Some Review, Some New Details and Explicit Time Evolution

Quantum Theory - How it works in general …… And some new details

Question: **How do we predict behavior of physical system using QM algorithms?**

General Approach

1. Identify vector space associated with system

    —> space where all possible physical states of system can be represented

    -—> find suitable basis set.

2. Identify operators associated with measurable properties of system

    —> calculate eigenvectors and eigenvalues of observables.

3. Map out specific correspondences

    between individual physical states and individual vectors

    –> decide how to label(what we can measure) state vectors(inside ket).

4. Ascertain present state vector of system by measurements

    —> determine initial labels

    —> state preparation.

5. Time evolution of systems determined by time evolution operator $\hat{U}$

—> **deterministic** equation until next measurement.

$\hat{U}$ is specified for each system.

6. Probabilities of particular outcomes of measurement

carried out at future time calculated by a Postulate

—> probability of measuring eigenvalue b of the observable $\hat{B}$ at t

when in state $|\psi\rangle$ at t given by $|\langle b|\psi\rangle|^2$.

7. Effects of measurement taken into account using a Postulate

–> state collapses to appropriate (as determined by value measured)

eigenvector of measured observable.

8. Then [5]-[7] are just repeated over and over again………that is how you use QM.

Remember:

Postulate —> state vector "collapsed"

into eigenvector of measured observable operator.

Postulate —> how state evolves in time

—> standard or Copenhagen interpretation of quantum theory

due to Bohr, Dirac, Born and von Neumann(he was misunderstood… as we will see!)

Now we present standard way of **talking**(students of physics must master) about superpositions

and deal with some apparent contradictions

that we saw earlier in the discussion of color and hardness measurements:

Right way to think about superpositions of, say, being green **and** being magenta

—> think of them as situations where **color predictions** cannot be made

—> situations where **color talk** is unintelligible.

Talking and inquiring about color of an electron

in such circumstances (in standard view) makes **no sense** whatsoever.

—> if we follow this rule

then earlier contradictions will go away.

—> it is just not so that hard electrons

are not green and not magenta and not both and not neither,

since color talk about hard electrons

—> **no meaning at all**

and same is true for all other incompatible observables.

This is the way the world works according to the quantum physicist!

Once electron is “measured”

—> green or magenta detected,

then **“is”** green or magenta

(color talk now applies) according to standard interpretation.

Measuring color of hard electron,

then, is **not** matter of determining what color of that hard electron is

... it has **none**

it **only** has **probabilities** to have color values

if color is measured.

—> matter of “collapsing” state of measured electron

into one where color talk **applies**,

and “then” determining color of newly created, color-applicable state.

Measurements in QM (in standard view) are very **active** processes.

Not processes of merely learning something about system

—> processes which **drastically** change measured system.

Most important rule:

If going to measure some observable $\hat{C}$,

then choose as basis vectors the eigenvectors of $\hat{C}$ operator

since only with these states does $\hat{C}$–talk makes sense,

i.e., **if discussing color measurements**

than use color basis where color-talk makes sense!

Same holds true if going to measure anything else!

Some Consequences

We are allowed to use **any**

orthonormal set (number = dimension) as basis.

If other sets exist,

then they **must be** eigenvectors of other observables (not color).

Examples.....

$$Set \quad \#1$$

$$|1\rangle = \frac{1}{2}|g\rangle + \frac{\sqrt{3}}{2}|m\rangle \text{ and } |2\rangle = \frac{\sqrt{3}}{2}|g\rangle - \frac{1}{2}|m\rangle$$

$$Set \quad \#2$$

$$|1\rangle = \frac{1}{2}|g\rangle - \frac{\sqrt{3}}{2}|m\rangle \text{ and } |2\rangle = \frac{\sqrt{3}}{2}|g\rangle + \frac{1}{2}|m\rangle$$

Both are basis sets

i.e., orthonormal sets

—> operators - let us call observables **direction** d and **truth** t

$\hat{d}$ —> set #1 = eigenstates with eigenvalues +1(up) and –1(down)

$\hat{t}$ —> set #2 = eigenstates of with eigenvalues +1(true) and –1(false)

$$Set \quad \#1$$
$$|1\rangle = |direction = +1\rangle = |up\rangle = \frac{1}{2}|g\rangle + \frac{\sqrt{3}}{2}|m\rangle$$
$$|2\rangle = |direction = -1\rangle = |down\rangle = \frac{\sqrt{3}}{2}|g\rangle - \frac{1}{2}|m\rangle$$

direction

$$Set \quad \#2$$
$$|1\rangle = |truth = +1\rangle = |true\rangle = \frac{1}{2}|g\rangle - \frac{\sqrt{3}}{2}|m\rangle$$
$$|2\rangle = |truth = -1\rangle = |false\rangle = \frac{\sqrt{3}}{2}|g\rangle + \frac{1}{2}|m\rangle$$

truth

Remember other basis sets:

$$|color = +1\rangle = |g\rangle = \frac{1}{\sqrt{2}}|h\rangle + \frac{1}{\sqrt{2}}|s\rangle$$
$$|color = -1\rangle = |m\rangle = \frac{1}{\sqrt{2}}|h\rangle - \frac{1}{\sqrt{2}}|s\rangle$$

$$|hardness = +1\rangle = |h\rangle = \frac{1}{\sqrt{2}}|g\rangle + \frac{1}{\sqrt{2}}|m\rangle$$
$$|hardness = -1\rangle = |s\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{1}{\sqrt{2}}|m\rangle$$

Assume states represent a real physical system and think about probabilities. —> can make these statements (based on our rules):

1. probability electron is “up” will be measured to be “magenta” = 3/4
2. probability electron is “false” will be measured to be “magenta” = 1/4
3. probability electron is “soft” will be measured to be “magenta” = 1/2
4. probability electron is “green” will be measured to be “hard” = 1/2

and so on.

Do #1:

$$P(up; magenta) = |\langle magenta \,|\, up\rangle|^2 = |\frac{1}{\sqrt{2}}\langle m \,|\, g\rangle + \frac{\sqrt{3}}{2}\langle m \,|\, m\rangle|^2$$
$$P(up; magenta) = |\frac{1}{\sqrt{2}}(0) + \frac{\sqrt{3}}{2}(1)|^2 = |\frac{\sqrt{3}}{2}|^2 = \frac{3}{4}$$

Now some details and examples —> apply all stuff and expand knowledge and capabilities.

