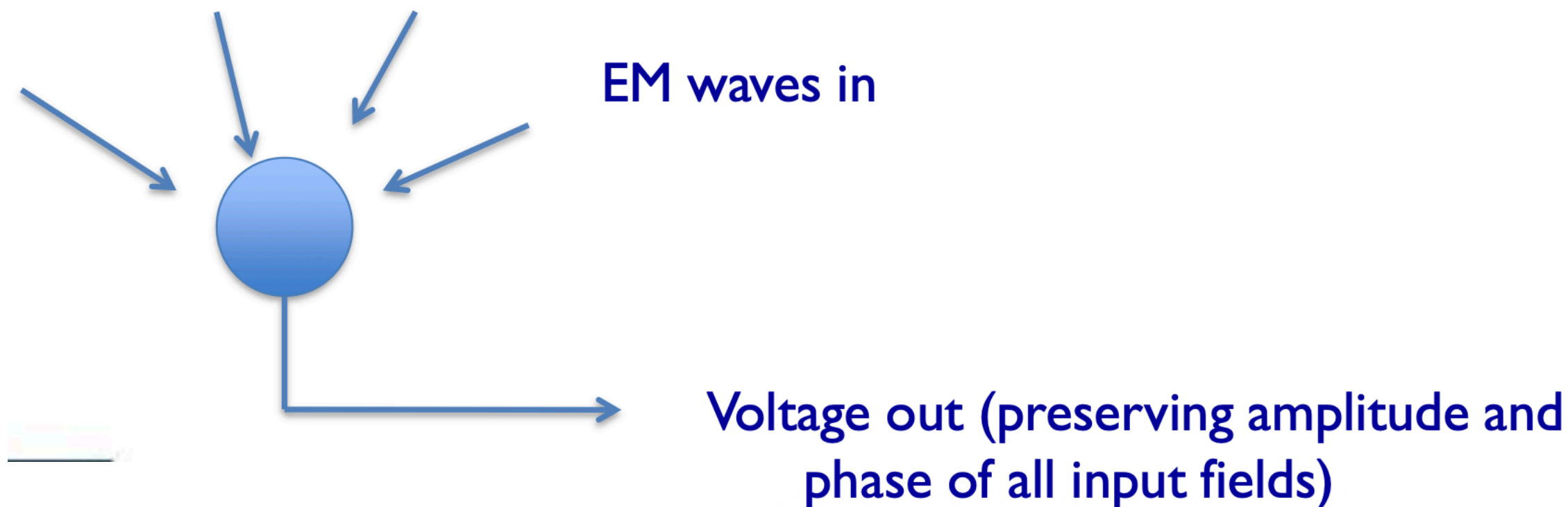
- A parabolic dish coherently sums EM fields at the focus.
- The same result can be obtained by adding voltages from individual elements with a wired network.
 - Note – they need not be adjacent.
- This is the basic concept of interferometry – coherent summation of voltages from separated antennas.
- Aperture Synthesis is an extension of this concept.



Signals summed here equivalent to those collected at prime focus

The Role of the Sensor (aka Antenna)

- Coherent interferometry is based on the ability to correlate the electric fields measured at spatially separated locations.
- To do this (without mirrors) requires conversion of the electric field $E(\mathbf{r}, \nu, t)$ at some place ($\mathbf{r}$) to a voltage $V(\nu, t)$ which can be conveyed to a central location for processing – without losing the phase information.
- For our purpose, the sensor (a.k.a. ‘antenna’) is simply a device which senses the electric field at some place and converts this to a voltage which faithfully retains the amplitudes and phases of the electric fields.

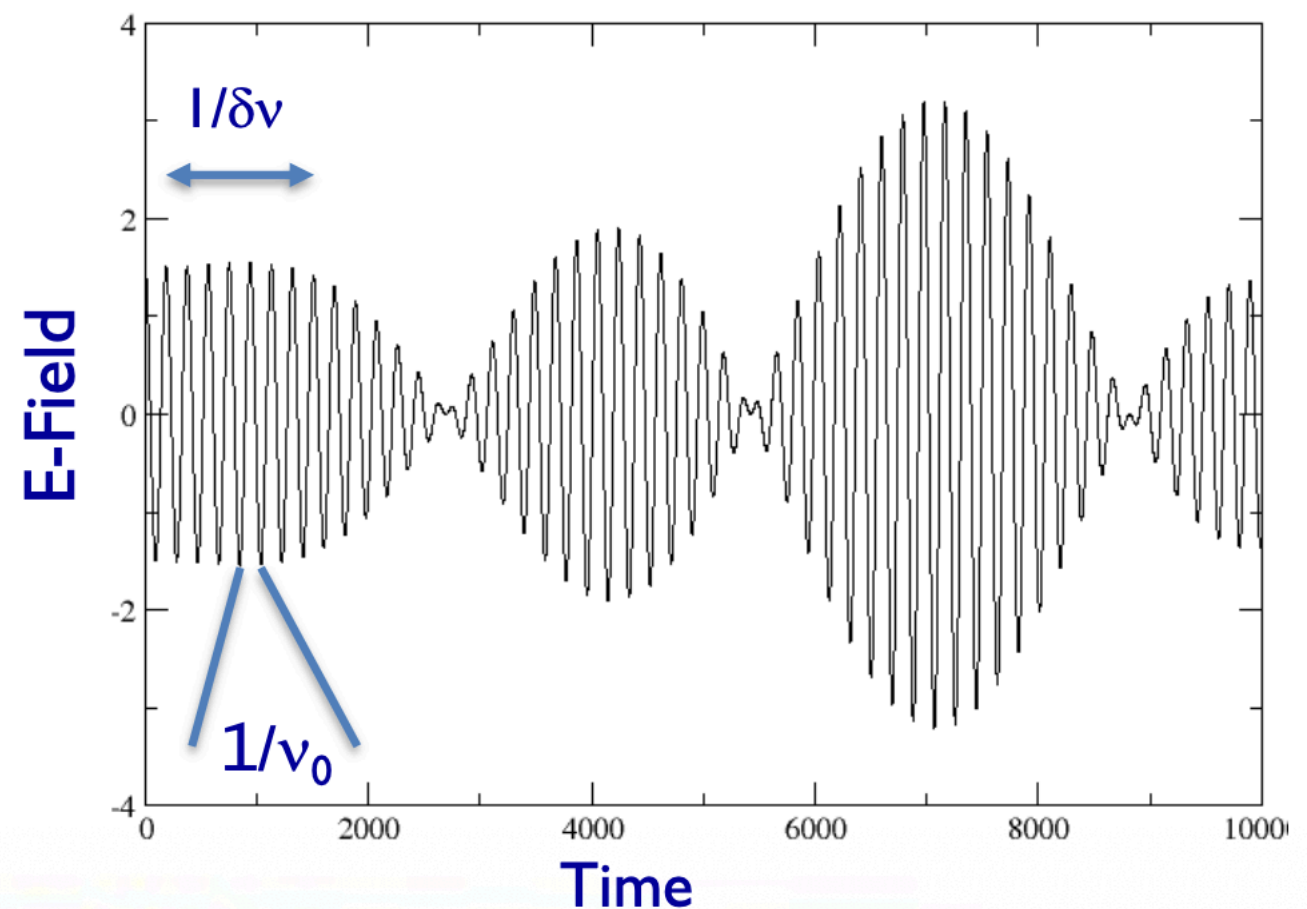


Quasi-Monochromatic Radiation

- Mathematical analysis of wideband noise is difficult, if not impossible.
- Analysis is simple if the fields are monochromatic.
 - Natural radiation is never monochromatic
- So we consider instead ‘quasi-monochromatic’ radiation, where the bandwidth $\delta\nu$ is very small, but not zero.
- Then, for a time $dt \sim 1/\delta\nu$, the electric fields will be sinusoidal, with unchanging amplitude E and phase ϕ , described by

$$E_\nu(t) = E \cos(2\pi\nu t + \phi)$$

The figure shows an ‘oscilloscope’ trace of a narrow bandwidth noise signal. The period of the wave is $T = 1/\nu_0$, the duration over which the signal is closely sinusoidal is $T \sim 1/\delta\nu$. There are $N \sim \nu_0/\delta\nu$ oscillations in a ‘wave packet’.



Simplifying Assumptions

- We now consider the most basic interferometer, and seek a relation between the characteristics of the product of the voltages from two separated antennas and the distribution of the brightness of the originating source emission.
- To establish the basic relations, the following simplifications are introduced:
 - Isotropic sensors fixed in space – no rotation or motion
 - Quasi-monochromatic radiation
 - Source in the far-field ($D \gg b^2/\lambda$)
 - No frequency conversions (an 'RF interferometer')
 - Single polarization
 - Propagation in vacuum, without distortions (no ionosphere, atmosphere ...)
 - Idealized electronics (perfectly linear, perfectly uniform in frequency and direction, perfectly identical for both elements, no added noise.

I will later relax most of these restrictions.

The Stationary, Quasi-Monochromatic Radio-Frequency Interferometer

$\mathbf{s}$ = Direction Unit Vector

$\mathbf{b}$ = Baseline Vector

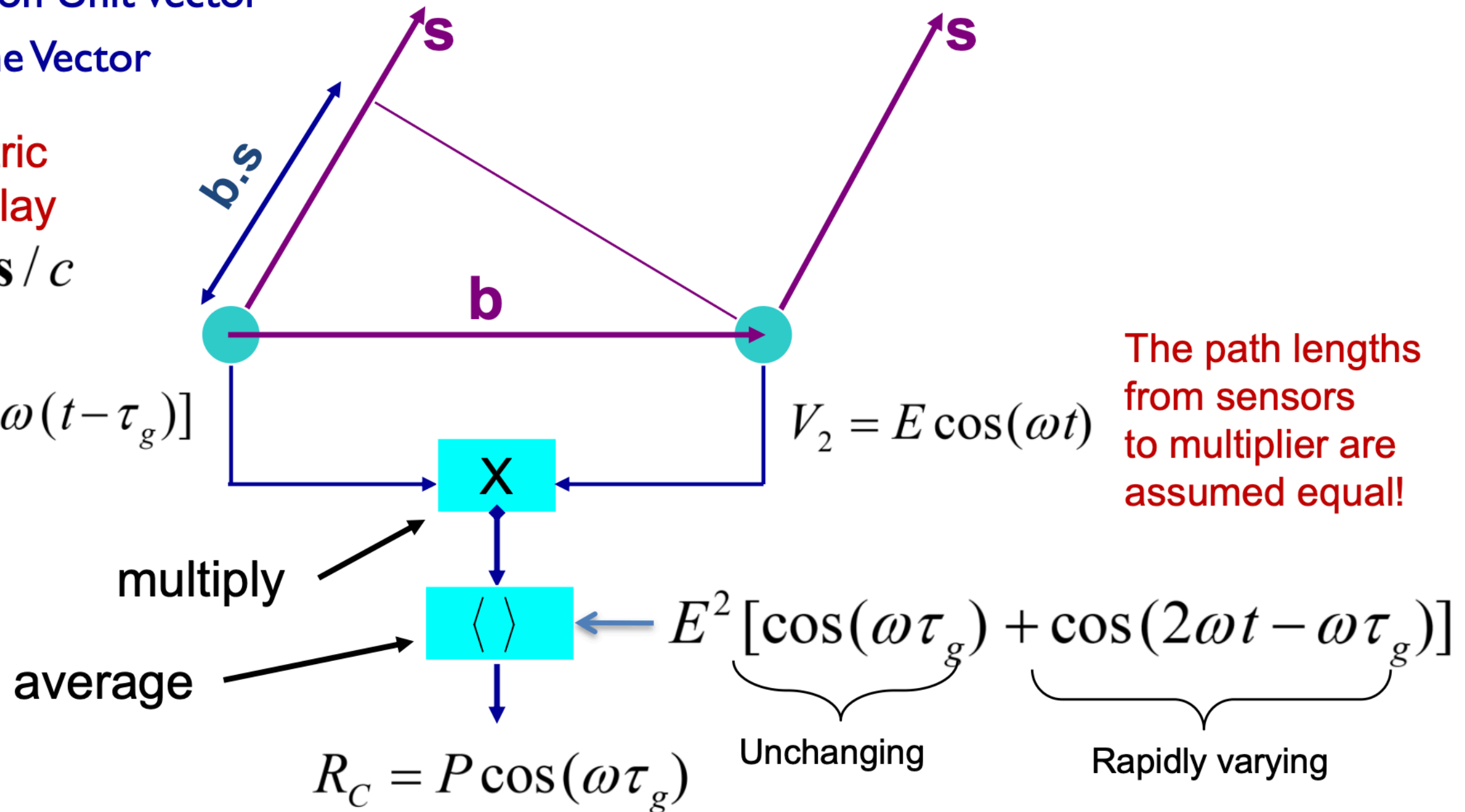
Geometric
Time Delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$

$$V_1 = E \cos[\omega(t - \tau_g)]$$

$$V_2 = E \cos(\omega t)$$

The path lengths
from sensors
to multiplier are
assumed equal!



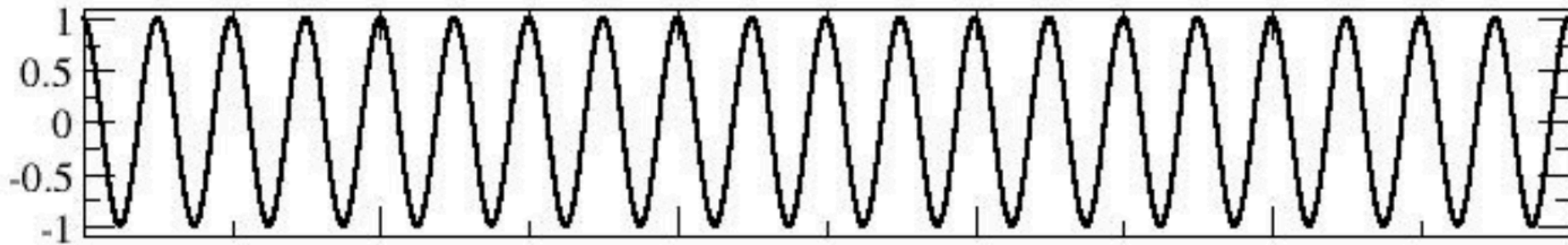
Where I replaced E^2 with P (power), and ignored factors of 2

Pictorial Example: Signals In Phase

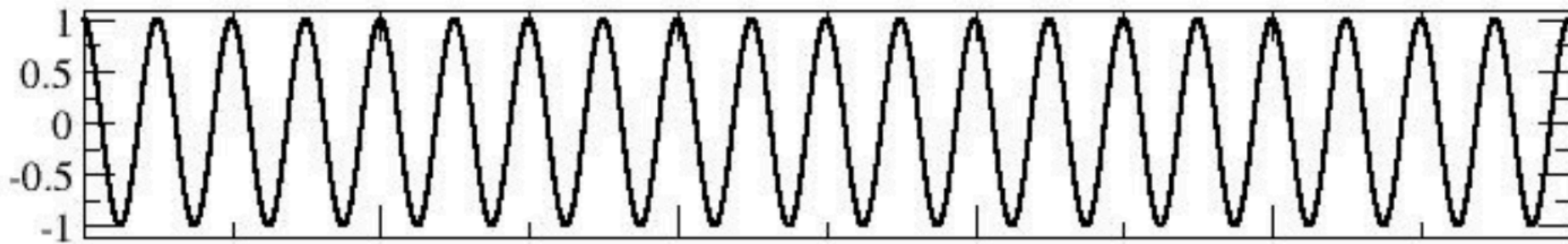
2 GHz Frequency, with voltages in phase:

$$\mathbf{b.s} = n\lambda, \text{ or } \tau_g = n/v$$

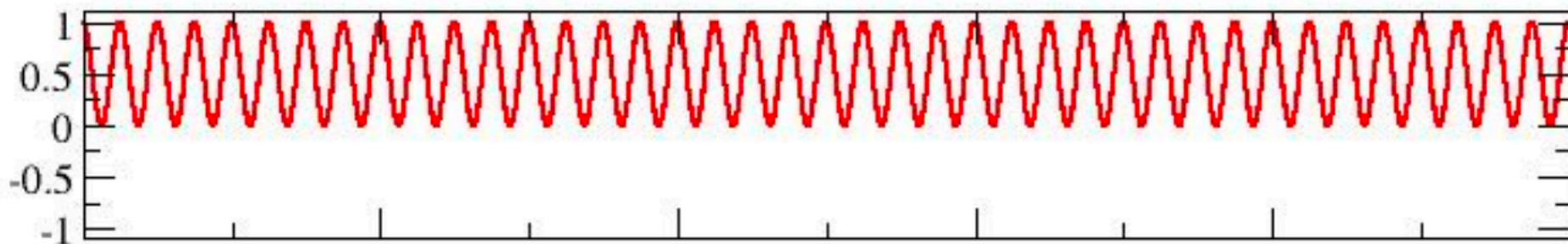
- Antenna 1 Voltage



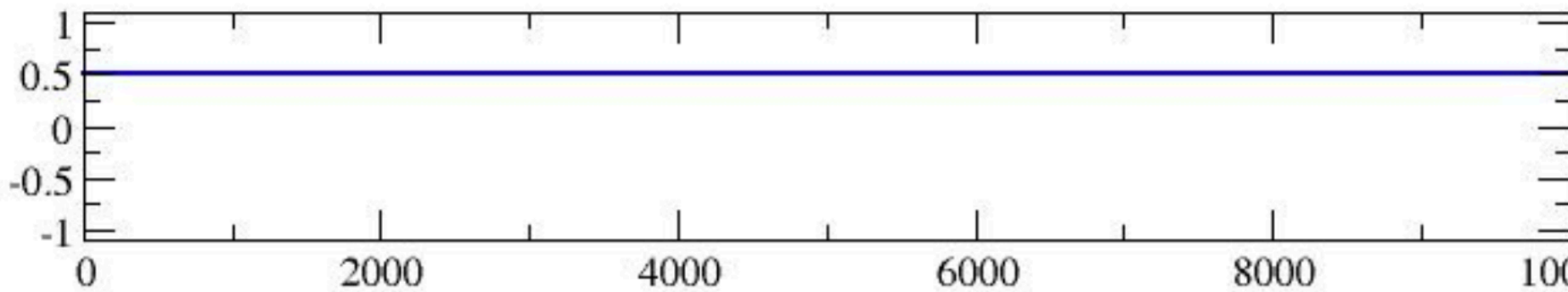
- Antenna 2 Voltage



- Product Voltage



- Average



Positive
constant

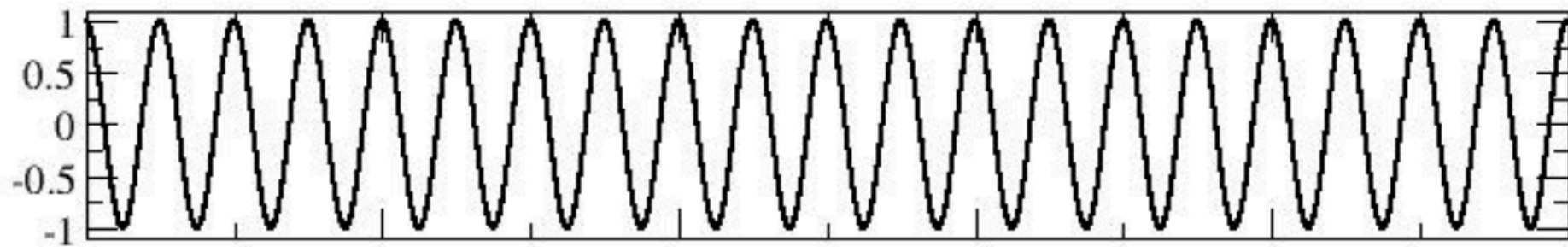
Time (ps)