We begin with an example that includes only familiar classical properties

like position, velocity, momentum, energy, etc,

then return to further explain the color-hardness experiments

We know behavior of big particles(large mass)

—> rocks well described by classical mechanics of Newton

—> whatever QM theory says about microworld particles

—> ought to predict that everyday particles, subject to everyday circumstances,

behave in a Newtonian way.

Now position operator $\hat{x}$ and momentum operator $\hat{p}$ are **incompatible**

(i.e., **not simultaneously measurable**)

—> Heisenberg uncertainty principle **exists** (as we will see later)

—> $[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar \neq 0$ **commutator**

Since $\hat{x}$ = operator representing a physical observable,

a possible basis of space is set of eigenvectors of $\hat{x}$ operator.

Similarly, for eigenvectors of $\hat{p}$ operator.

and since each are Hermitian operators —> each form a possible basis.

Because corresponding operators do not commute,

a state of definite momentum (eigenvector of $\hat{p}$)

will be a **superposition** of states of definite position(eigenvectors of $\hat{x}$)

or state of definite momentum can only have **probabilities** of having definite x–values.

Similarly, state of definite position (eigenvector of $\hat{x}$)

—> **superposition** of states of definite momentum (eigenvectors of $\hat{p}$).

or state of definite position can only have **probabilities** of having definite p–values.

Vector space when $\hat{x}$ eigenvectors are basis —> coordinate representation.

Vector space when $\hat{p}$ eigenvectors are basis —> momentum representation.

Most important operator in quantum mechanics is the energy operator or Hamiltonian $\hat{H}$

—> will determine time evolution operator as we will see later.

Vector space when $\hat{H}$ eigenvectors are basis —> energy representation.

All are **equivalent** basis sets.

Choice depends on questions being asked or measurements being carried out.

Remember: Choosing an appropriate basis is called "going to HOME SPACE".

—> coordinate-talk, momentum-talk, energy-talk ....

Possible eigenvalues(allowed measured values) of $\hat{x}$ and $\hat{p}$

form **continuum** extending from $-\infty$ to ∞ (**continuous** spectrum).

Basis is infinite(non-denumerable) dimensional

—> many difficult mathematical problems during discussion in full theory.

However, for simple systems we will discuss, infinite dimension causes **no difficulties**

—> can treat almost all properties as if spaces had finite dimension.

(One exception <—> normalization - only do in an advanced class).

State $|x\rangle$ corresponds to system with definite value of position, namely, x

—> eigenvector of $\hat{x}$ operator.

State $|p\rangle$ corresponds to system with definite value of momentum, namely, p

—> eigenvector of $\hat{p}$ operator.

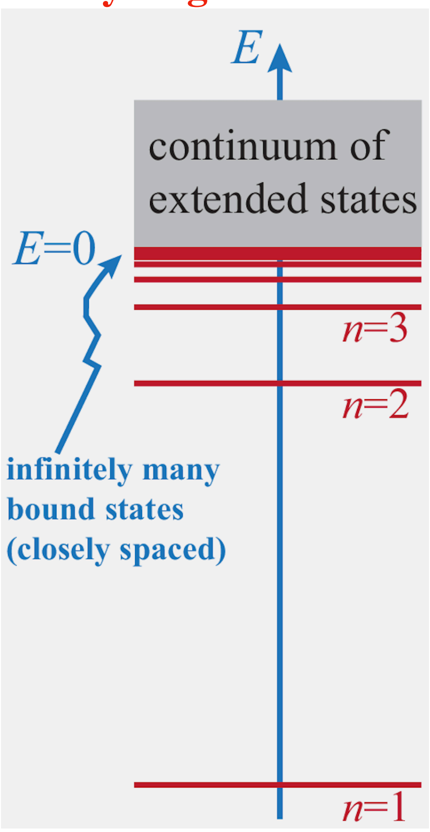
State $|E\rangle$ corresponds to system with definite value of energy, namely, E

—> eigenvector of $\hat{H}$ operator.

In general, set of energy eigenvalues

are both a discrete set over some range **and** a continuous set over a disjoint range.

Hydrogen Atom



Operators position $\hat{x}$ and $\hat{C}$ (color) are **compatible**

—> can simultaneously measure these observables.

—> can say that magenta electron located at x = 7 and that makes sense.

We write state representing this case as $|color - value\rangle\, |x - value\rangle$

i.e., $|m\rangle\, |7\rangle$ for magenta electron located at x = 7. (formal name for product is tensor-product).

When operators act on such states their operation takes the form

$$\hat{C}\,|m\rangle\,|7\rangle = -\,|m\rangle\,|7\rangle \text{ i.e., } \hat{C} \text{ only acts on the color space part}$$
$$\hat{x}\,|m\rangle\,|7\rangle = 7\,|m\rangle\,|7\rangle \text{ i.e., } \hat{x} \text{ only acts on the position space part}$$

Similar statements hold for

position and hardness, momentum and color, momentum and hardness,

energy and color, and energy and hardness.

All represent simultaneously measurable pairs.

Operator corresponding to observable energy = Hamiltonian $\hat{H}$ where $\hat{H}\,|E\rangle = E\,|E\rangle$

energy eigenvalues

energy eigenvectors

Energy basis = most fundamental in quantum theory.

We will see why shortly.

Now the quantity $\psi_E(x) = \langle x \mid E\rangle$

for state vector |E⟩ = energy **wave function** (just a "bracket" —> nothing mysterious!)

since it takes on different values for each value of x it is **function** of x —> limited use.

Satisfies famous equation that governs time and space dependence in QM,

namely, the Schrodinger equation.

Schrodinger equation in 1-dimension is an "ordinary differential equation" given by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E(x,t)}{dx^2} + V(x)\psi_E(x,t) = E\psi_E(x,t) = i\hbar\frac{d\psi_E(x,t)}{dt}$$

Time evolution operator for simple systems

is expressed in terms of Hamiltonian operator $\hat{H}$ by relation $\hat{U} = e^{-i\hat{H}t/\hbar}$

—> **operator function** of $\hat{H}$.

In simple cases, functions of operators are easy to deal with.