Pictorial Example: Signals out of Phase

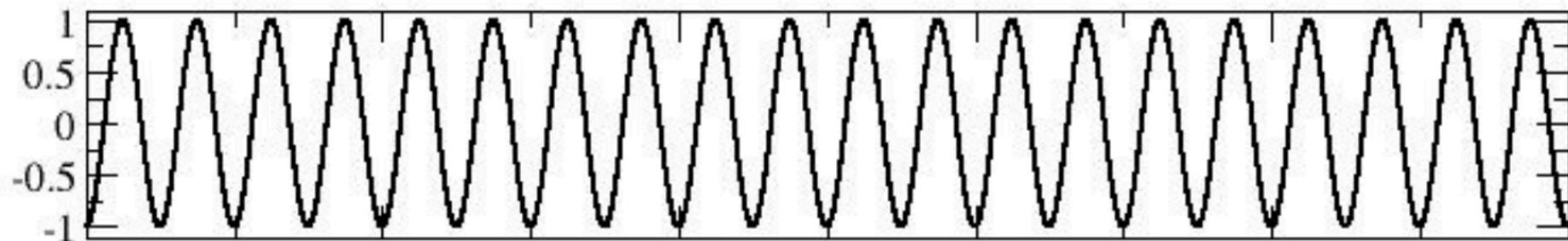
2 GHz Frequency, with voltages out of phase:

$$b.s = (n \pm \frac{1}{2})\lambda \quad \tau_g = (n \pm \frac{1}{2})/\nu$$

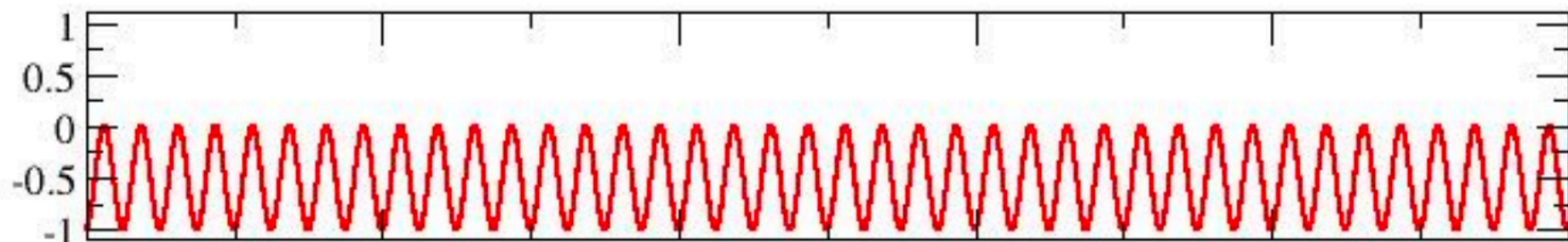
- Antenna 1 Voltage



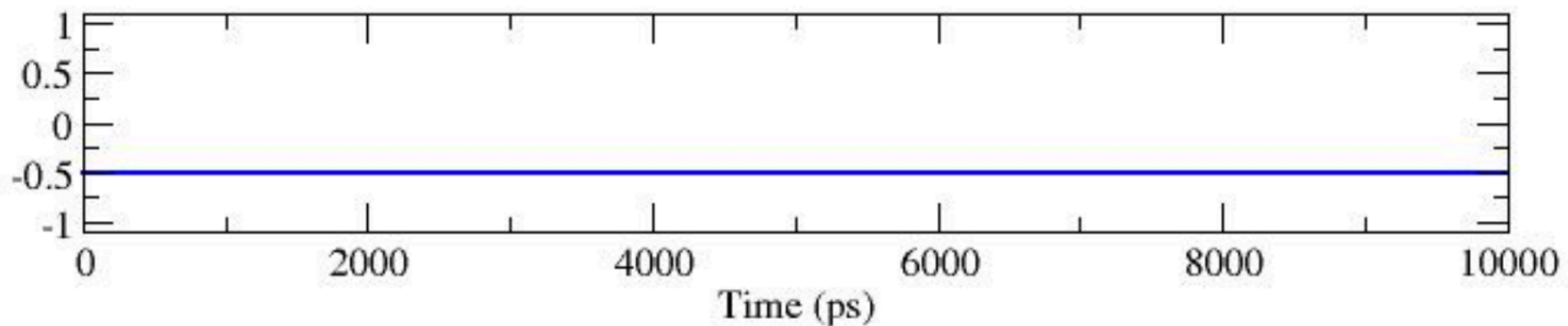
- Antenna 2 Voltage



- Product Voltage



- Average



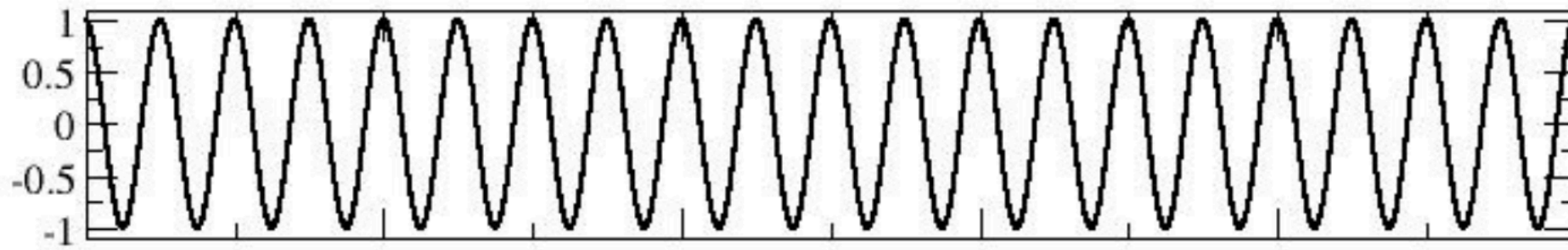
Negative constant

Pictorial Example: Signals in Quad Phase

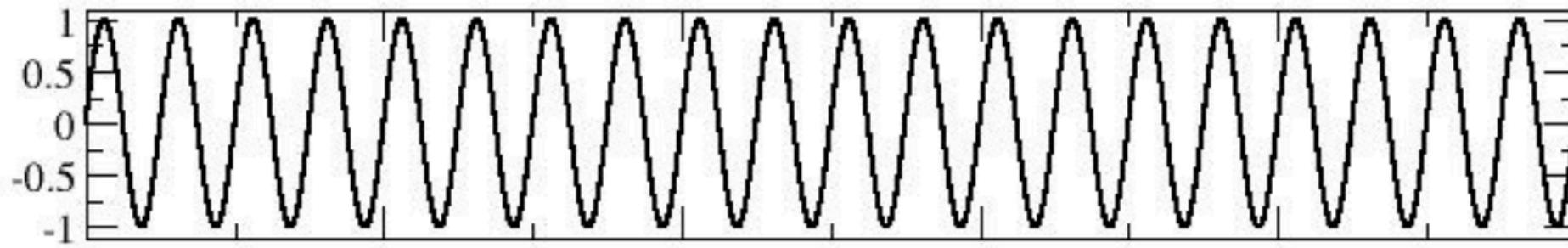
2 GHz Frequency, with voltages in quadrature phase:

$$b.s = (n \pm 1/4)\lambda, \tau_g = (n \pm 1/4)/v$$

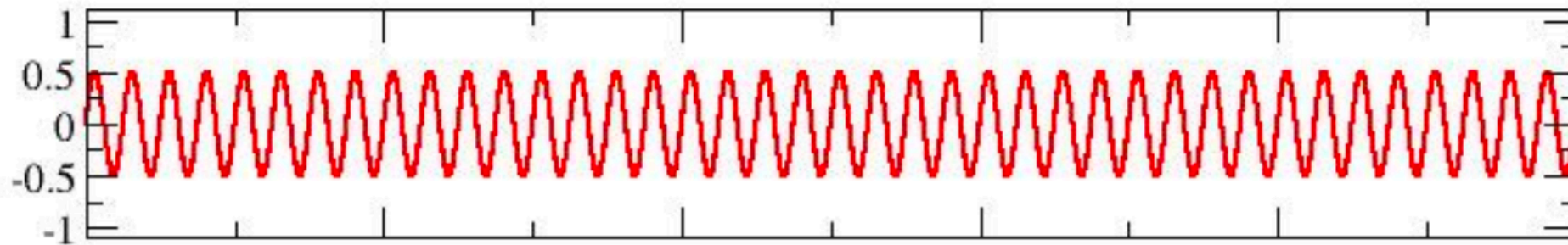
- Antenna 1 Voltage



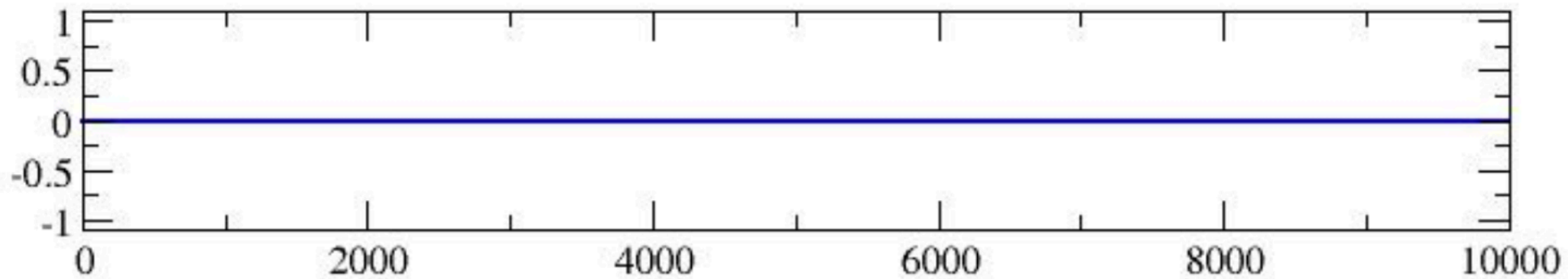
- Antenna 2 Voltage



- Product Voltage



- Average



Zero
constant

Time (ps)

General Comments on this Product

- The averaged product R_C is dependent on the received power, $P = E^2/2$ and geometric delay, τ_g , and hence on the baseline orientation and source direction. From slide 10, we have:

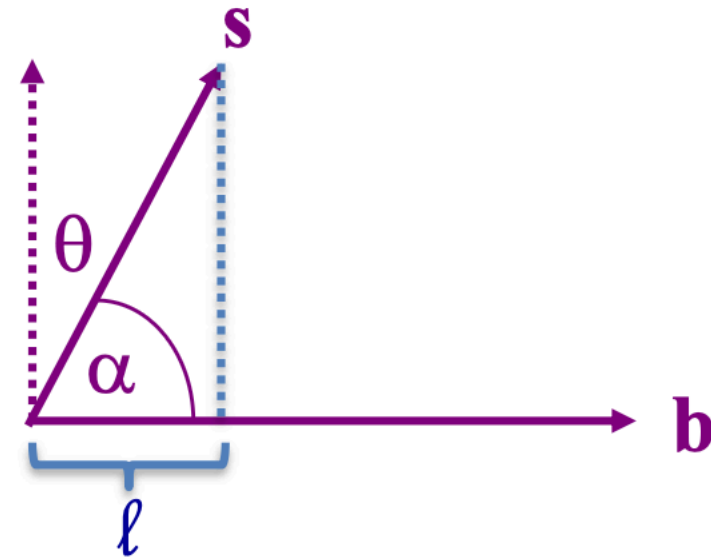
$$R_C = P \cos(\omega \tau_g) = P \cos\left(2\pi \frac{\mathbf{b} \cdot \mathbf{s}}{\lambda}\right)$$

- Note that R_C is not a function of:
 - The time of the observation -- provided the source itself is not variable!
 - The location of the baseline -- provided the emission is in the far-field.
 - The actual phase of the incoming signal – the distance to the source -- provided the source is in the far-field.
- The strength of the product is also dependent on practical factors such as the antenna sizes, electronic gains, bandwidth and time averaging – but these factors are ignored here (and can be calibrated for.)

Define the Direction Cosine

- Consider a single baseline, and define the x-axis to extend along this baseline.
- Write $\mathbf{b} = u \hat{\mathbf{x}}$ where $u = |\mathbf{b}|/\lambda$ is the baseline length in wavelengths, and $\hat{\mathbf{x}}$ is the unit direction vector.
- Define the 'direction cosine' as: $l = \hat{\mathbf{x}} \cdot \mathbf{s} = \cos \alpha = \sin \theta$
- Then:

$$\frac{\mathbf{b} \cdot \mathbf{s}}{\lambda} = u \cos \alpha = u \sin \theta = ul$$



- So the interferometer response is:

$$R_C = P \cos(\omega \tau_g) = P \cos\left(2\pi \frac{\mathbf{b} \cdot \mathbf{s}}{\lambda}\right) = P \cos(2\pi ul)$$

Two Illustrative Examples

- Consider the response R_c , as a function of angle, for two different baselines with $u = b/\lambda = 10$, and $u = 25$ wavelengths.
- Since $R_c = \cos(2\pi ul)$
- We have, for $u = 10$: $R_c = \cos(20\pi l)$
- And, for $u = 25$: $R_c = \cos(50\pi l)$
- These are simple functions of angle on the sky.

Remember:

u = baseline length in wavelengths

$l = \sin \theta$,

θ = angular offset from perpendicular plane

Whole-Sky Response

- Top:

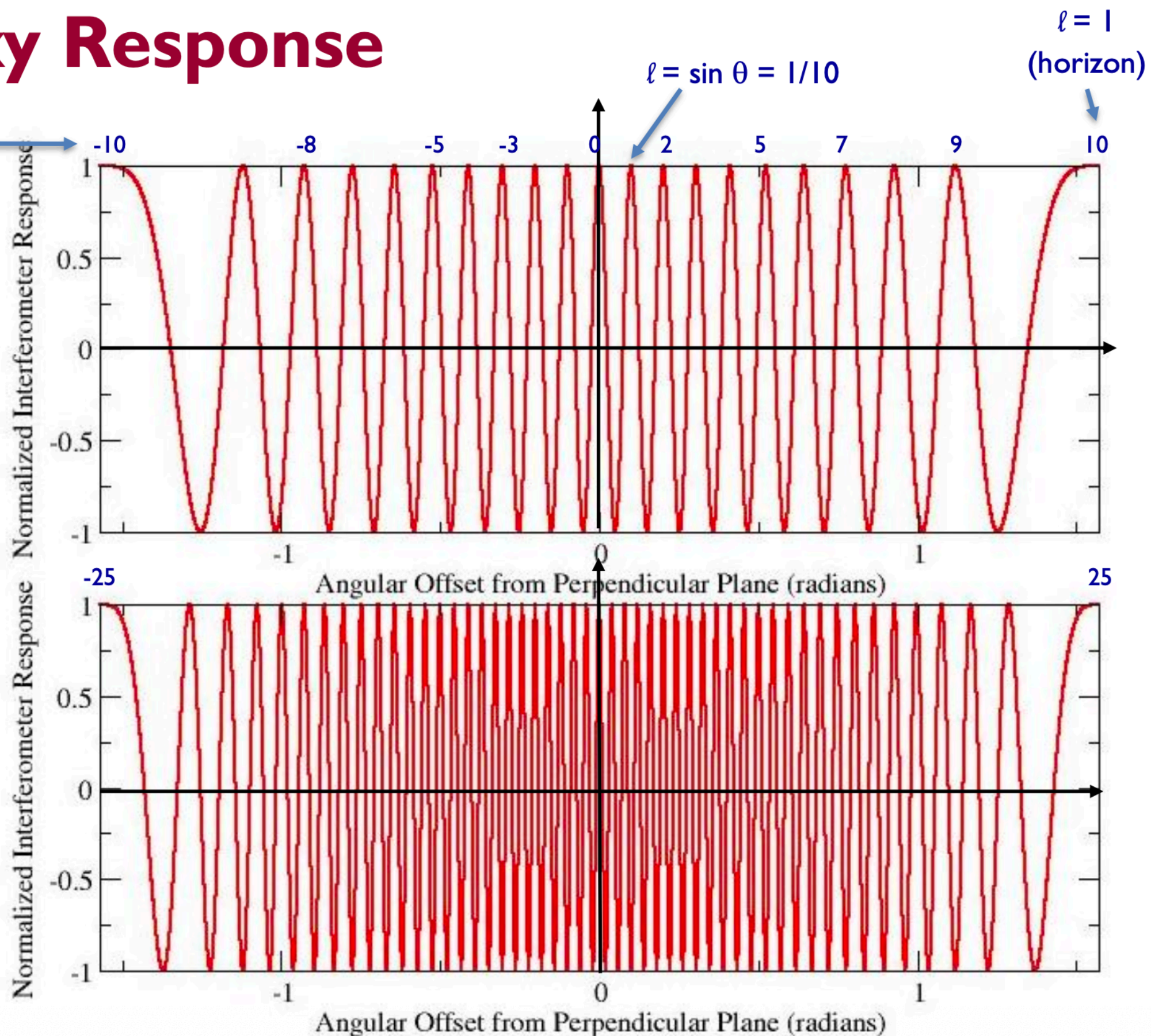
$$u = 10$$

There are 21 fringe maxima, and 20 fringe minima over the sphere.