If $$\hat{B}\,|b\rangle = b\,|b\rangle$$

i.e., |b⟩ = eigenstate of $\hat{B}$, then have

$$f(\hat{B})\,|b\rangle = f(b)\,|b\rangle$$ for function of operator.

—> operator argument is replaced by eigenvalue inside the function

—> energy eigenvectors have simple time dependence or time evolution, i.e.,

$$|E,t\rangle = \hat{U}\,|E,0\rangle = e^{-i\hat{H}t/\hbar}\,|E,0\rangle = e^{-iEt/\hbar}\,|E,0\rangle$$ **just replace operator with eigenvalue**

where complex exponential form —> $$e^{-iEt/\hbar} = \cos\frac{E}{\hbar}t - i\sin\frac{E}{\hbar}t$$

Using eigenvectors/eigenvalues of Hamiltonian, namely, $$\hat{H}\,|E_n\rangle = E_n\,|E_n\rangle$$

as basis —> time dependence of arbitrary state vector can be derived as follows:

This is one of the major techniques used in QM calculations.

Initial state $|\psi(0)\rangle$

Expand in energy basis (HOME space of time-evolution operator)

$$|\psi(0)\rangle = \sum_n c_n |E_n\rangle$$

Operate with time evolution operator

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle = \hat{U}(t)\sum_n c_n |E_n\rangle = \sum_n c_n \hat{U}(t)|E_n\rangle$$

Now

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

Therefore

$$|\psi(t)\rangle = \sum_n c_n e^{-i\hat{H}t/\hbar} |E_n\rangle$$

and we finally get (substitute eigenvalue for operator)

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

Short digression for the mathematically inclined(otherwise just sit and listen) to show you that you can actually already do real QM calculations.
Now repeat some earlier algebra for fun(to see it really is not difficult)

In the discrete spectrum case, we have, using a basis set {|n⟩}, that

$$|\psi\rangle = \sum_n c_n |n\rangle \quad , \quad c_m = \langle m | \psi \rangle$$

so that

same / same

$$|\psi\rangle = \sum_n \langle n | \psi \rangle |n\rangle = \sum_n |n\rangle \langle n | \psi \rangle = \left(\sum_n |n\rangle \langle n| \right) |\psi\rangle$$

which says that

$$\sum_n |n\rangle \langle n| = \hat{I}$$ **identity operator** as derived earlier

and if

$$\hat{A} |a_n\rangle = a_n |a_n\rangle \quad , \quad |\psi\rangle = \sum_n c_n |a_n\rangle$$

then

$$\hat{A} |\psi\rangle = \sum_n c_n \hat{A} |a_n\rangle = \sum_n c_n a_n |a_n\rangle$$

now using clever math trick

$$\sum_n c_n a_n |a_n\rangle = \sum_n c_n \sum_m a_m |a_m\rangle \delta_{nm} = \sum_{n,m} c_n a_m |a_m\rangle \delta_{nm}$$

and substituting $\delta_{nm} = \langle a_m | a_n \rangle$ we get

$$\sum_n c_n (\hat{A}) |a_n\rangle = \sum_{n,m} c_n a_m |a_m\rangle \langle a_m | a_n \rangle = \sum_n c_n \left(\sum_m a_m |a_m\rangle \langle a_m| \right) |a_n\rangle$$

so that

$$\hat{A} = \sum_m a_m |a_m\rangle \langle a_m|$$

as derived earlier

which is **spectral representation** of operator $\hat{A}$ in terms of its eigenvalues and eigenvectors.

Now you are learning math language and we are beginning to get somewhere!

Next derivation more mathematical, but very important. **Result is important thing!**

Now we have all needed tools.

General procedure for figuring out time dependence of states

and then answering questions posed in experiments

goes as follows (go slowly — tricky algebra - **just follow along**):

1. Ask experiment/theoretical question — the typical form is

   - if system in state $|\phi\rangle$ at t = 0, what is probability of system being in state $|b_7\rangle$ at t?

2. Assume $|b_7\rangle$ one of eigenvectors of measurable operator $\hat{B}$.

3. Solve for energy eigenvectors of system,

   i.e. find Hamiltonian (energy operator) and do mathematics.

4. Write $|\phi\rangle$ in terms of energy eigenvector basis $|\phi\rangle = \sum_E c_E |E\rangle$ where $c_{E'} = \langle E' \mid \phi\rangle$

   component

5. Since time dependence of energy eigenstates is easy,

   can write down time dependence of state $|\phi\rangle$ as

$$|\phi, t\rangle = \sum_E c_E |E, t\rangle = \sum_E c_E e^{-iEt/\hbar} |E\rangle$$

it is that easy!

6. Write |E⟩ in terms of $\hat{B}$ eigenvector basis $|E\rangle = \sum_j d_j |b_j\rangle$ where $d_\mathrm{k} = \langle b_k \mid E\rangle$

7. Write |ϕ, t⟩ in terms of $\hat{B}$ eigenvector basis

$$|\phi, t\rangle = \sum_E c_E e^{-iEt/\hbar} |E\rangle = \sum_E \langle E \mid \phi\rangle\, e^{-iEt/\hbar} \sum_j \langle b_j \mid E\rangle\, |b_j\rangle$$

8. Then probability of finding b_7 at time t given by(a postulate)

$$P = |\,\langle b_7 \,|\, \phi, t\rangle\,|^2$$

We get

$$P = \left| \langle b_7| \left(\sum_E \langle E \,|\, \phi\rangle\, e^{-iEt/\hbar} \sum_j \langle b_j \,|\, E\rangle\, |b_j\rangle \right) \right|^2$$

or

$$P = \left| \left(\sum_E \langle E \,|\, \phi\rangle\, e^{-iEt/\hbar} \sum_j \langle b_j \,|\, E\rangle\, \langle b_7 \,|\, b_j\rangle \right) \right|^2$$

Now

$$\langle b_7 \,|\, b_j\rangle = \delta_{j7}$$

This means that the sum over j disappears and all j's get replaced by 7 . We get

$$P = \left| \sum_E \langle b_7 \mid E \rangle \langle E \mid \phi \rangle e^{-iEt/\hbar} \right|^2$$

Final result expresses answer

in terms of initial and final states

and properties of energy eigenvectors and energy eigenvalues,

and all are known!!

Thus after several passes(to better understand things) through various topics involved,

we see that our formulation of quantum theory

is capable of making the necessary predictions.

We won't be doing this in general(that is for a regular QM course),

but it is important to know that we can!

Some more mathematical material is on the website in the the supplementary readings.

We now look at some intriguing experiments exhibiting QM phenomena in order to get an idea of how the strangely quantum world behaves and to also start to learn about non-locality which will be a central to understanding measurement.

Now to look at some important experiments —> give us hints as to how QM works!

An Interference Experiment with Photons

Direct laser beam at a half-silvered mirror.

For **intense** light beams,

such mirrors reflect half of light and allow half to pass straight through.

When intensity of laser beam high,

two beams are seen emerging from mirror,

each having 1/2 intensity of incoming beam.

Arrangement is called a **beam-splitter**.

If turn intensity of laser down(photons emerge with time gaps)

-> only one photon around at any given time,

and use pair of C(harged)C(oupled)D(device) detectors

to detect reflected and transmitted photons,

something very interesting happens.

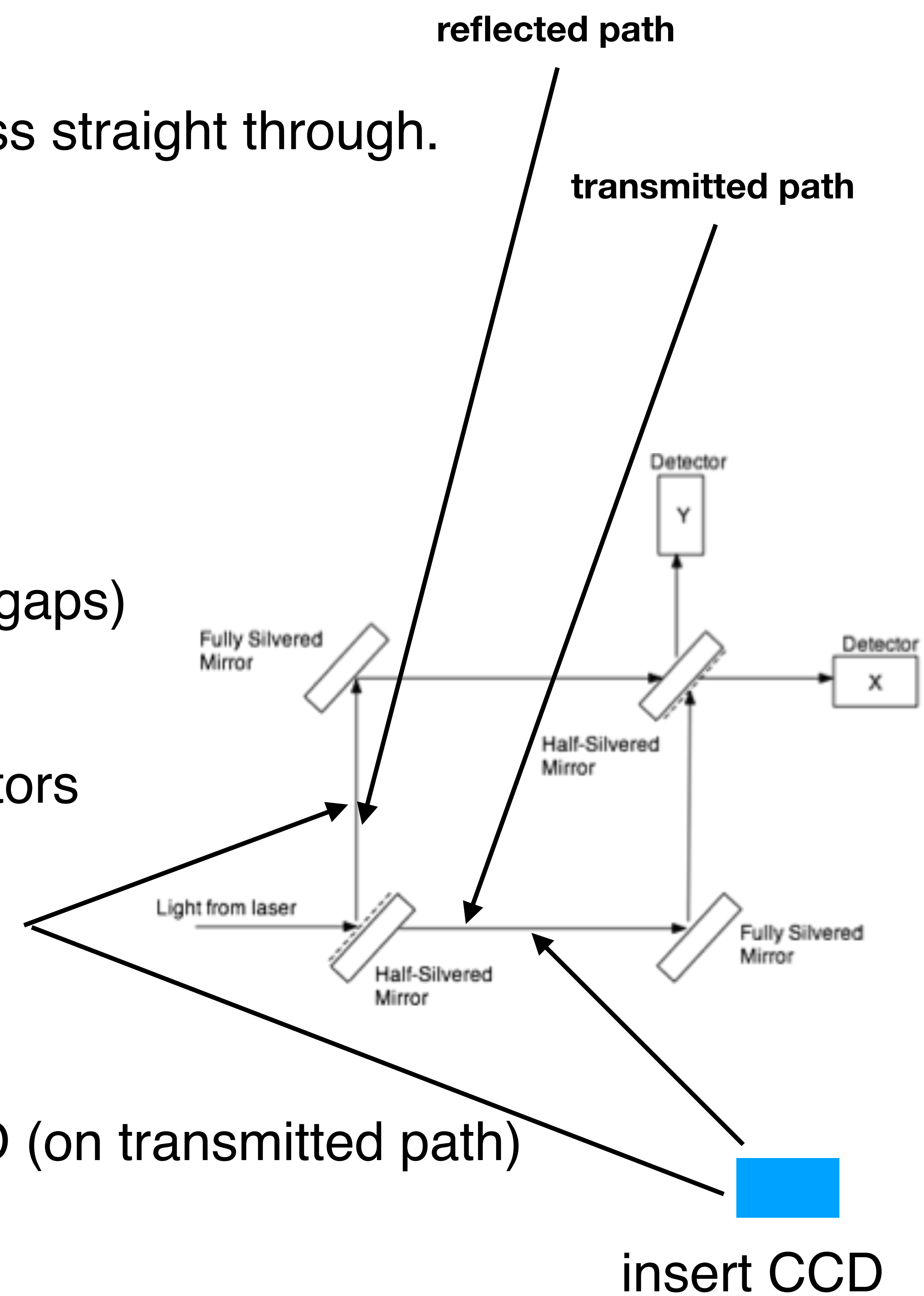
For each photon that leaves laser,

one photon is detected either at transmission CCD (on transmitted path)

or at reflection CCD (on reflected path).

Photons are not split in some odd manner

so that half photon goes one way at mirror and half other way. See figure.



Instead, seems to be 50:50 chance(probability)

that photon transmitted or reflected by half-silvered mirror.

No measurable difference between photons as they approach mirror,

i.e., no property seems to determine which way they will go (sound familiar).

—> Fundamental point that will come up repeatedly in context of quantum theory.

Next step is to remove detectors and replace them with two mirrors (fully silvered)

that divert two beams(change their directions by 90°)

to a second half-silvered mirror as in figure.

There are now two beams arriving at the 2nd half-silvered mirror.

One from each path through 1st silvered mirror.