- Bottom:

$$u = 25$$

There are 51 fringe maxima over the sphere

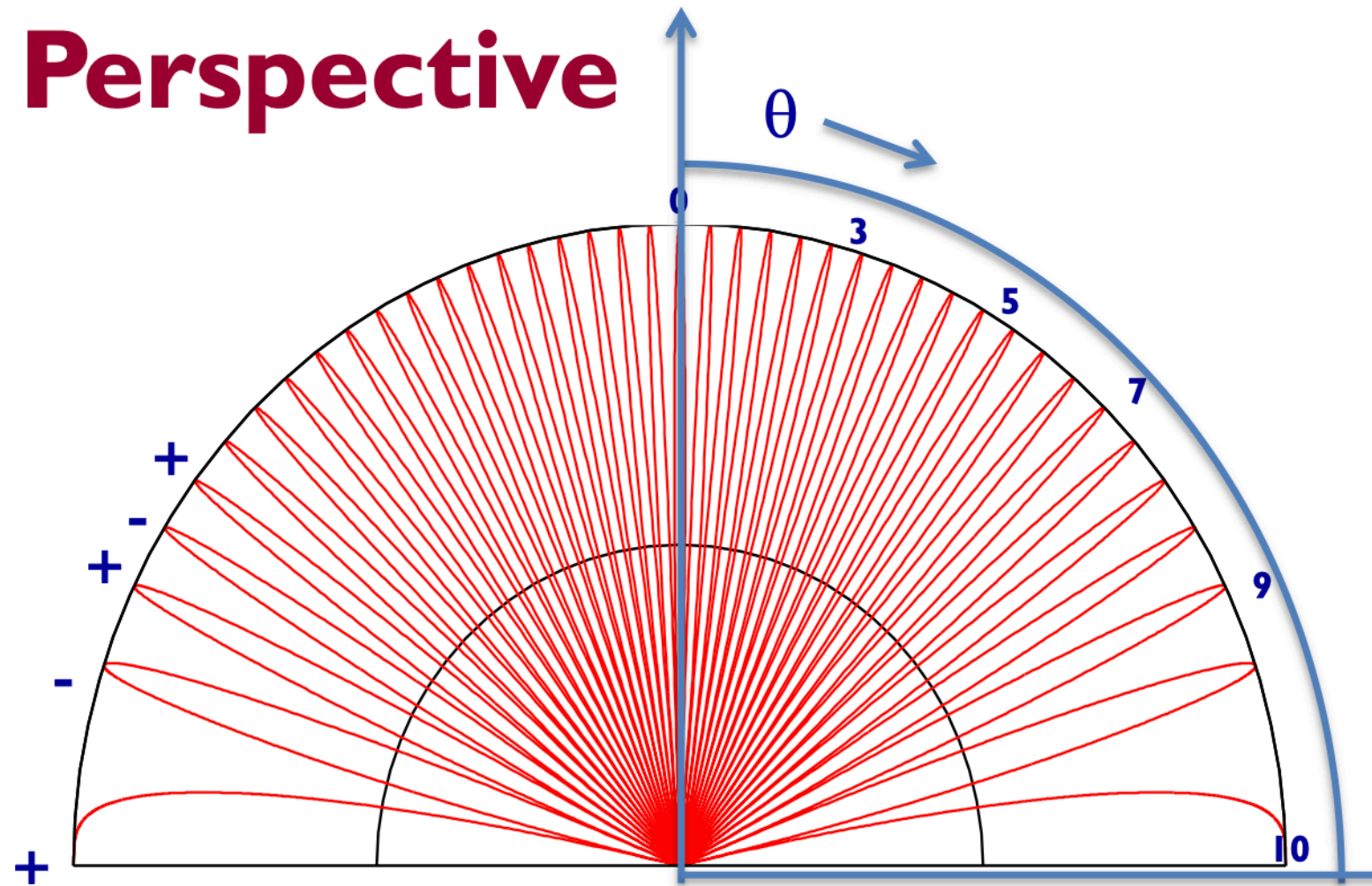


From an Angular Perspective

Top Panel:

The absolute value of the response for $u = 10$, as a function of angle.

The 'lobes' of the response pattern alternate in sign.

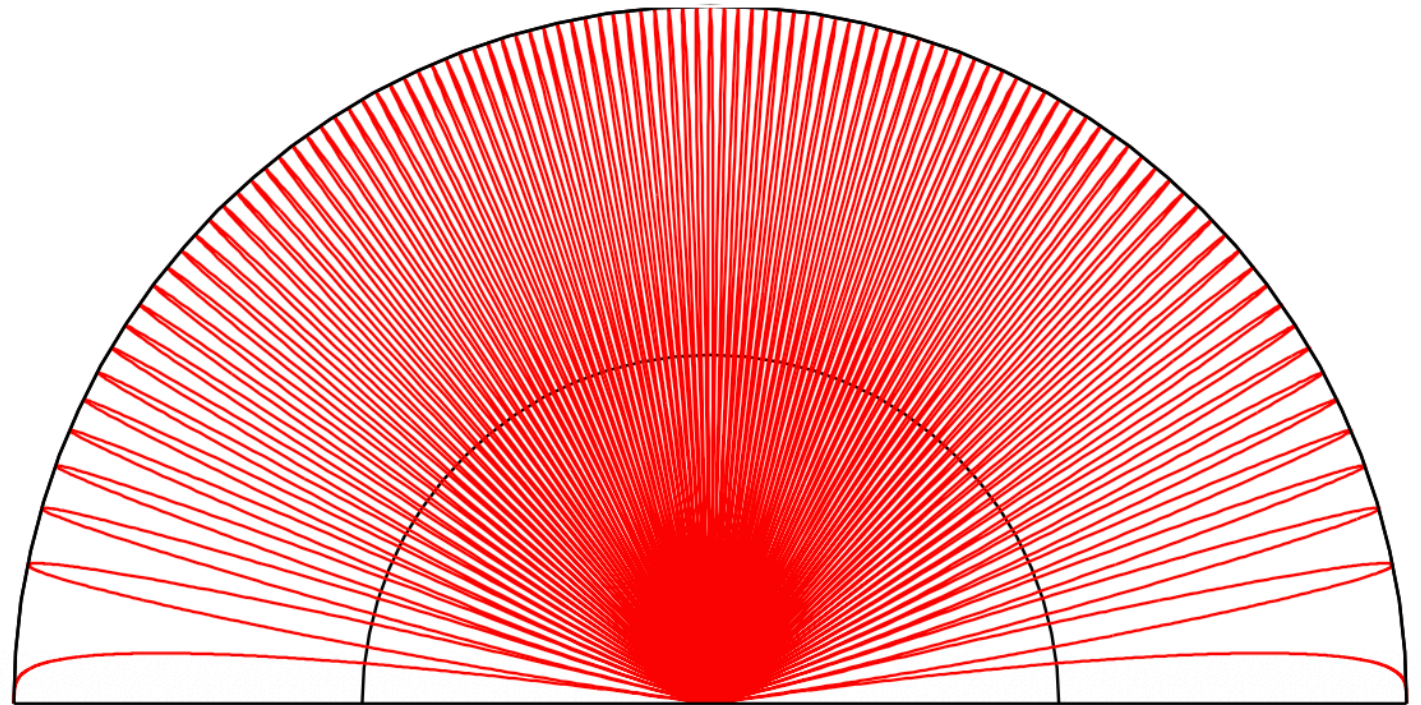


Bottom Panel:

The same, but for $u = 25$.

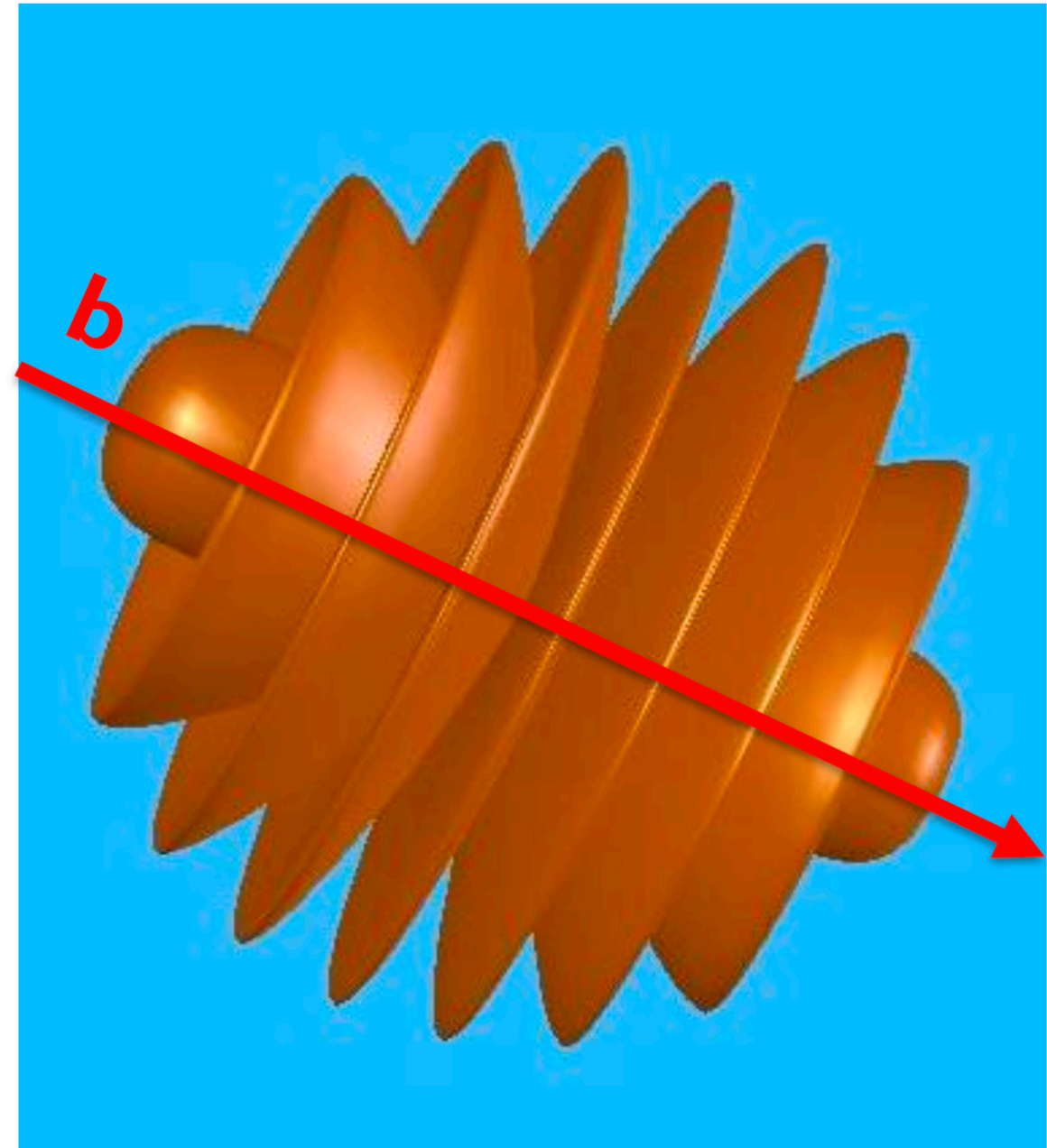
Angular separation between lobes (of the same sign) is

$$\delta\theta \sim 1/u = \lambda/b \text{ radians.}$$



Hemispheric Pattern

- The preceding plot is a meridional cut through the hemisphere, oriented along the baseline vector **b**.
- In the three-dimensional space, the fringe pattern consists of a series of coaxial cones, oriented along the baseline vector.
- The figure is a two-dimensional representation when $u = 4$.
- As viewed along the baseline vector, the fringes show a 'bulls-eye' pattern – concentric circles.

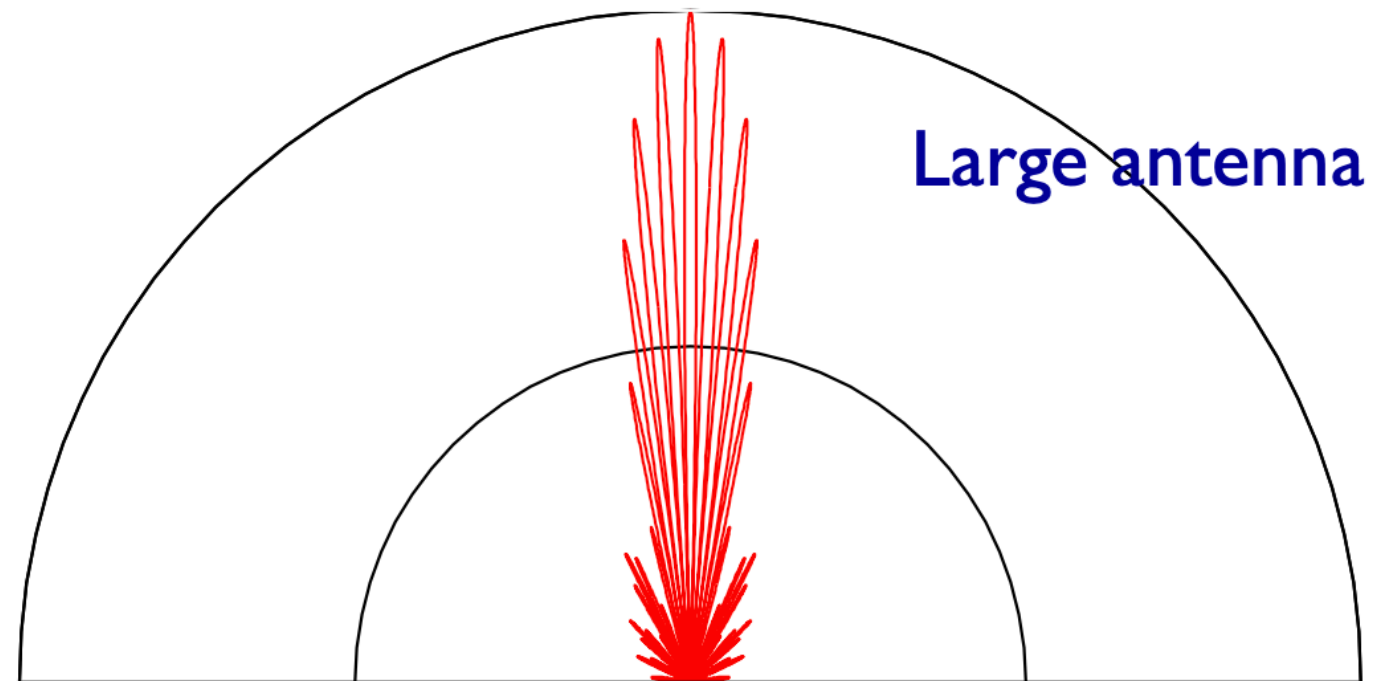
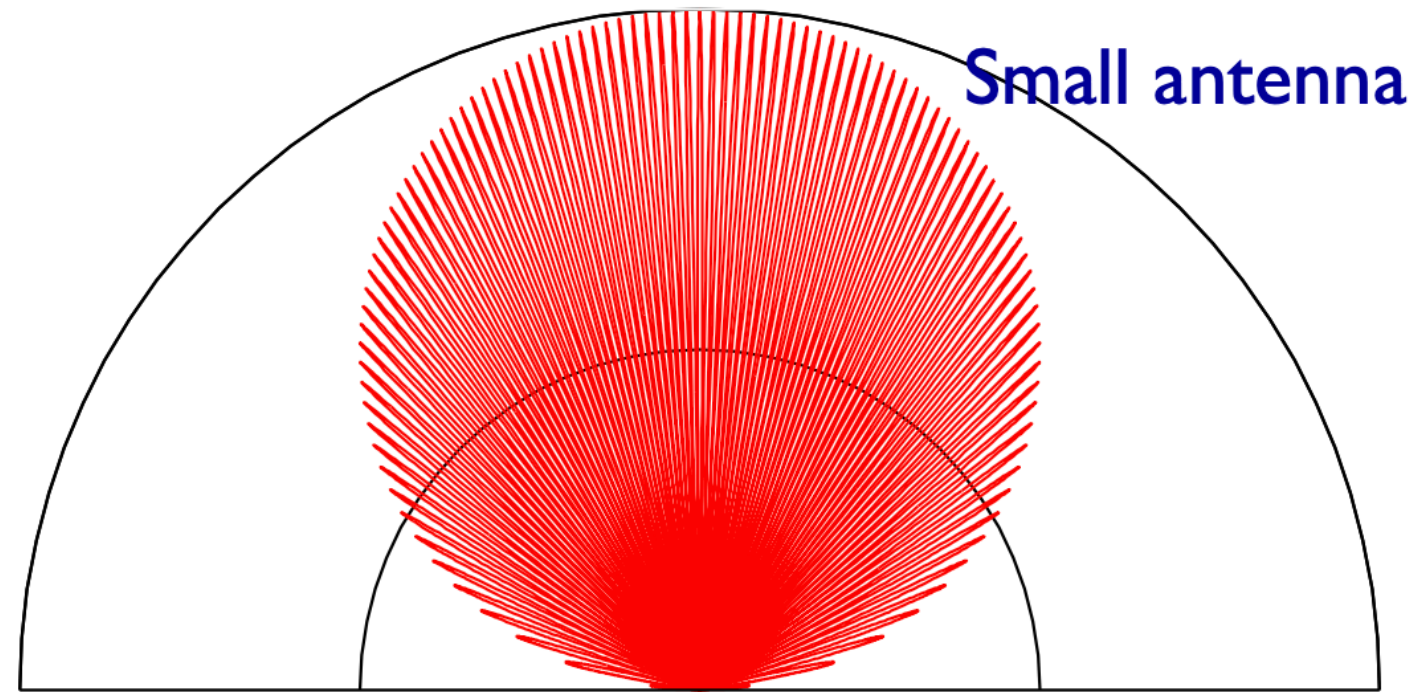


The Effect of the Sensor (aka Antenna)

- The patterns shown presume the sensor has isotropic response.
- This is a convenient assumption, but doesn't represent reality.
- Real sensors impose their own patterns, which modulate the amplitude and phase of the interferometer response.
- Large sensors have good angular resolution and high gain
 - very useful for some applications like imaging individual objects.
- Small sensors have poor angular resolution and low gain
 - useful for wide-angle surveys.
- Emphasis: The fringe pattern is a function of the baseline length (in wavelengths) and orientation, and is not affected by the antenna patterns.

The Effect of Sensor Patterns

- Sensors (or antennas) are not isotropic, and have their own responses – both amplitude and phase.
- **Top Panel:** The interferometer pattern with a $\cos(\theta)$ -like sensor response.
- **Bottom Panel:** A multiple-wavelength aperture antenna has a narrow beam, but also sidelobes.
- **Emphasis:** The fringe pattern is a function of the array geometry. The angular modulation is from the sensor.



From Point Sources to Extended Emission

- By construction, everything I stated earlier holds for ‘point sources’. What are these?
- Point Source: Emission from an object whose angular extent is much, much smaller than the interferometer fringe scale.
- That is: $\theta_{rad} \ll \lambda / B$
where θ = angular size, λ = wavelength, B = baseline
- What happens when the source angular size is comparable to, or larger than, the fringe scale?
- Short answer: Under nearly all conditions, you just ‘add up’ the individual responses from the components of the extended source.