At this point,

same thing happens at 2nd silvered mirror,

with 1/2 of light arriving at 2nd half-silvered

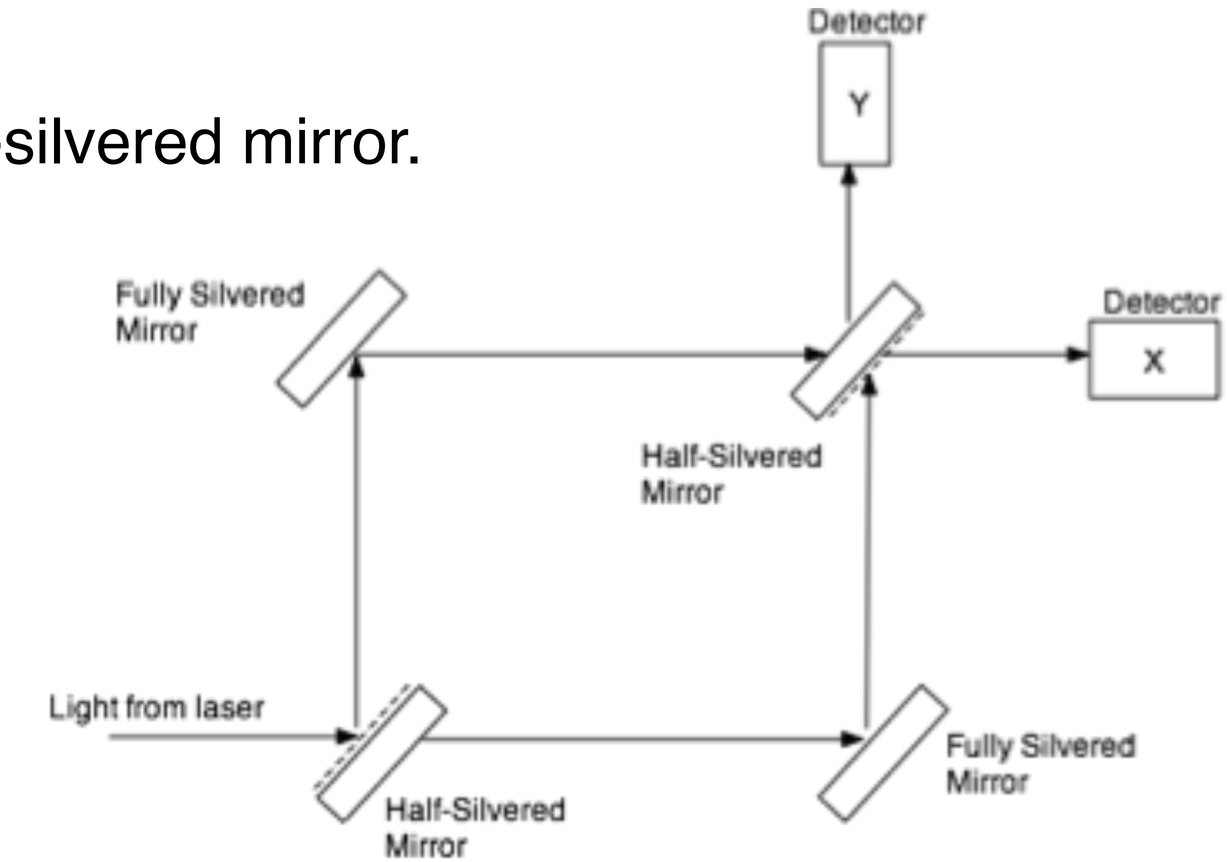
mirror passing straight through

and other half being reflected.

Two new beams emerge

and eventually travel to pair of detectors

placed at X and Y. See figure



Beam heading to detector X

is combination of light that was reflected by 1st half-silvered mirror (travelled top path),

then transmitted by 2nd half-silvered mirror,

with light that transmitted by 1st half-silvered mirror(along bottom path)

and reflected by 2nd.

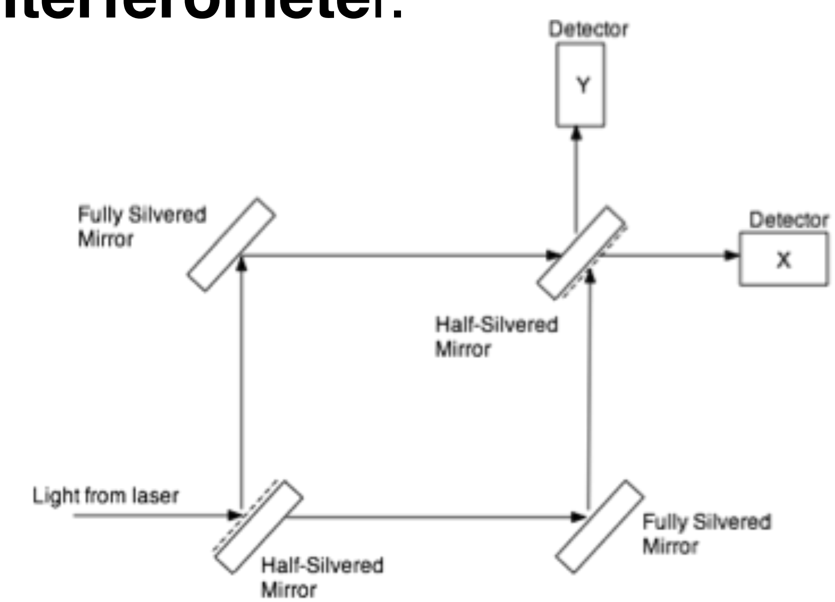
Detector Y collects light that is similar mixed combination.

Arrangement of mirrors and detectors is called **Mach-Zehnder interferometer**.

Once set up, it is easy to confirm that

intensity of light reaching each detector depends critically

on **distances travelled** by light along top and bottom paths.



If equipment is finely adjusted

so that paths are exactly same length,

detector Y records no light

while detector X records all of intensity (all photons) entering experiment.

Without critical adjustment,

X and Y collect light in varying amounts:

more light at X -> less reaches Y (and vice versa).

Using classical physics methods these effects can be completely explained by saying that light is a **wave —- but this argument works only when the beam intensity is high**.

Interference as Wave Effect (reminder)

Consider ripples crossing surface of lake.

Ripples consist of places where water level is higher than normal (peaks)

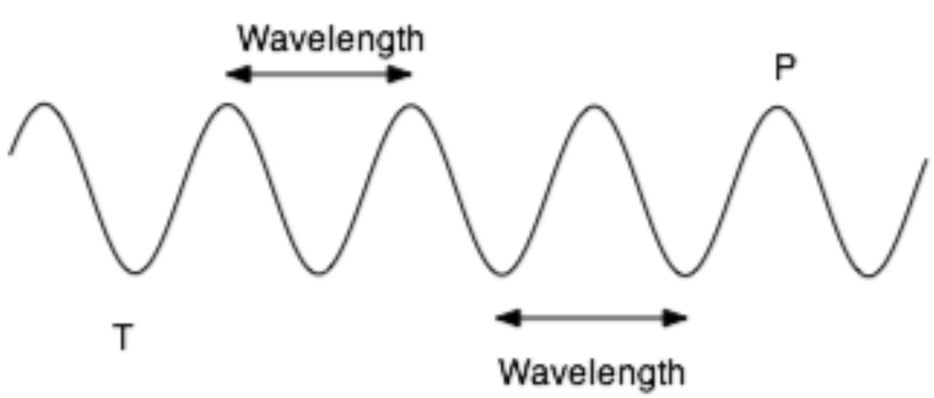
and places where dropped below normal (troughs).

Wavelength of ripple is distance between successive peaks(P),

same as distance between successive troughs(T).

Frequency of wave is rate at which complete cycles (peak to trough to peak again)

pass fixed point, and period is time taken for one cycle.



Light(E/M fields) is more complicated than a water wave.

Peaks and troughs of light wave are not physical distances (like height of water wave)

but are variations in strength of fields(E and B).

Mathematically, however, they are the **same** phenomena since the equations are identical.

Thus, light waves are very sensitive measures of distance.

In interference experiment with interferometer,

divide distance travelled by light wave on route to detector into sections,

each having length equal to wavelength of wave.

Distance probably not whole number of wavelengths.

Furthermore, two different possible routes through experiment

have to be precisely same length to be precisely same number of wavelengths long.

If distances not precisely same,

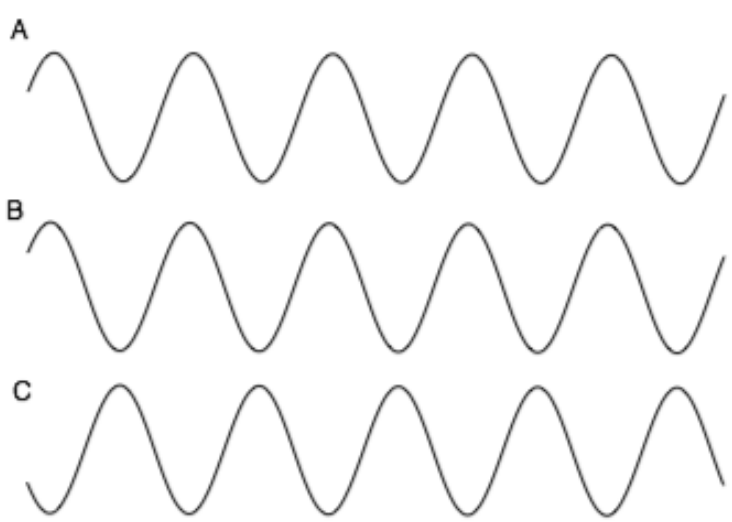
light traveling along each route consequently will

have gone through different number of complete waves when gets to the detector.

As light in two beams has common source at first half-silvered mirror,

two beams will set off on different routes in phase

(i.e., in step - simplest definition of phase - both at same point on wave) with each other.



Waves labelled A and B in phase(peak to peak),

waves B and C exactly out of phase (peak to trough).

Above statement ->> set off peak for peak -

so were in phase.

By time they get to detector

two beams may no longer be in phase (different distances travelled).

One could be arriving at peak, and other at trough (B and C).

If this happens, then waves will cancel each out —> no energy entering detector

called **destructive interference**

If still in phase would add up

called **constructive interference**).

Exact cancellation only happens if waves meet precisely peak to trough

not possible for any extended length of time

due to small variations in distance (mirrors shaking slightly)

and fluctuations in laser output.

Detailed analysis of interference experiment

also takes into account what happens to light at various mirrors

-> also influence phase of waves.

When light reflects off mirror,

reflected wave out of phase with incoming wave by half wavelength.

Using λ = wavelength, wave has undergone $\lambda/2$ phase shift (shift by 1/2 wavelength)

on refection.

Slightly different with half-silvered mirror,

surface that reflects from either side mounted on thin block of glass.

Dashed line in figures indicates reflecting surface.

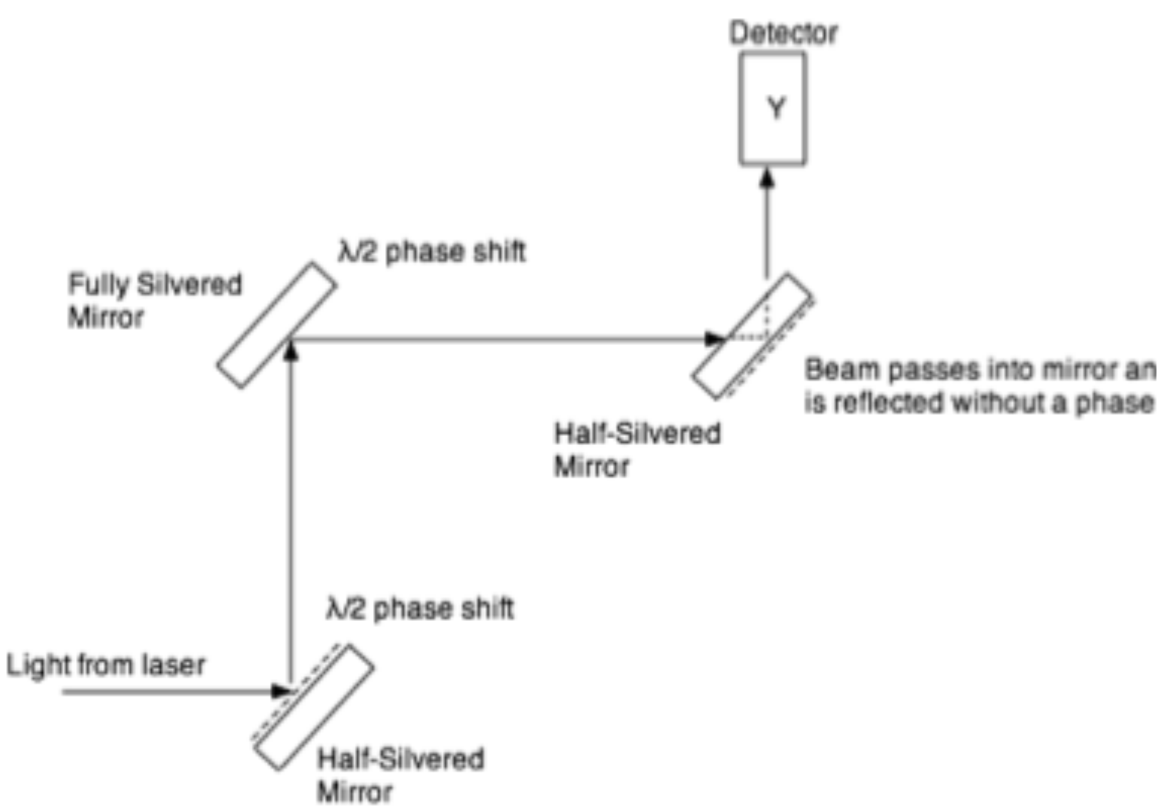
If reflection takes place off surface before light enters glass block,