The Response from an Extended Source

- For an extended source, the voltage output from a single antenna is the spatial integration of the E-field emission over the primary beam:

$$V = \iint E(\mathbf{s}) d\Omega$$

- So the correlator response becomes

$$R_C = \left\langle \iint E_1(\mathbf{s}) d\Omega_1 \times \iint E_2(\mathbf{s}) d\Omega_2 \right\rangle$$

- The averaging and integrals can be interchanged and, **providing the emission is spatially incoherent**, we get

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

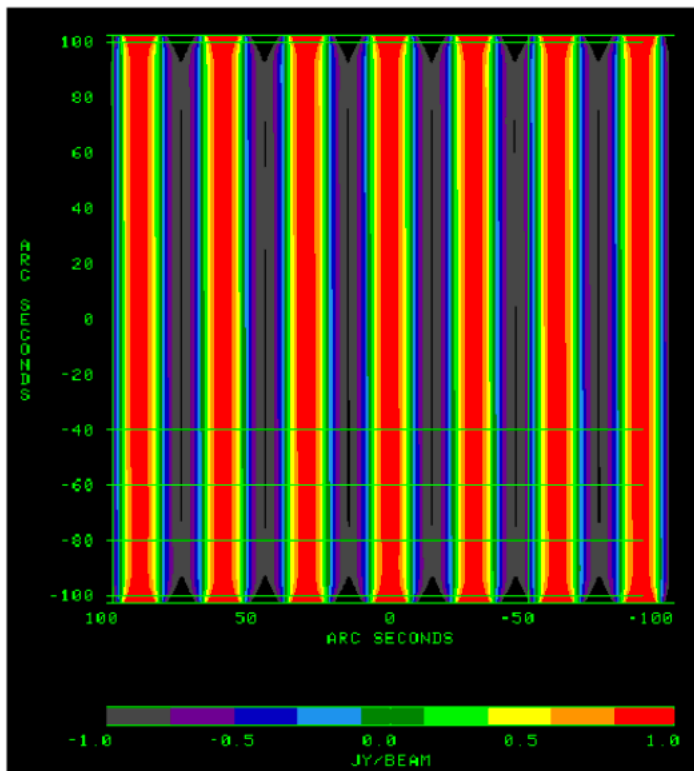
- The response is the sum of the spatial integration of the brightness modulated by the cosinusoidal interferometer pattern.
- This expression links what we want: the brightness on the sky, $I_\nu(\mathbf{s})$, to something we can measure - R_C , the interferometer response.
- Can we recover $I_\nu(\mathbf{s})$ from observations of R_C ?

A picture is worth 1000 words ...

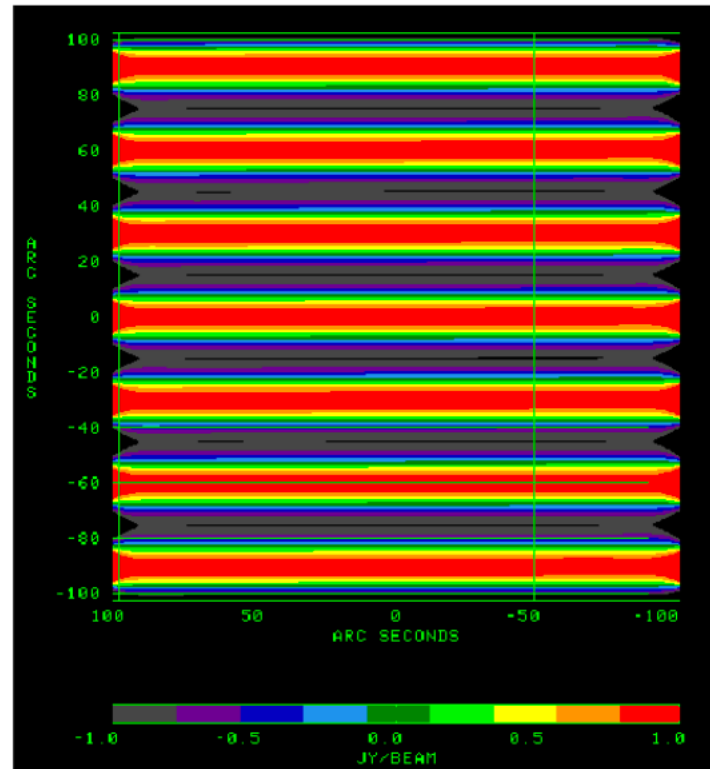
- As stated earlier, these concepts are not difficult, but are unfamiliar. We need to think in new ways, to get a deeper understanding of how all this works.
- To aid, I have generated images of interferometer fringes, of various baseline lengths and orientations.
- I then 'observe' a real source (Cygnus A, of course), to show what the interferometer actually measures.
- For all these, the 'observations' are made at 2052 MHz. The Cygnus A image is taken from real VLA data.
- To keep things simple, all simulations are done at meridian transit.

'Real' Fringes ... 1 Km Baseline at 2052 MHz

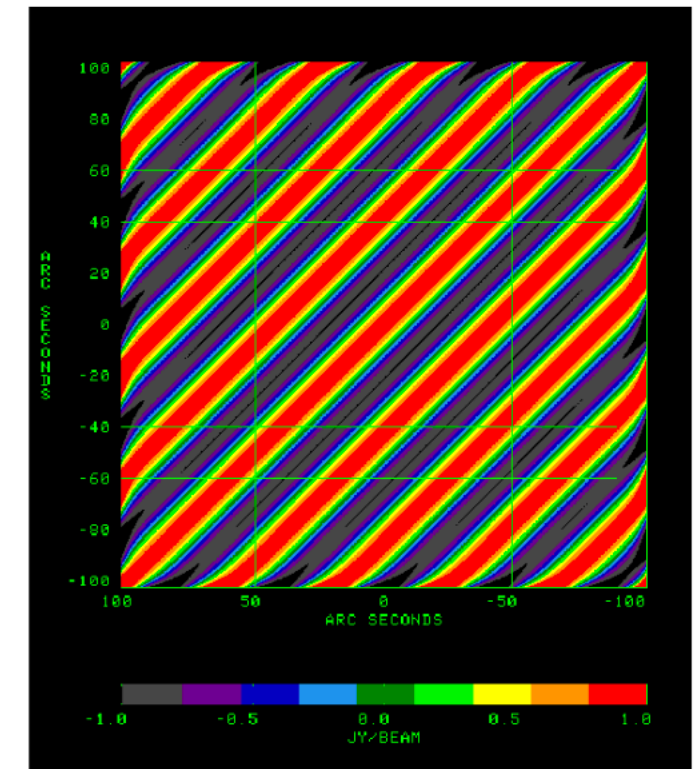
- The fringe separation given by baseline length in wavelengths, the orientation given by the orientation of the baseline.



East-West baseline
makes vertical fringes



North-South baseline
makes horizontal fringes



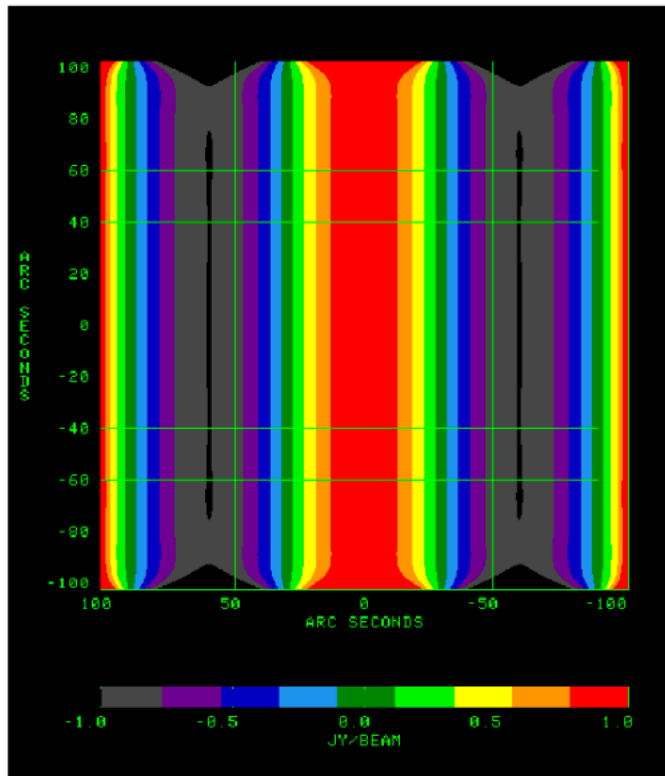
Rotated baseline makes
rotated fringes

- Red = positive maximum. Black = negative maximum. Green = zero
- Fringe angular spacing given by baseline length in wavelengths:

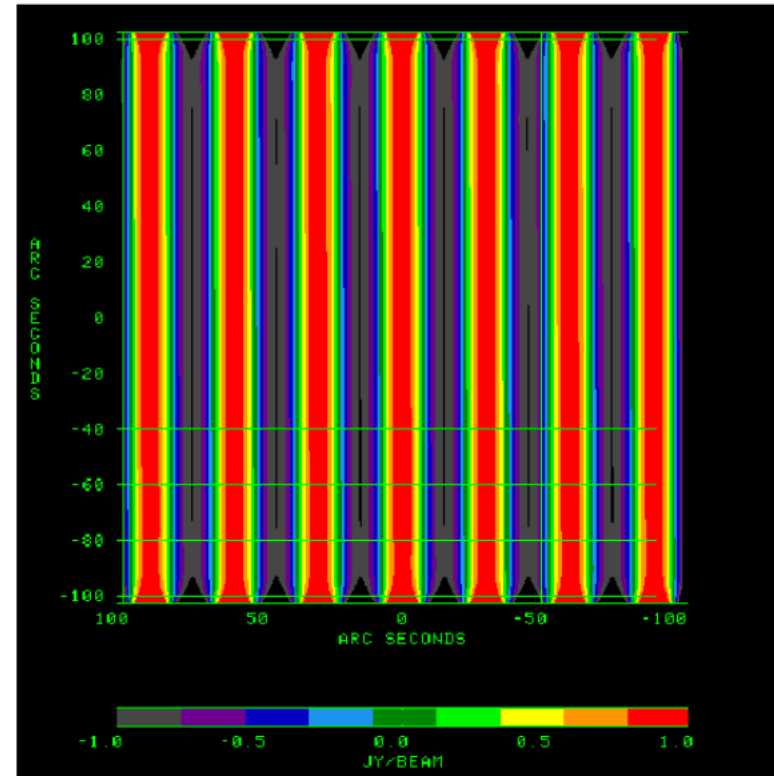
$$\Delta\theta = \lambda / B = 30.2''$$

Longer Baselines => Smaller Fringe Spacing

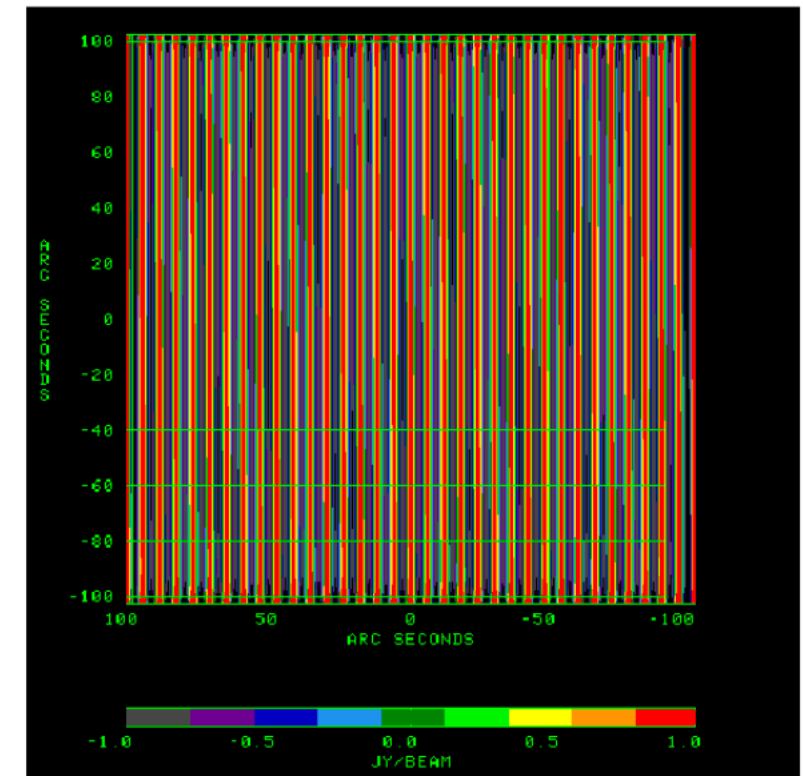
- With longer baselines (in wavelengths!) come finer fringes:



250 meter baseline
120 arcsecond fringe



1000 meter baseline
30 arcsecond fringe

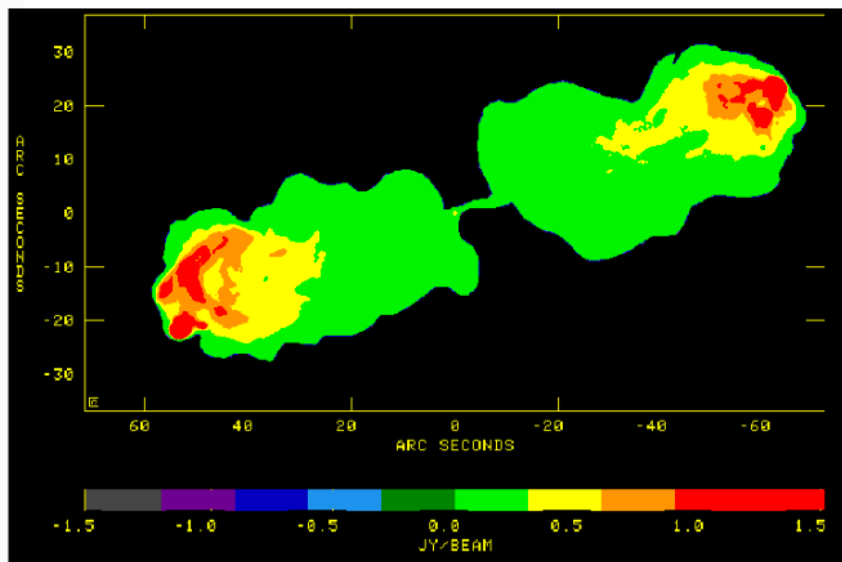


5000 meter baseline
6 arcsecond fringe

- What the interferometer measures is the integral (sum) of the product of this pattern with the actual brightness.

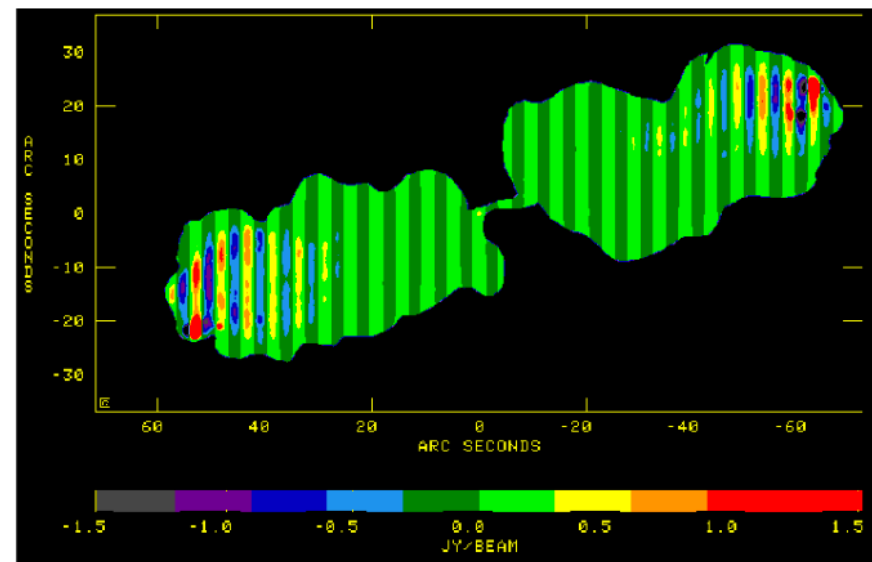
For a Real Source (Cygnus A = 3C405)

- Cygnus A is a powerful, nearby radio galaxy.
- The left panel shows the color-coded brightness at 2052 MHz. ($2''$ resⁿ)
- The other two panels show how 6.9 km EW and NS interferometers 'see' the source.



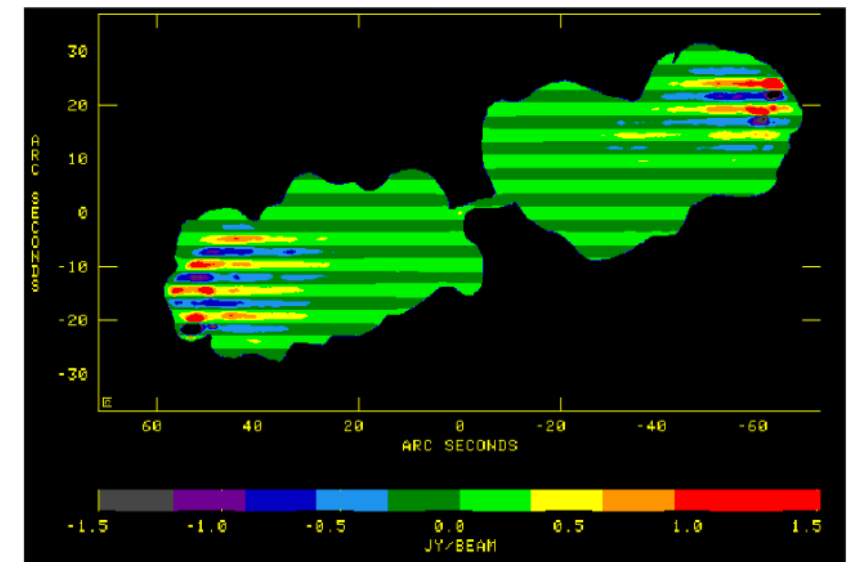
Zero-Spacing Image

Sum = 1029 Jy



6.3 km EW spacing

Sum = 70 Jy



6.3 km NS spacing

Sum = -19 Jy

- Dark green, blue, and purples => Negative correlation
- Light green, yellow, and reds => Positive correlation
- Don't be alarmed by the negative flux in the third panel.

So ... What Next?

- The interferometer casts a cosinusoidal pattern on the sky, with the result that we obtain a response which is some function of the source brightness and the fringe separation and orientation.
- By definition, 'Cosine' fringes have a peak passing through the central position of the source ('phase center').
- But in fact, something is missing. 'Cosine' fringes are not sufficient to allow recovery of the sky brightness.
- To answer why ...
- Time for some mathematics.... Starting with a seeming digression about odd and even functions.
- (All will be clear shortly...)