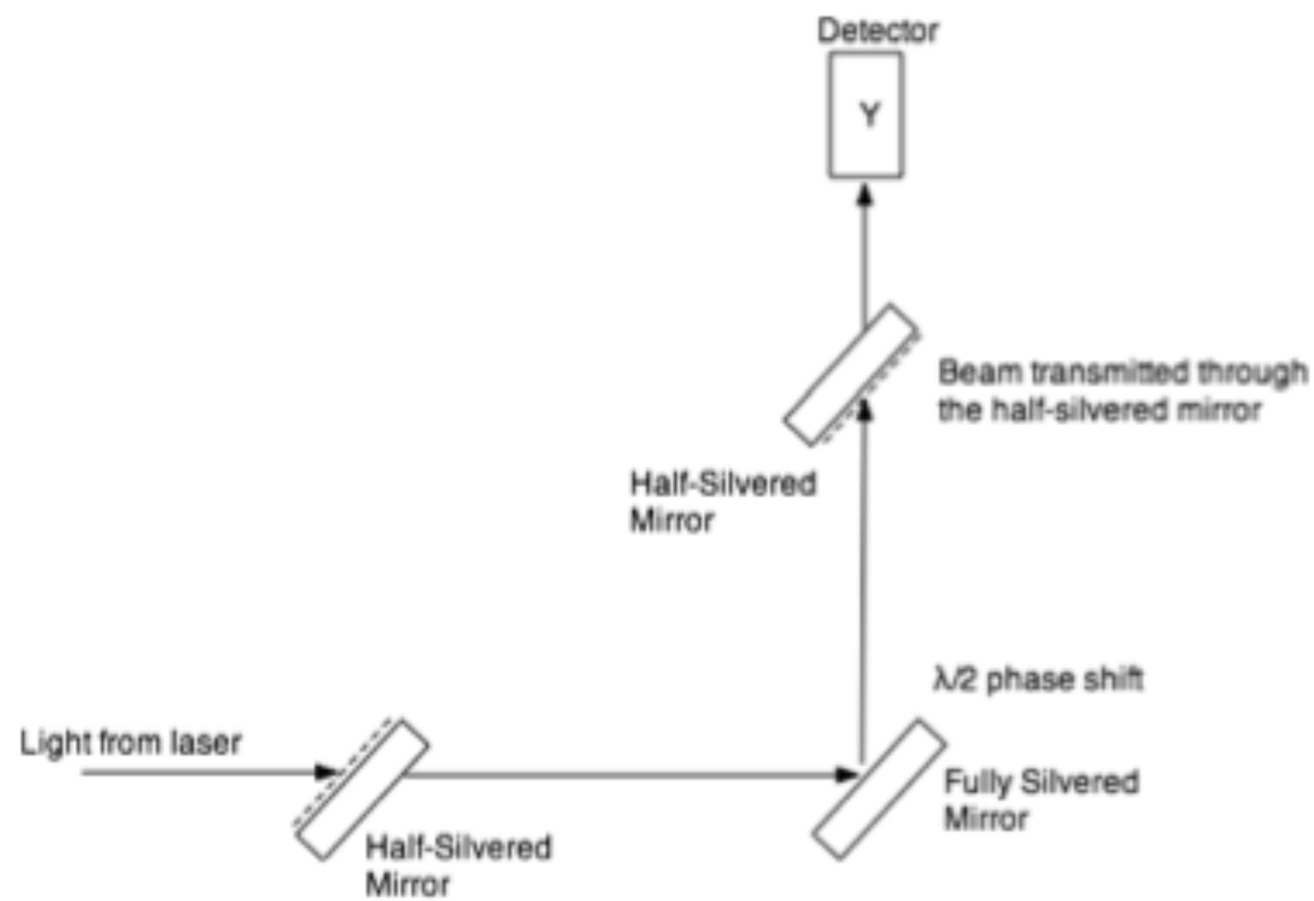
then ordinary λ/2 phase shift takes place.

However, any light that passes through block

before reaching reflecting surface not phase shifted on reflection.

See two figures below.





Two light beams reaching detector Y will be (overall) $\lambda/2$ out of phase.

Consequently, waves B and C and will completely cancel (if travelled equal distances).

If carry out same analysis for detector X,

i.e., chart progress of waves through interferometer to detector X,

find arrive in phase provided travelled equal distances.

Most experimental setups,

paths through interferometer not exactly equal

so waves not exactly in/out of phase.

Consequently, some light reaches both X and Y.

If equipment allowed movement of fully silvered mirrors,

so relative path lengths were changed,

then variation in brightness of light in X and Y

could be studied as mirror moved - as relative paths changed.

That is the classical way of thinking using interference.

Modern version of Mach-Zehnder interferometer with Single Photons

Suppose we are able to turn down laser intensity

so that light beam made up of single photons entering apparatus at any time.

Assume have done so —> only one photon in experimental apparatus at any time.

Also have control over average rate. Note that “interference” argument is no longer valid!!

Expect photons arriving at half-silvered mirror

to have 50:50 chance of going through/reflecting off.

Another possibility is

two reduced energy photons emerge from mirror, in each direction.

Can determine experimentally what happens:

place photon detectors just after mirror in path of each possible beam.

Simple experiment produces interesting result.

Half the time, photon is reflected, and half the time it is transmitted;

and never get two photons coming out at same time.

However, no inherent difference

between those entering photons that get through and those that reflect.

No pattern to sequence,

except that after long time half reflect and half get through.

Sound familiar!

Effect is common in quantum physics.

Some aspects of nature's behavior lie beyond ability to predict (e.g., which way photon will go).