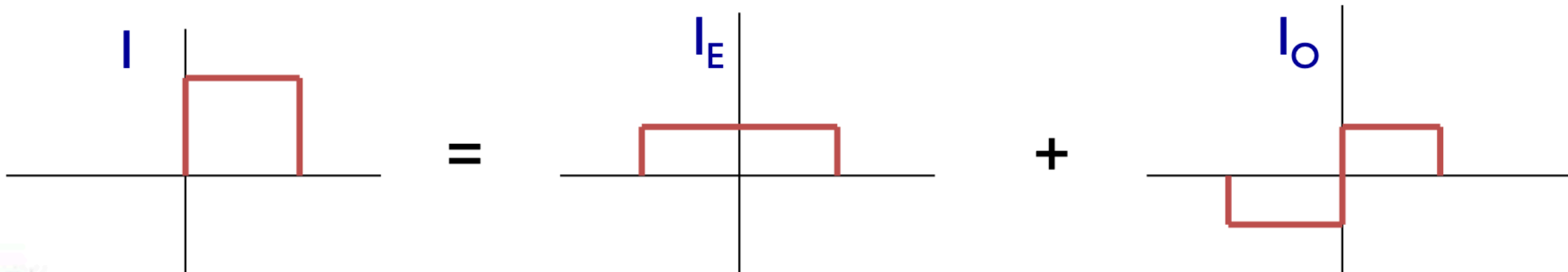
A Short Mathematics Digression – Odd and Even Functions

- Any real function, $I(x,y)$, can be expressed as the sum of two real functions which have specific symmetries:

$$I(x, y) = I_E(x, y) + I_O(x, y)$$

An even part: $I_E(x, y) = \frac{I(x, y) + I(-x, -y)}{2} = I_E(-x, -y)$

An odd part: $I_O(x, y) = \frac{I(x, y) - I(-x, -y)}{2} = -I_O(-x, -y)$



The Cosine Correlator is Blind to Odd Structure

- Suppose that the source of emission has a component with odd symmetry, for which

$$I_o(\mathbf{x}) = -I_o(-\mathbf{x})$$

- Since the cosine fringe pattern is even, the response of our interferometer to the odd brightness distribution is 0!

$$R_c = \iint I_o(\mathbf{s}) \cos(2\pi\nu \mathbf{b} \cdot \mathbf{s}/c) d\Omega = 0$$

- Thus, the cosine correlator response R_c :

$$R_c = \iint (I(\mathbf{s})) \cos(2\pi\nu \mathbf{b} \cdot \mathbf{s}/c) d\Omega = \iint (I_E(\mathbf{s})) \cos(2\pi\nu \mathbf{b} \cdot \mathbf{s}/c) d\Omega$$

is sensitive to the even component of the source structure.

Hence, we need more information if we are to completely recover the source brightness.

Thus: Two Correlations are Needed !!!

- The integration of the cosine response, R_c , over the source brightness is sensitive to only the even part of the brightness:

$$R_c = \iint I(\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega = \iint I_E(\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

since the integral of an odd function (I_o) with an even function ($\cos x$) is zero.

- To recover the 'odd' part of the brightness, I_o , we need an 'odd' fringe pattern.
- Let us replace the 'cos' with 'sin' in the integral, to get

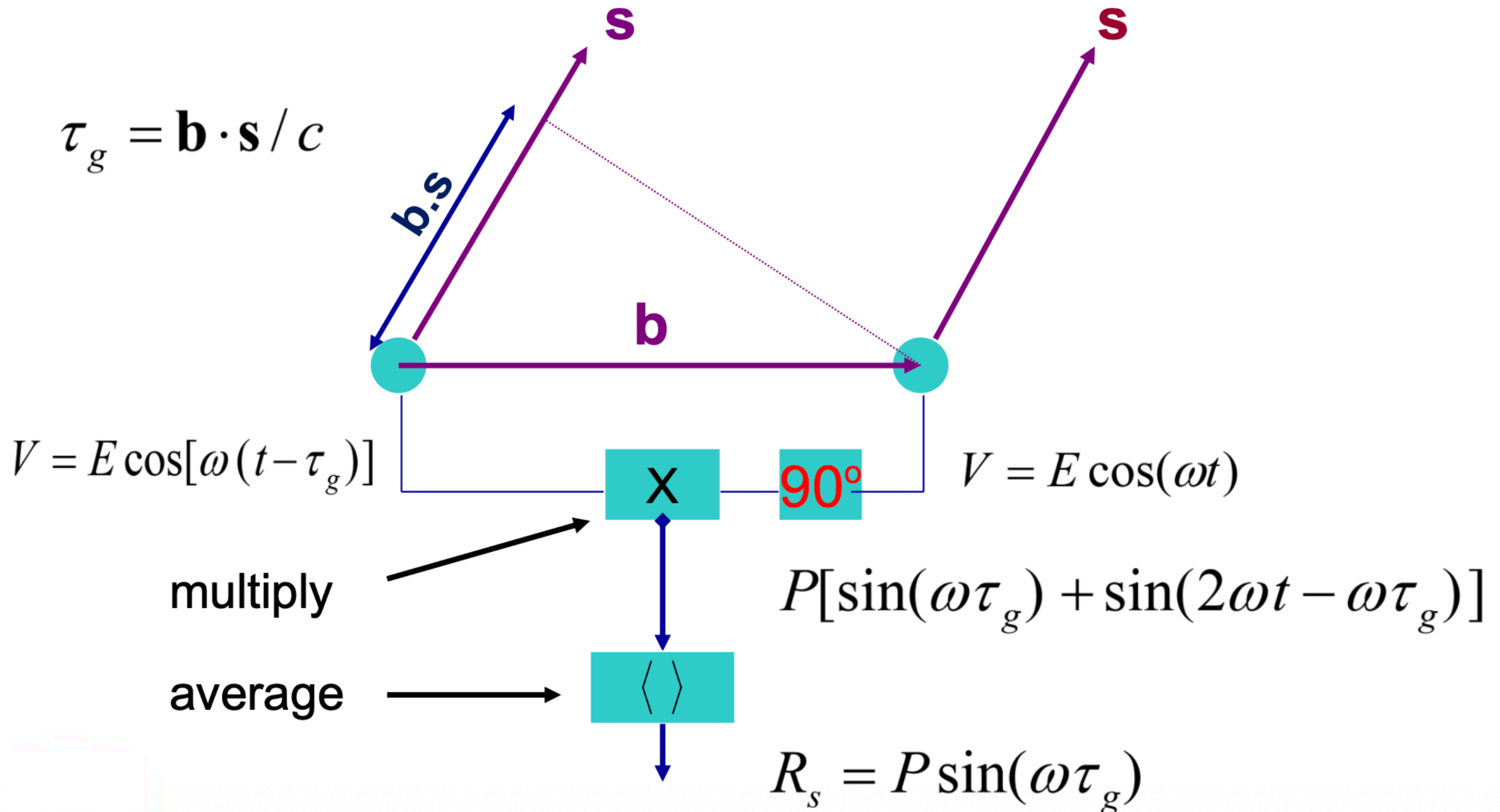
$$R_s = \iint I(\mathbf{s}) \sin(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega = \iint I_o(\mathbf{s}) \sin(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

since the integral of an even times an odd function is zero.

- To obtain this necessary component, we must make a 'sine' pattern.

Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.



Define the Complex Visibility

- We now DEFINE a complex function, the complex visibility, V , from the two independent (real) correlator outputs R_C and R_S :

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

- This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:

$$V_v(\mathbf{b}) = R_C - iR_S = \iint I_v(\mathbf{s}) e^{-2\pi i \nu \mathbf{b} \cdot \mathbf{s} / c} d\Omega$$

- Under some circumstances, this is a 2-D Fourier transform, giving us a well established way to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

The Complex Correlator and Complex Notation

- A correlator which produces both 'Real' and 'Imaginary' parts – or the Cosine and Sine fringes, is called a 'Complex Correlator'
 - For a complex correlator, think of two independent sets of projected sinusoids, separated by 1/4 fringe spacing.
 - In our scenario, both components are necessary, because we have assumed there is no motion – the 'fringes' are fixed on the source emission, which is itself stationary.
- The complex output of the complex correlator also means we can use complex analysis throughout: Let:

$$V_1 = A \cos(\omega t) \rightarrow A e^{-i\omega t}$$

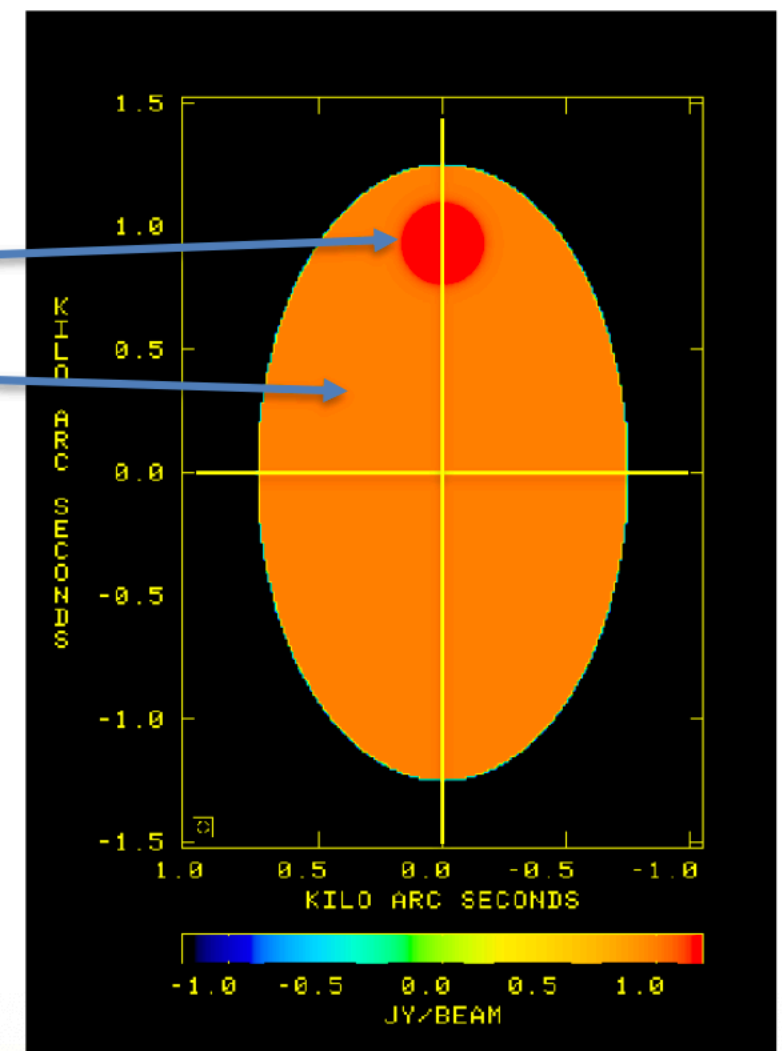
$$V_2 = A \cos[\omega(t - \mathbf{b} \cdot \mathbf{s} / c)] \rightarrow A e^{-i\omega(t - \mathbf{b} \cdot \mathbf{s} / c)}$$

- Then:

$$P_{corr} = \langle V_1 V_2^* \rangle = A^2 e^{-i\omega \mathbf{b} \cdot \mathbf{s} / c}$$

Visualizing Visibilities

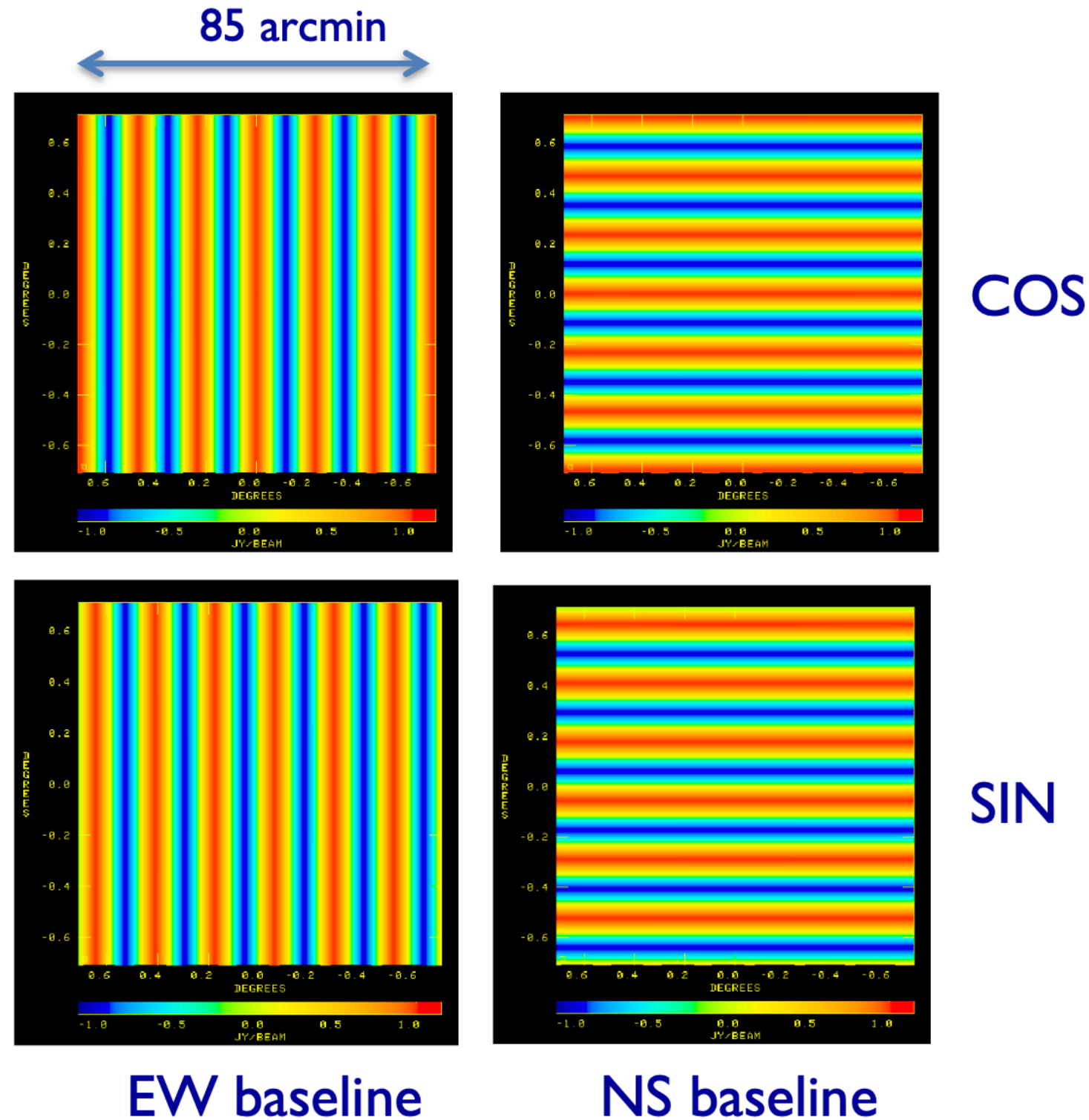
- An intimate familiarity with interferometer visibilities is essential in understanding how interferometers work.
- Fortunately, the concepts can be easily grasped through pictures.
- To illustrate, I have generated a mock source, consisting of a flat elliptical disk, (1500'' x 2500'') and a bright, circular gaussian 'spot' of width 150'' near one end.
 - Flux of disk: 1283 Jy.
 - Peak brightness of hotspot: 6 Jy/beam
 - Brightness of disk: 1 Jy/beam
 - Flux of hotspot: 56 Jy
 - Beamsize = 45 arcseconds
 - The odd color wedge is chosen to illustrate the main points
 - Color wedge runs from -1.25 Jy/beam (black) to +1.25 Jy/beam (red).



Our Mock Interferometer Response

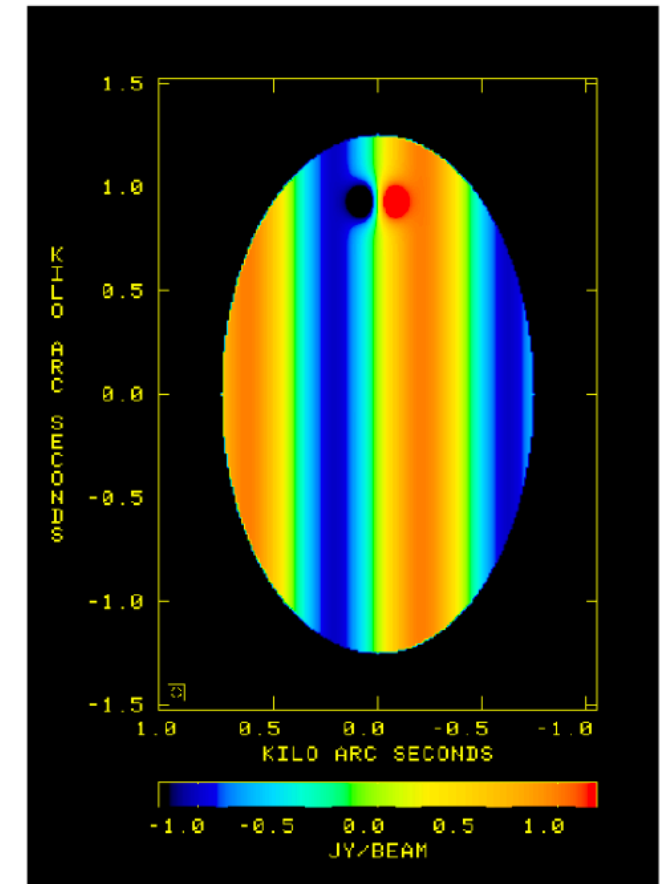
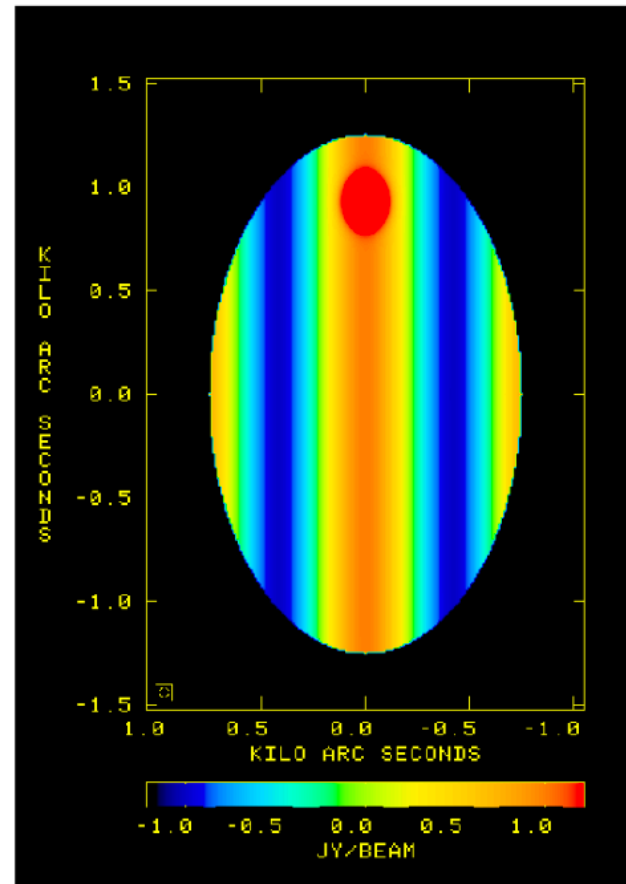
- We now 'observe' this source with an interferometer, with seven baselines running from 12 through 525 meters, at a frequency of 1471 MHz.
 - These numbers are appropriate for the MeerKAT interferometer, but can be scaled to any other.
- The mock interferometer has only EW and NS spacings.
- Shown here are the two fringe patterns (Cos and Sin) for the two 50-meter baselines.
- Fringe spacing = $841''$

$$= 300 \times 206265 / (1471 \times 50)$$



What does the interferometer 'do'?

- Recall that the interferometer multiplies the actual brightness by the fringe pattern (both COS and SIN), and integrates (adds) over the field of view.
- The complex visibility is made from these products as:
 - COS $\Rightarrow$ Real Part
 - SIN $\Rightarrow$ Imaginary Part
- Shown are the COS and SIN products for the 50-m EW baseline.



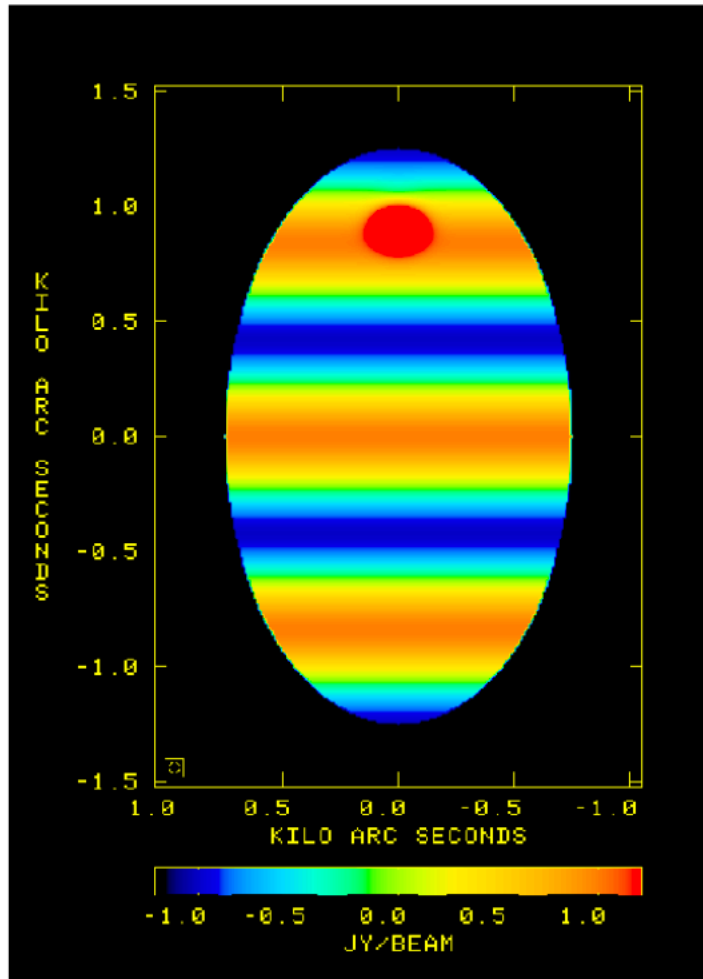
$$\text{Cos } \Sigma = -1.04 \text{ Jy} \quad \text{Sin } \Sigma = -0.14 \text{ Jy}$$

Thus: $A = 1.04 \text{ Jy}$, $\phi = 180 \text{ degrees}$.

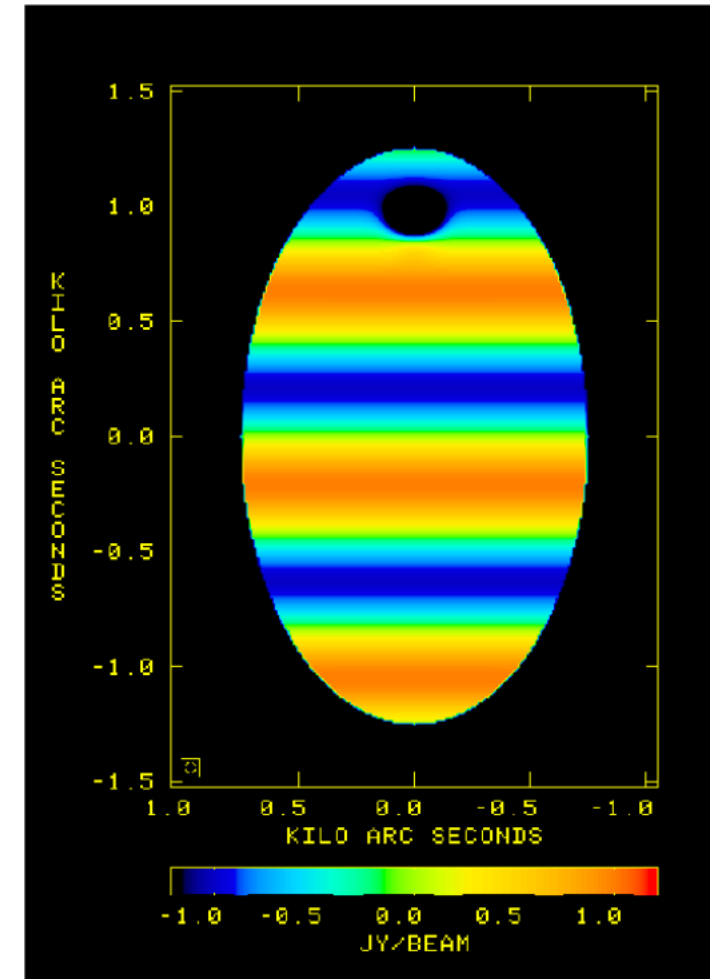
NB: The EW even symmetry requires the SIN integral = 0, so the phase must be either 0 or 180.

While for the NS baseline...

- The N-S asymmetry of the model means the NS baselines 'see' a wider range in the SIN (imaginary) component:



$$\text{Cos } \Sigma = 91.9 \text{ Jy}$$



$$\text{Sin } \Sigma = -30.6 \text{ Jy}$$

Thus: $A = 96.9 \text{ Jy}$, $\phi = -18.4 \text{ degrees}$.

NB: There are no symmetries here, so phases can be any value.

Visibilities as a function of baseline length

- Showing a sequence as the E-W baselines gets longer:

Baseline: 12.5m

25m

50m

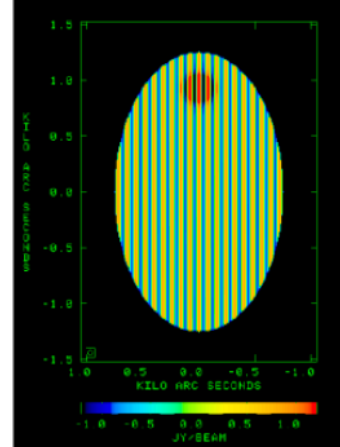
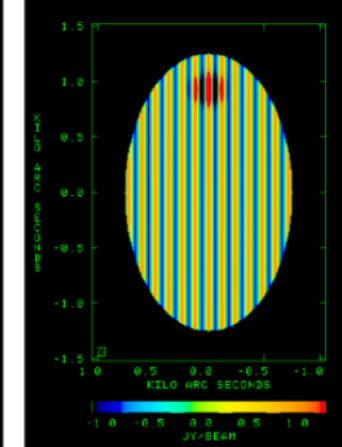
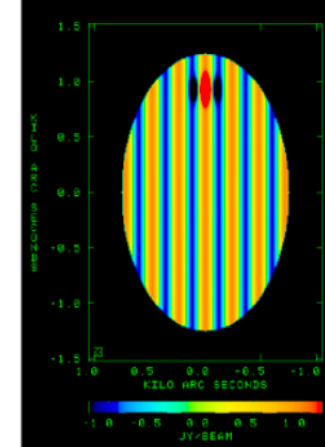
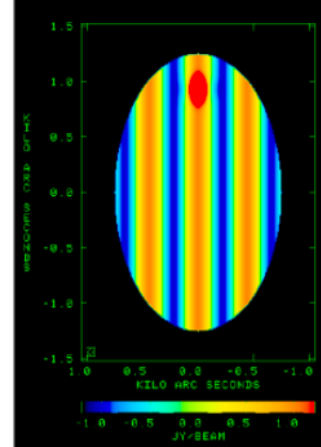
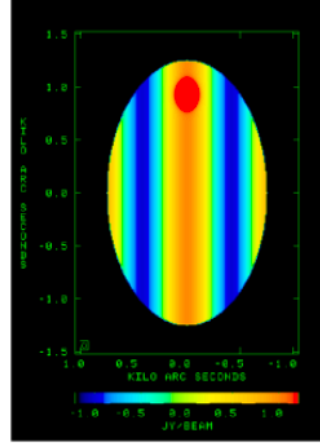
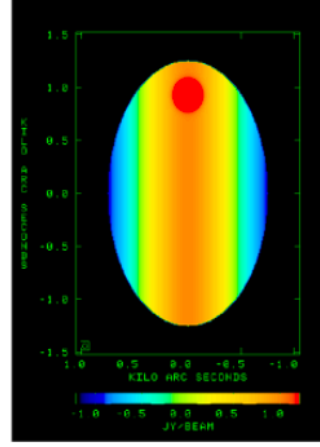
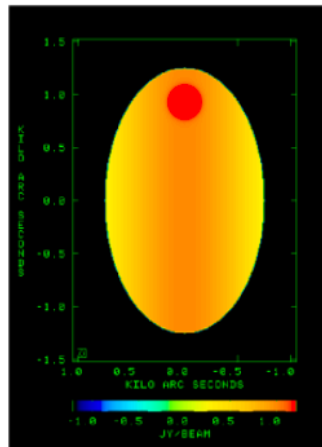
95m

175m

350m

525m

COS



Σ

1069

429

-104

8.6

13.3

6.0

5.15

Δ

1069

429

104

8.6

13.3

6.0

5.15

ϕ

-0.3

0.1

180

0.0

0.0

0.0

0.0

Σ

-6.3

0.6

-0.1

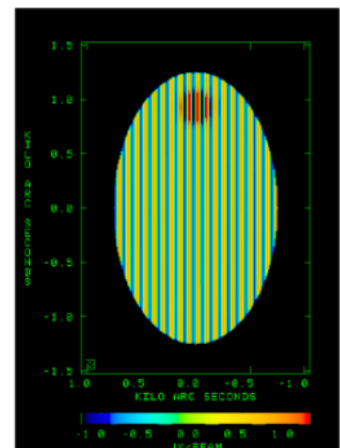
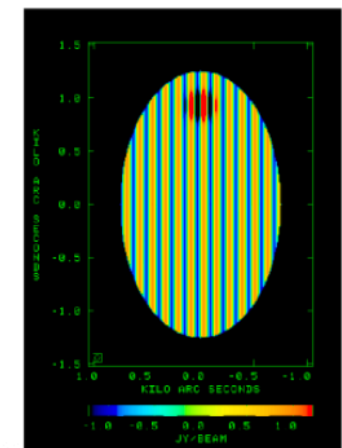
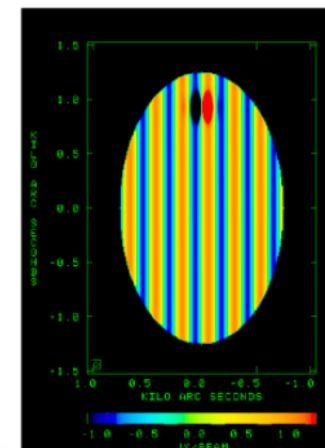
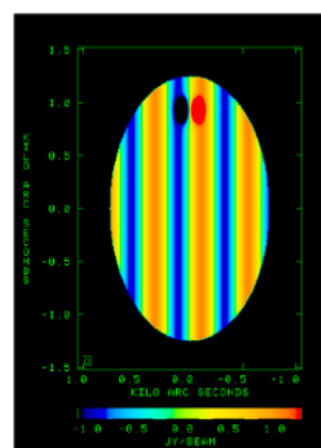
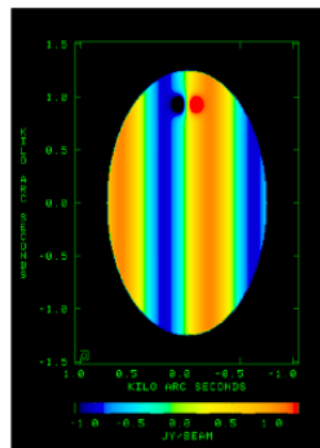
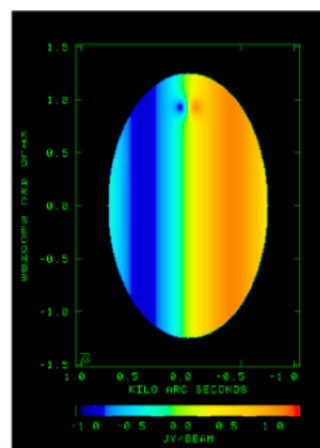
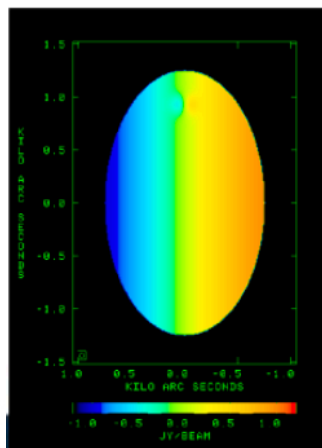
0.0

0.1

0.0

0.1

SIN



- Note that the 'SIN' fluxes are all very low, so the visibility phases are $\sim$ zero or 180 degrees.

As a function of N-S baseline ...

- Showing a sequence as the baseline gets longer:

Baseline: 12.5m

25m

50m

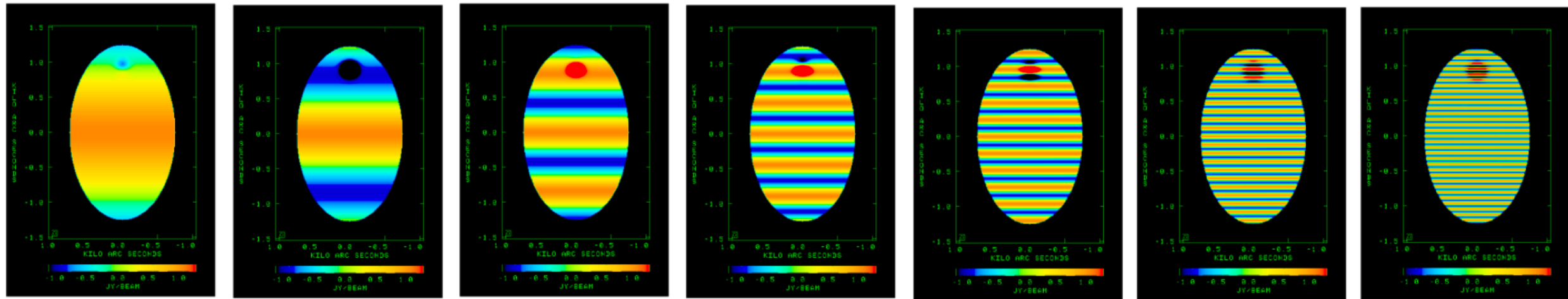
95m

175m

350m

525m

COS



Σ

624

-201

91.9

4.2

14.9

3.5

6.8

Δ

627

202

96.9

21.2

17.7

3.5

6.8

ϕ

-5.4

175

-18

-78

34

3.6

0

Σ

-59

17.4

-30.6

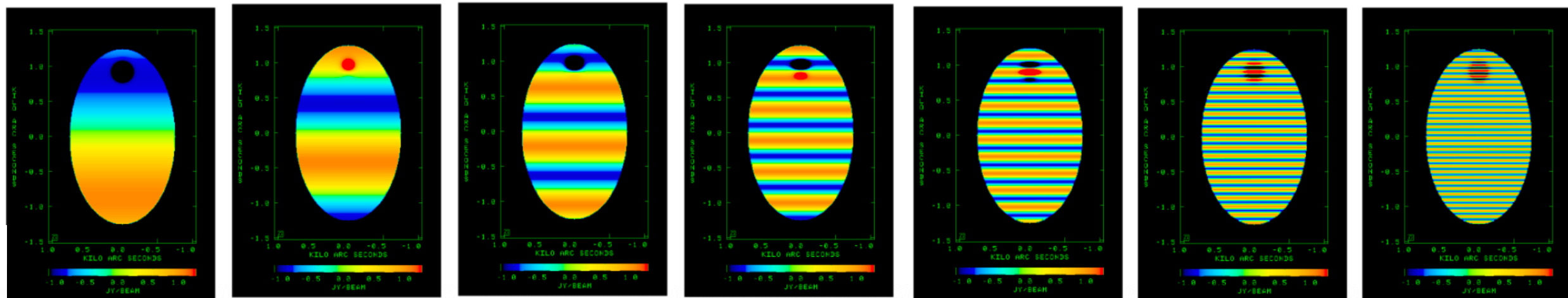
-20.8

10.0

0.2

0.0

SIN



- Note that the 'SIN' fluxes are now much larger, and the visibility phases far from zero or 180.

Some Take-Aways...

- In general, as the baseline gets longer, the visibility amplitude declines.
- The decline need not be smooth – complicated emission will often have an oscillating, decaying, visibility function.
- The visibility from a ‘zero-spacing’ interferometer (aka ‘a single dish’) equals the total flux density of the source.
 - For Stokes ‘I’, this cannot be less than all visibilities at longer spacings.
- The visibilities for a source with even symmetry about some axis must be real (phase 0 or 180) for fringes parallel to that axis.
 - An extreme case of this is a smooth circular source: The emission is even about all axes (through the center), so all visibilities are real.
- The visibilities for a source with odd symmetry about some axis must be imaginary (phase 90 or -90) for fringes parallel to that axis.
 - This is not possible for Stokes ‘I’, but is possible for Q, U, or V.

Some Thoughts to Ponder

- The complex visibility **amplitude** is independent of the source location*, and linearly related to source flux density.
- The complex visibility **phase** is a function of source location, and independent of source flux density.
- These two statements, restated, are:
 - 1) Doubling the source brightness doubles the visibility amplitude, but doesn't change the visibility phase
 - 2) Shifting the source position changes the phase, but does not change the visibility amplitude.*
- Reversing the elements of an interferometer negates the phase of the complex visibility, and leaves the amplitude unchanged.
- For those of you familiar with Fourier transforms, the equivalent statement is that:
 - 'As the source brightness is a real function, its Fourier transform is Hermitian'.

* Ignoring any attenuation due to the primary beam.

Visibilities Lead to Images ...

- The Visibility is a unique function of the source brightness.
- The two functions are related through a Fourier transform. $V_v(u, v) \Leftrightarrow I(l, m)$
- How we go from visibilities to images is the subject of a later lecture.
- An interferometer, at any one time, makes one measure of the visibility, at baseline coordinate (u,v).
- 'A sufficient number of measures' of the visibility function (as derived from an interferometer) will provide us a 'reasonable estimate' of the source brightness.
- How many is 'sufficient', and how good is 'reasonable'?
- These simple questions do not have easy answers...

Final Comments ...

- The formalism presented here presumes much ... including that there is no motion between source and interferometer.
- Real interferometers:
 - Are on a rotating platform
 - Use wide bandwidths
 - Average data over time
 - Employ frequency downconversion
 - Have to deal with corruptions due to many causes...

Michelson–Morley experiment

(For fun I include all the nitty-gritty!)

The **Michelson–Morley experiment** was an attempt to measure the motion of the Earth relative to the luminiferous aether, a supposed medium permeating space that was thought to be the carrier of light waves.

The experiment was performed between April and July 1887 by American physicists Albert A. Michelson and Edward W. Morley at what is now Case Western Reserve University in Cleveland, Ohio.

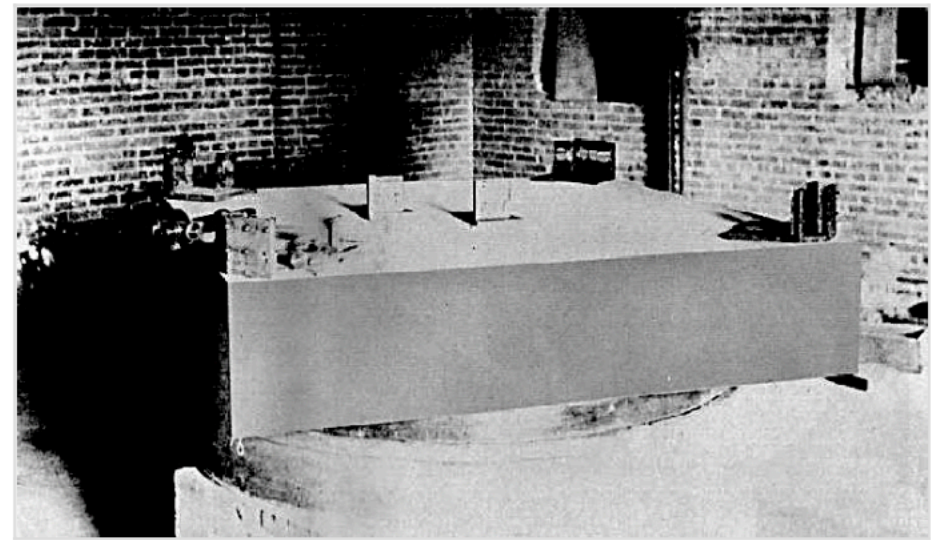
The experiment compared the speed of light in perpendicular directions in an attempt to detect the relative motion of matter, including their laboratory, through the luminiferous aether, or "aether wind" as it was sometimes called.

The result was negative, in that Michelson and Morley found no significant difference between the speed of light in the direction of movement through the presumed aether, and the speed at right angles.

This result is generally considered to be the first strong evidence against some ether theories, as well as initiating a line of research that eventually led to special relativity, which rules out motion against an aether.

Of this experiment, Albert Einstein wrote, "If the Michelson–Morley experiment had not brought us into serious embarrassment, no one would have regarded the relativity theory as a (halfway) redemption.

Michelson–Morley type experiments have been repeated many times with steadily increasing sensitivity.



Michelson and Morley's interferometric setup, mounted on a stone slab that floats in an annular trough of mercury.

These include experiments from 1902 to 1905, and a series of experiments in the 1920s.

More recently, in 2009, optical resonator experiments confirmed the absence of any aether wind at the 10^{-17} level.

Together with the Ives–Stilwell and Kennedy–Thorndike experiments, Michelson–Morley type experiments form one of the fundamental tests of special relativity.

Detecting the aether

Physics theories of the 19th century assumed that just as surface water waves must have a supporting substance, i.e., a "medium", to move across (in this case water), and audible sound requires a medium to transmit its wave motions (such as air or water), so light must also require a medium, the “luminiferous aether”, to transmit its wave motions.

Because light can travel through a vacuum, it was assumed that even a vacuum must be filled with aether.

Because the speed of light is so great, and because material bodies pass through the aether without obvious friction or drag, it was assumed to have a highly unusual combination of properties.

Designing experiments to investigate these properties was a high priority of 19th-century physics.

Earth orbits around the Sun at a speed of around 30 km/s (19 mi/s), or 107,000 km/h (66,000 mph).

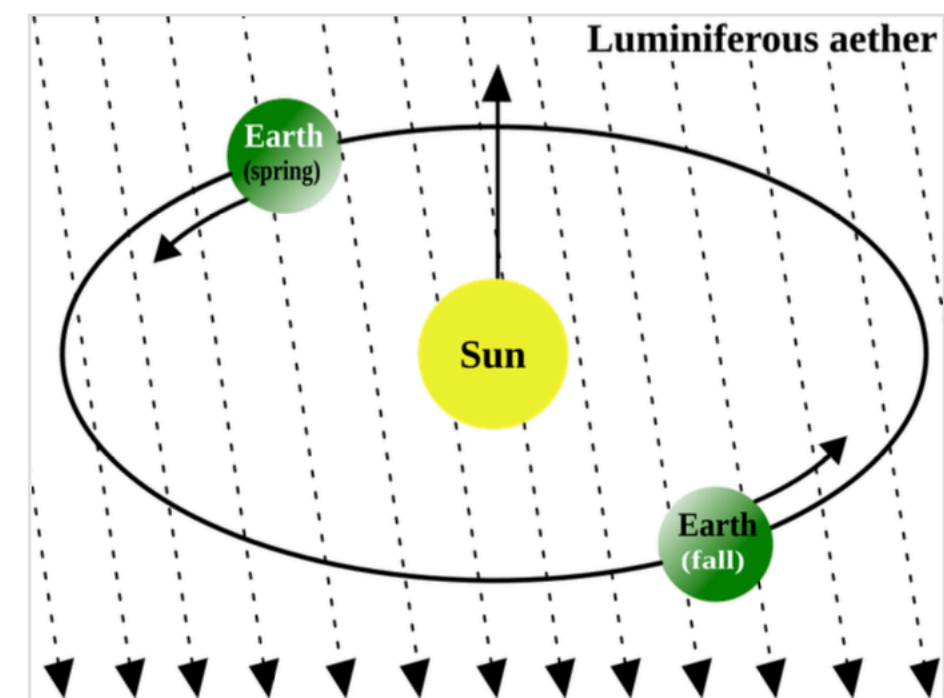
The Earth is in motion, so two main possibilities were considered: (1) The aether is stationary and only partially dragged by Earth (proposed by Augustin-Jean Fresnel in 1818), or (2) the aether is completely dragged by Earth and thus shares its motion at Earth's surface (proposed by George Stokes in 1844).

In addition, James Clerk Maxwell (1865) recognized the electromagnetic nature of light and developed what are now called Maxwell's equations, but these equations were still interpreted as describing the motion of waves through an aether, whose state of motion was unknown.

Eventually, Fresnel's idea of an (almost) stationary aether was preferred because it appeared to be confirmed by the Fizeau experiment (1851) and the aberration of star light.

According to the stationary and the partially dragged aether hypotheses, Earth and the aether are in relative motion, implying that a so-called "aether wind" (Figure) should exist.

Although it would be theoretically possible for the Earth's motion to match that of the aether at one moment in time, it was not possible for the Earth to remain at rest with respect to the aether at all times, because of the variation in both the direction and the speed of the motion.



A depiction of the concept of the "aether wind"

At any given point on the Earth's surface, the magnitude and direction of the wind would vary with time of day and season.

By analyzing the return speed of light in different directions at various different times, it was thought to be possible to measure the motion of the Earth relative to the aether.

The expected relative difference in the measured speed of light was quite small, given that the velocity of the Earth in its orbit around the Sun has a magnitude of about one hundredth of one percent of the speed of light.

During the mid-19th century, measurements of aether wind effects of first order, i.e., effects proportional to v/c (v being Earth's velocity, c the speed of light) were thought to be possible, but no direct measurement of the speed of light was possible with the accuracy required.

For instance, the Fizeau wheel could measure the speed of light to perhaps 5% accuracy, which was quite inadequate for measuring directly a first-order 0.01% change in the speed of light.

A number of physicists therefore attempted to make measurements of indirect first-order effects not of the speed of light itself, but of variations in the speed of light.

The Hoek experiment, for example, was intended to detect interferometric fringe shifts due to speed differences of oppositely propagating light waves through water at rest.

The results of such experiments were all negative.

This could be explained by using Fresnel's dragging coefficient, according to which the aether and thus light are partially dragged by moving matter.

Partial aether-dragging would thwart attempts to measure any first order change in the speed of light.

As pointed out by Maxwell (1878), only experimental arrangements capable of measuring second order effects would have any hope of detecting aether drift, i.e., effects proportional to v^2/c^2 .

Existing experimental setups, however, were not sensitive enough to measure effects of that size.

1881 and 1887 experiments

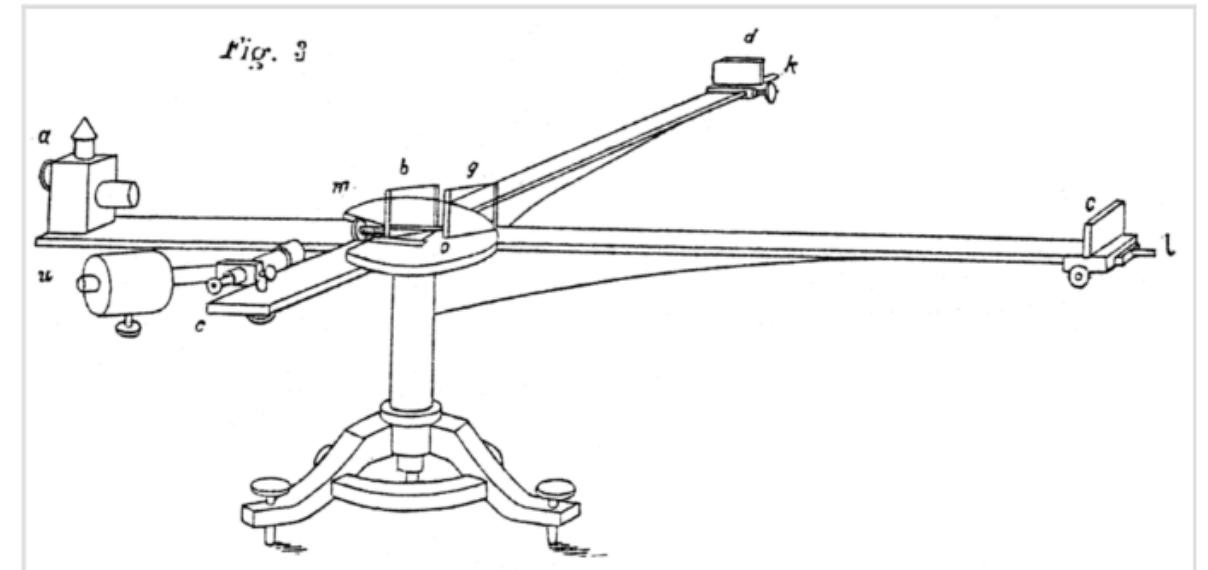
Michelson experiment (1881)

Michelson had a solution to the problem of how to construct a device sufficiently accurate to detect aether flow.

In 1877, while teaching at his alma mater, the United States Naval Academy in Annapolis, Michelson conducted his first known light speed experiments as a part of a classroom demonstration.

In 1881, he left active U.S. Naval service while in Germany concluding his studies.

In that year, Michelson used a prototype experimental device to make several more measurements.



Michelson's 1881 interferometer. Although ultimately it proved incapable of distinguishing between differing theories of aether-dragging, its construction provided important lessons for the design of Michelson and Morley's 1887 instrument.

The device he designed, later known as a Michelson interferometer, sent yellow light from a sodium flame (for alignment), or white light (for the actual observations), through a half-silvered mirror that was used to split it into two beams traveling at right angles to one another.

After leaving the splitter, the beams traveled out to the ends of long arms where they were reflected back into the middle by small mirrors.

They then recombined on the far side of the splitter in an eyepiece, producing a pattern of constructive and destructive interference whose transverse displacement would depend on the relative time it takes light to transit the longitudinal vs. the transverse arms.

If the Earth is traveling through an aether medium, a light beam traveling parallel to the flow of that aether will take longer to reflect back and forth than would a beam traveling perpendicular to the aether, because the increase in elapsed time from traveling against the aether wind is more than the time saved by traveling with the aether wind.

Michelson expected that the Earth's motion would produce a fringe shift equal to 0.04 fringes—that is, of the separation between areas of the same intensity.

He did not observe the expected shift; the greatest average deviation that he measured (in the northwest direction) was only 0.018 fringes; most of his measurements were much less.

His conclusion was that Fresnel's hypothesis of a stationary aether with partial aether dragging would have to be rejected, and thus he confirmed Stokes' hypothesis of complete aether dragging.

However, Alfred Potier (and later Hendrik Lorentz) pointed out to Michelson that he had made an error of calculation, and that the expected fringe shift should have been only 0.02 fringes.

Michelson's apparatus was subject to experimental errors far too large to say anything conclusive about the aether wind.

Definitive measurement of the aether wind would require an experiment with greater accuracy and better controls than the original.

Nevertheless, the prototype was successful in demonstrating that the basic method was feasible.

Michelson–Morley experiment (1887)

In 1885, Michelson began a collaboration with Edward Morley, spending considerable time and money to confirm with higher accuracy Fizeau's 1851 experiment on Fresnel's drag coefficient, to improve on Michelson's 1881 experiment, and to establish the wavelength of light as a standard of length.

John Brashear made the high-quality optics for the Interferometer in his Allegheny-Observatory-affiliated shop.

At this time Michelson was professor of physics at the Case School of Applied Science, and Morley was professor of chemistry at Western Reserve University (WRU), which shared a campus with the Case School on the eastern edge of Cleveland.

Michelson suffered a mental health crisis in September 1885, from which he recovered by October 1885.

Morley ascribed this breakdown to the intense work of Michelson during the preparation of the experiments.

In 1886, Michelson and Morley successfully confirmed Fresnel's drag coefficient—this result was also considered as a confirmation of the stationary aether concept.

This result strengthened their hope of finding the aether wind.

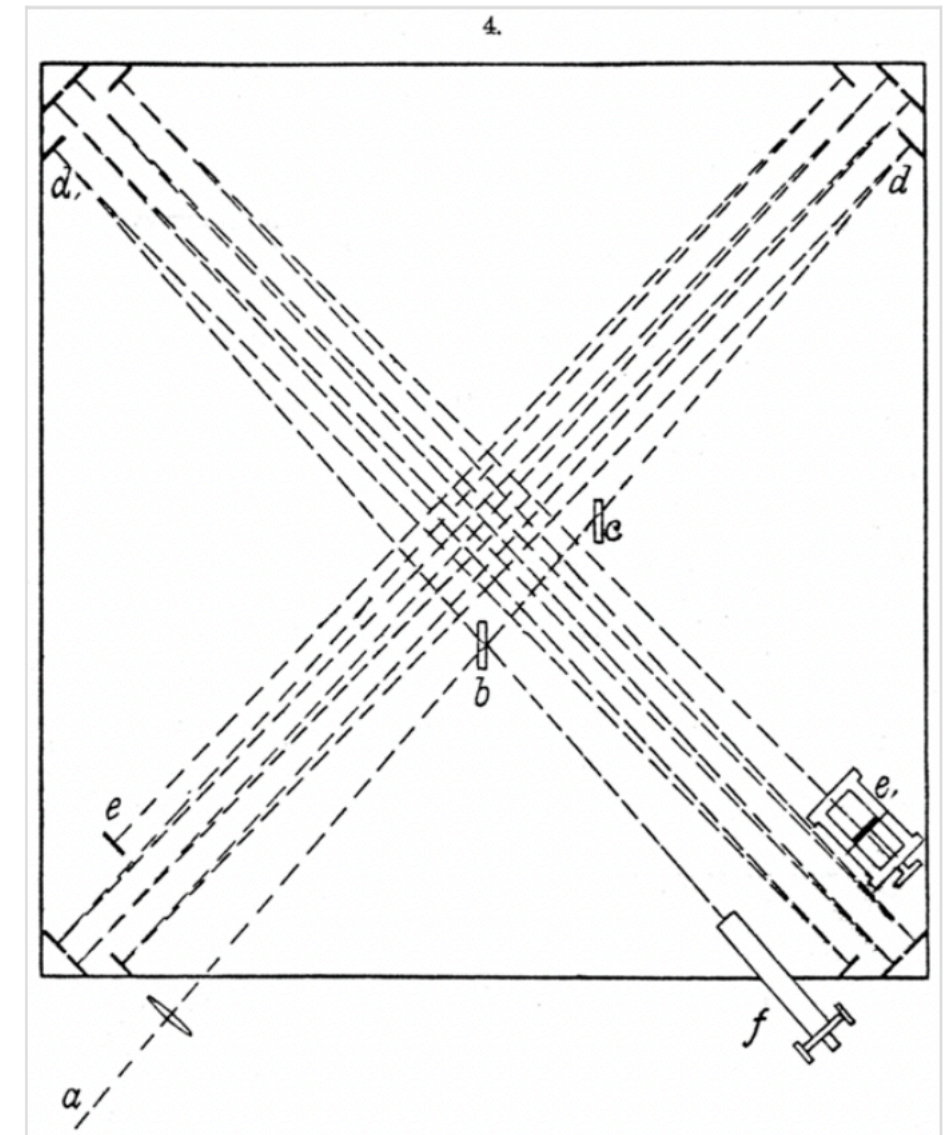
Michelson and Morley created an improved version of the Michelson experiment with more than enough accuracy to detect this hypothetical effect.

The experiment was performed in several periods of concentrated observations between April and July 1887.

As shown in the diagram to the right, the light was repeatedly reflected back and forth along the arms of the interferometer, increasing the path length to 11 m (36 ft).

At this length, the drift would be about 0.4 fringes.

To make that easily detectable, the apparatus was assembled in a closed room in the basement of the heavy stone dormitory, eliminating most thermal and vibrational effects.



This diagram illustrates the folded light path used in the Michelson–Morley interferometer that enabled a path length of 11 m. *a* is the light source, an oil lamp. *b* is a beam splitter. *c* is a compensating plate so that both the reflected and transmitted beams travel through the same amount of glass (important since experiments were run with white light which has an extremely short coherence length requiring precise matching of optical path lengths for fringes to be visible;

monochromatic sodium light was used only for initial alignment———, *d*, *d'* and *e* are mirrors. *e'* is a fine adjustment mirror. *f* is a telescope.

Vibrations were further reduced by building the apparatus on top of a large block of sandstone (earlier figure), about a foot thick and five feet (1.5 m) square, which was then floated in a circular trough of mercury.

They estimated that effects of about 0.01 fringe would be detectable.

Michelson and Morley and other early experimentalists using interferometric techniques in an attempt to measure the properties of the luminiferous aether, used (partially) monochromatic light only for initially setting up their equipment, always switching to white light for the actual measurements.

The reason is that measurements were recorded visually.

Purely monochromatic light would result in a uniform fringe pattern.

Lacking modern means of environmental temperature control, experimentalists struggled with continual fringe drift even when the interferometer was set up in a basement.

Because the fringes would occasionally disappear due to vibrations caused by passing horse traffic, distant thunderstorms and the like, an observer could easily "get lost" when the fringes returned to visibility.

The advantages of white light, which produced a distinctive colored fringe pattern, far outweighed the difficulties of aligning the apparatus due to its low coherence length.

As Dayton Miller wrote, "White light fringes were chosen for the observations because they consist of a small group of fringes having a central, sharply defined black fringe which forms a permanent zero reference mark for all readings.

Use of partially monochromatic light (yellow sodium light) during initial alignment enabled the researchers to locate the position of equal path length, more or less easily, before switching to white light.

The mercury trough allowed the device to turn with close to zero friction, so that once having given the sandstone block a single push it would slowly rotate through the entire range of possible angles to the "aether wind", while measurements were continuously observed by looking through the eyepiece.

The hypothesis of aether drift implies that because one of the arms would inevitably turn into the direction of the wind at the same time that another arm was turning perpendicularly to the wind, an effect should be noticeable even over a period of minutes.

The expectation was that the effect would be graphable as a sine wave with two peaks and two troughs per rotation of the device.

This result could have been expected because during each full rotation, each arm would be parallel to the wind twice (facing into and away from the wind giving identical readings) and perpendicular to the wind twice.

Additionally, due to the Earth's rotation, the wind would be expected to show periodic changes in direction and magnitude during the course of a sidereal day.

Because of the motion of the Earth around the Sun, the measured data were also expected to show annual variations.

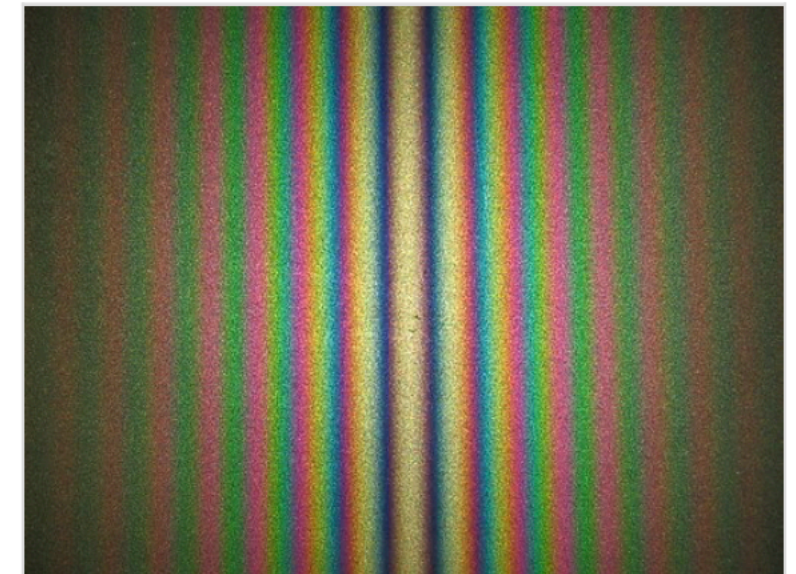
Most famous "failed" experiment

After all this thought and preparation, the experiment became what has been called the most famous failed experiment in history.

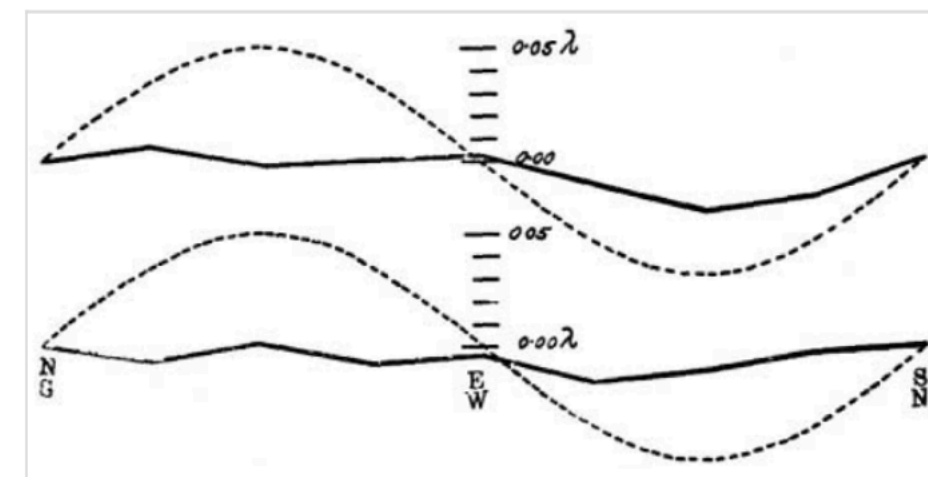
Instead of providing insight into the properties of the aether, Michelson and Morley's article in the American Journal of Science reported the measurement to be as small as one-fortieth of the expected displacement, but "since the displacement is proportional to the square of the velocity" they concluded that the measured velocity was "probably less than one-sixth" of the expected velocity of the Earth's motion in orbit and "certainly less than one-fourth".

Although this small "velocity" was measured, it was considered far too small to be used as evidence of speed relative to the aether, and it was understood to be within the range of an experimental error that would allow the speed to actually be zero.

For instance, Michelson wrote about the "decidedly negative result" in a letter to Lord Rayleigh in August 1887:



Fringe pattern produced with a Michelson interferometer using white light. As configured here, the central fringe is white rather than black.



Michelson and Morley's results. The upper solid line is the curve for their observations at noon, and the lower solid line is that for their evening observations. Note that the theoretical curves and the observed curves are not plotted at the same scale: the dotted curves, in fact, represent only one-eighth of the theoretical displacements.

The Experiments on the relative motion of the earth and ether have been completed and the result decidedly negative. The expected deviation of the interference fringes from the zero should have been 0.40 of a fringe—the maximum displacement was 0.02 and the average much less than 0.01—and then not in the right place. As displacement is proportional to squares of the relative velocities it follows that if the ether does slip past the relative velocity is less than one sixth of the earth's velocity.

From the standpoint of the then current aether models, the experimental results were conflicting.

The Fizeau experiment and its 1886 repetition by Michelson and Morley apparently confirmed the stationary aether with partial aether dragging, and refuted complete aether dragging.

On the other hand, the much more precise Michelson–Morley experiment (1887) apparently confirmed complete aether dragging and refuted the stationary aether.

In addition, the Michelson–Morley null result was further substantiated by the null results of other second-order experiments of different kind, namely the Trouton–Noble experiment (1903) and the experiments of Rayleigh and Brace (1902–1904).

These problems and their solution led to the development of the Lorentz transformation and special relativity.

After the "failed" experiment Michelson and Morley ceased their aether drift measurements and started to use their newly developed technique to establish the wavelength of light as a standard of length.

Light path analysis and consequences

Observer resting in the aether

The beam travel time in the longitudinal direction can be derived as follows: Light is sent from the source and propagates with the speed of light in the aether.

It passes through the half-silvered mirror at the origin at $T = 0$.

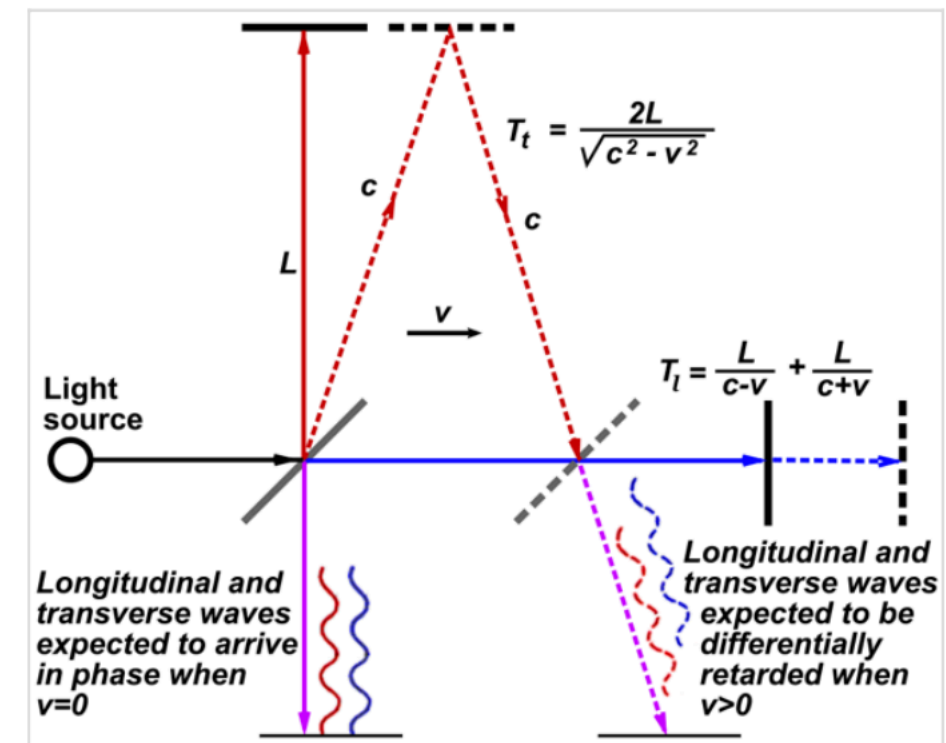
The reflecting mirror is at that moment at distance L (the length of the interferometer arm) and is moving with velocity v .

The beam hits the mirror at time T_1 and thus travels the distance $c T_1$.

At this time, the mirror has traveled the distance $v T_1$.

Thus $c T_1 = L + v T_1$ and consequently the travel time $T_1 = L/(c-v)$.

The same consideration applies to the backward journey, with the sign of v reversed, resulting in $cT_2 = L - vT_2$ and $T_2 = L/(c+v)$.



Expected differential phase shift between light traveling the longitudinal versus the transverse arms of the Michelson–Morley apparatus

The total travel time $T_\ell = T_1 + T_2$ is:

$$T_\ell = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c} \frac{1}{1 - \frac{v^2}{c^2}} \approx \frac{2L}{c} \left(1 + \frac{v^2}{c^2} \right)$$

Michelson obtained this expression correctly in 1881, however, in transverse direction he obtained the incorrect expression

$$T_t = \frac{2L}{c},$$

because he overlooked the increase in path length in the rest frame of the aether.

This was corrected by Alfred Potier (1882) and Hendrik Lorentz (1886).

The derivation in the transverse direction can be given as follows (analogous to the derivation of time dilation using a light clock):

The beam is propagating at the speed of light c and hits the mirror at time T_3 , traveling the distance cT_3 .

At the same time, the mirror has traveled the distance vT_3 in the x direction.

So in order to hit the mirror, the travel path of the beam is L in the y direction (assuming equal-length arms) and vT_3 in the x direction.

This inclined travel path follows from the transformation from the interferometer rest frame to the aether rest frame.

Therefore, the Pythagorean theorem gives the actual beam travel distance of $\sqrt{L^2 + (vT_3)^2}$.

Thus $cT_3 = \sqrt{L^2 + (vT_3)^2}$ and consequently the travel time $T_3 = L/\sqrt{c^2 - v^2}$ which is the same for the backward journey.

The total travel time $T_t = 2T_3$ is:

$$T_t = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx \frac{2L}{c} \left(1 + \frac{v^2}{2c^2} \right)$$

The time difference between T_ℓ and T_t is given by

$$T_\ell - T_t = \frac{2L}{c} \left(\frac{1}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

To find the path difference, simply multiply by c ;

$$\Delta\lambda_1 = 2L \left(\frac{1}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

The path difference is denoted by $\Delta\lambda$ because the beams are out of phase by a some number of wavelengths (λ).

To visualize this, consider taking the two beam paths along the longitudinal and transverse plane, and lying them straight.

One path will be longer than the other, this distance is $\Delta\lambda$.

Alternatively, consider the rearrangement of the speed of light formula $c\Delta T = \Delta\lambda$.

If the relation $v^2/c^2 \ll 1$ is true (if the velocity of the aether is small relative to the speed of light), then the expression can be simplified using a first order binomial expansion;

$$(1 - x)^n \approx 1 - nx$$

So, rewriting the above in terms of powers;

$$\Delta\lambda_1 = 2L \left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \right)$$

Applying binomial simplification;

$$\Delta\lambda_1 = 2L \left(\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{v^2}{2c^2}\right) \right) = 2L \frac{v^2}{2c^2}$$

Therefore;

$$\Delta\lambda_1 = L \frac{v^2}{c^2}$$

The derivation above shows that the presence of an aether wind would produce a difference in optical path lengths between the two arms of the interferometer.

This path difference depends on the orientation of the interferometer relative to the aether wind.

Specifically, the derivation assumes that the longitudinal arm is aligned parallel to the presumed direction of the aether wind.

If instead the longitudinal arm is oriented perpendicular to the aether wind, the resulting path difference would have the opposite sign.

The magnitude of the path difference can vary continuously and may represent any fraction of the wavelength, depending on both the angle between the apparatus and the aether wind and the wind's speed.

To detect the existence of the aether, Michelson and Morley aimed to observe a "fringe shift" in the interference pattern.

The underlying principle is straightforward: when the interferometer is rotated by 90° , the roles of the two arms are exchanged, altering the path difference due to the aether wind.

The fringe shift is determined by calculating the difference in path differences between the two orientations, and then dividing that value by the wavelength.

$$n = \frac{\Delta\lambda_1 - \Delta\lambda_2}{\lambda} \approx \frac{2Lv^2}{\lambda c^2}.$$

Note the difference between $\Delta\lambda$, which is some number of wavelengths, and λ which is a single wavelength.

As can be seen by this relation, fringe shift n is a unitless quantity.

Since $L \approx 11$ meters and $\lambda \approx 500$ nanometers, the expected fringe shift was $n \approx 0.44$.

The negative result led Michelson to the conclusion that there is no measurable aether drift.

However, he never accepted this on a personal level, and the negative result haunted him for the rest of his life.

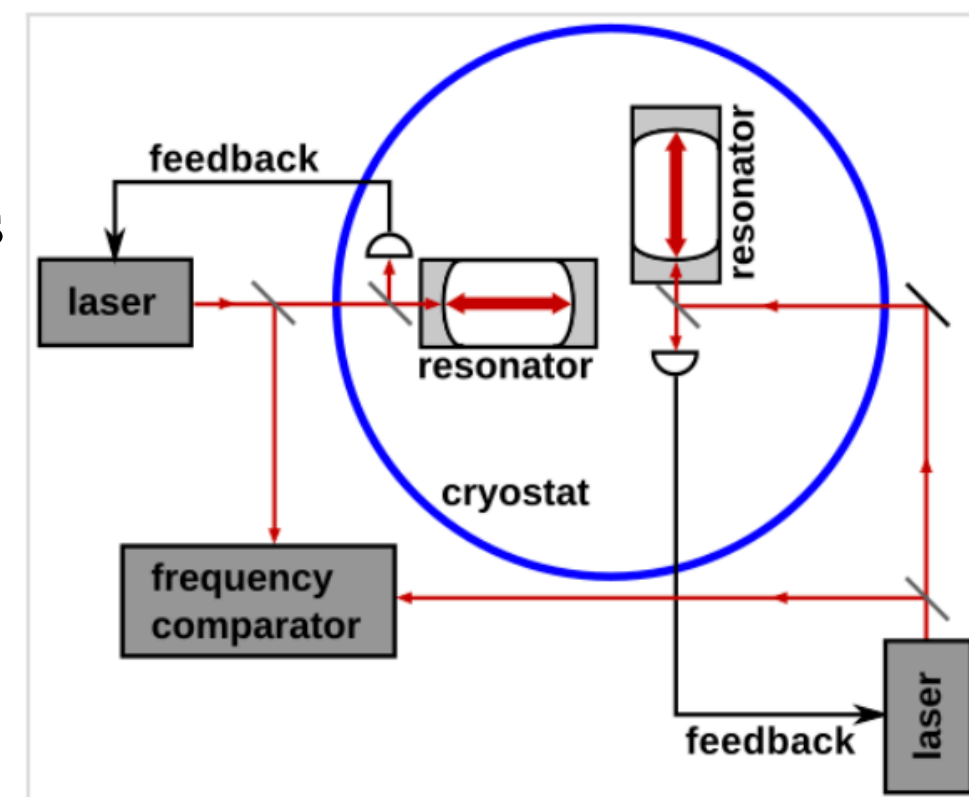
Recent optical resonator experiments

During the early 21st century, there has been a resurgence in interest in performing precise Michelson–Morley type experiments using lasers, masers, cryogenic optical resonators, etc.

This is in large part due to predictions of quantum gravity that suggest that special relativity may be violated at scales accessible to experimental study.

The first of these highly accurate experiments was conducted by Brillet & Hall (1979), in which they analyzed a laser frequency stabilized to a resonance of a rotating optical Fabry–Pérot cavity.

They set a limit on the anisotropy of the speed of light resulting from the Earth's motions of $\Delta c/c \approx 10^{-15}$, where Δc is the difference between the speed of light in the x- and y-directions.



Michelson–Morley experiment with cryogenic optical resonators of a form such as was used by Müller et al. (2003).

As of 2015, optical and microwave resonator experiments have improved this limit to $\Delta c/c \approx 10^{-18}$.

In some of them, the devices were rotated or remained stationary, and some were combined with the Kennedy–Thorndike experiment.

In particular, Earth's Michelson–Morley experiment with cryogenic optical resonators of a form such as was used by Müller et al. (2003). direction and velocity (ca. 368 km/s (229 mi/s)) relative to the CMB rest frame are ordinarily used as references in these searches for anisotropies.