Question - does this reflect fundamentally random aspect to nature,

or is something more subtle going on that have not discovered yet?

Having established that photon reaching

1st half-silvered mirror in Mach-Zehnder interferometer

will either reflect and travel top path through device,

or transmit and follow bottom path,

Now turn attention to what happens at detector end of device.

First find, between them, detectors pick all photons that enter experiment.

Number of photons arriving at either detector in given time depends on two path lengths,

i.e., if exactly equal then no photons ever arrive at Y.

If paths not exactly equal, then find that detection rate at each detector

reflects intensity of interference pattern

that would be observed when intensity is turned up.

What do we mean by that?

Let's imagine that had arranged for path lengths

such that 70% of total light intensity entering experiment arrives at X and 30% at Y.

No double photon firings.

Experiment done under well-controlled conditions and no doubt

that photon arrival rate directly reflects an interference pattern between paths.

Doesn't sound like a problem, but there is.

If photon is small particle of light,

then how can different path lengths have any effect on one single photon?

Confirmed that photons randomly choose reflection or transmission at half-silvered mirror.

After that, surely they proceed along one path or other to detector.

Hard to imagine single photon going along both paths at same time.

Remember that is rejected by experimental results

(detectors only registered one photon at a time).

Now a wave can do this.

Can spread out throughout experiment (ripples on lake)

so that parts of wave travel along each path at same time

(i.e., wave energy divides between paths).

When two parts of wave combine at far side of experiment,

information about both paths is being compared,

which leads to interference pattern.

A single photon however, must surely have information about only one path,

so how can single photon experiments produce interference patterns?

Flaw in arguments.

The flaw in these arguments is extremely subtle

and leads to the primary issue physicists face when dealing with quantum world.

We confirmed that photons divert at half-silvered mirror by placing detectors in two paths.

However, doing this eliminated any chance of picking up an interference pattern.

If detectors have stopped photons, then they **have not** travelled any paths.

In principle, experiment does not tell anything about what happens when no detectors present.

Common sense to assume that photons do same thing with or without detectors,

but as already seen,

the interference pattern for photons(all of QM) is not a matter of common sense.

In addition, color/hardness experiments

say that it is important whether or not detectors are present!

Must investigate this further.

Place one photon detector after half-silvered mirror - in path of reflected beam.

If detect photon there, then would not get one at far side of experiment.

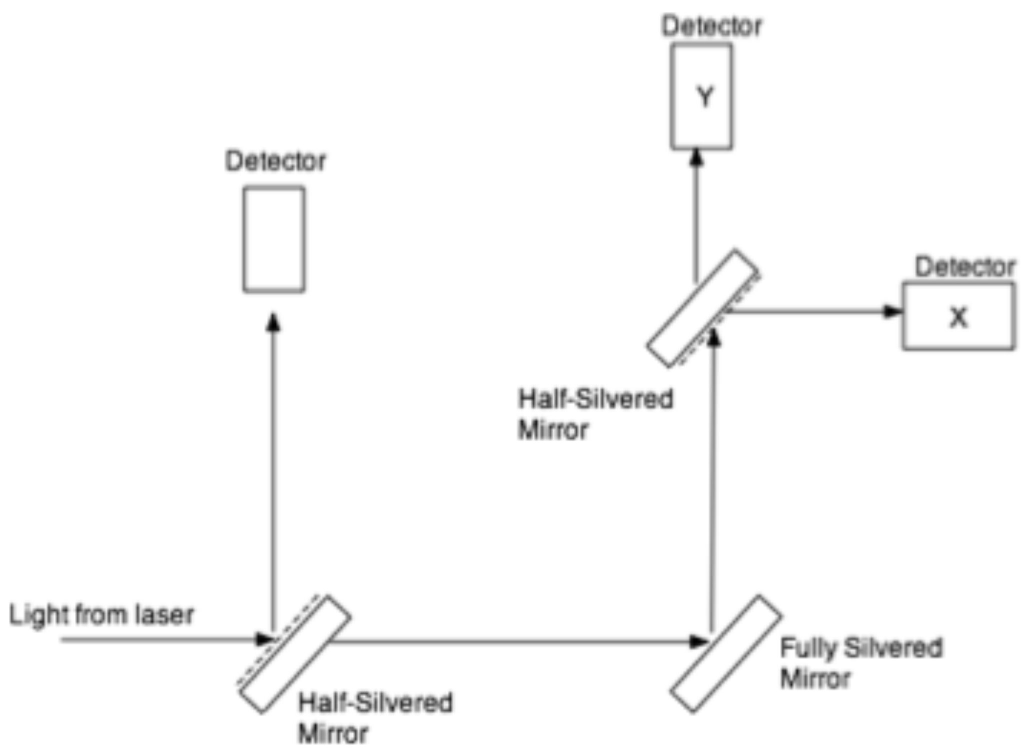
On other hand, if do not pick one up at detector

then it has passed through mirror rather than reflecting and should see it at far end.

Experiment is easily done and confirms

that for every photon leaving the laser the detectors pick one up either at far end

or in reflected beam as below.



Find for transmitted photons

half of arrive at Y and other half at X,

no matter what the length of paths is.

In other words, no interference takes place.

Removing detector on reflected path

(and replacing the corresponding half-silvered

mirror) opens up that route

to far side of experiment again.

At same time

it removes any direct knowledge that we might have

about behavior of photons at the half-silvered mirror.

We observe that doing this restores the interference pattern!

Remember what happened in the color/hardness experiments.

Summarizing the logic so we can expose what is happening and find the flaw.

(1) Rate of photons arriving at far side of experiment related to intensity of bright beam.

(2) Moving mirror with bright beam maps out interference pattern in detectors.

(3) Reducing intensity of beam does not affect interference pattern - instead arrival rate of photons now depends on position of mirror

(4) If set up experiment so that can tell which path taken by photon (directly or indirectly), then interference pattern is destroyed.

(5) If unable to tell paths of photons, then there is interference pattern, implies photons arriving have information about both routes through experiment.

(6) Opening up top path (removing detector) can reduce number of photons arriving at Y.

In fact, if path lengths are same, opening up top path means never get any photons at Y.

All experimental results equivalent to Young-type two-slit interference experiment

and the two-path color-hardness experiment we discussed earlier.

Changing experiment to a Delayed Choice configuration makes result even stronger!

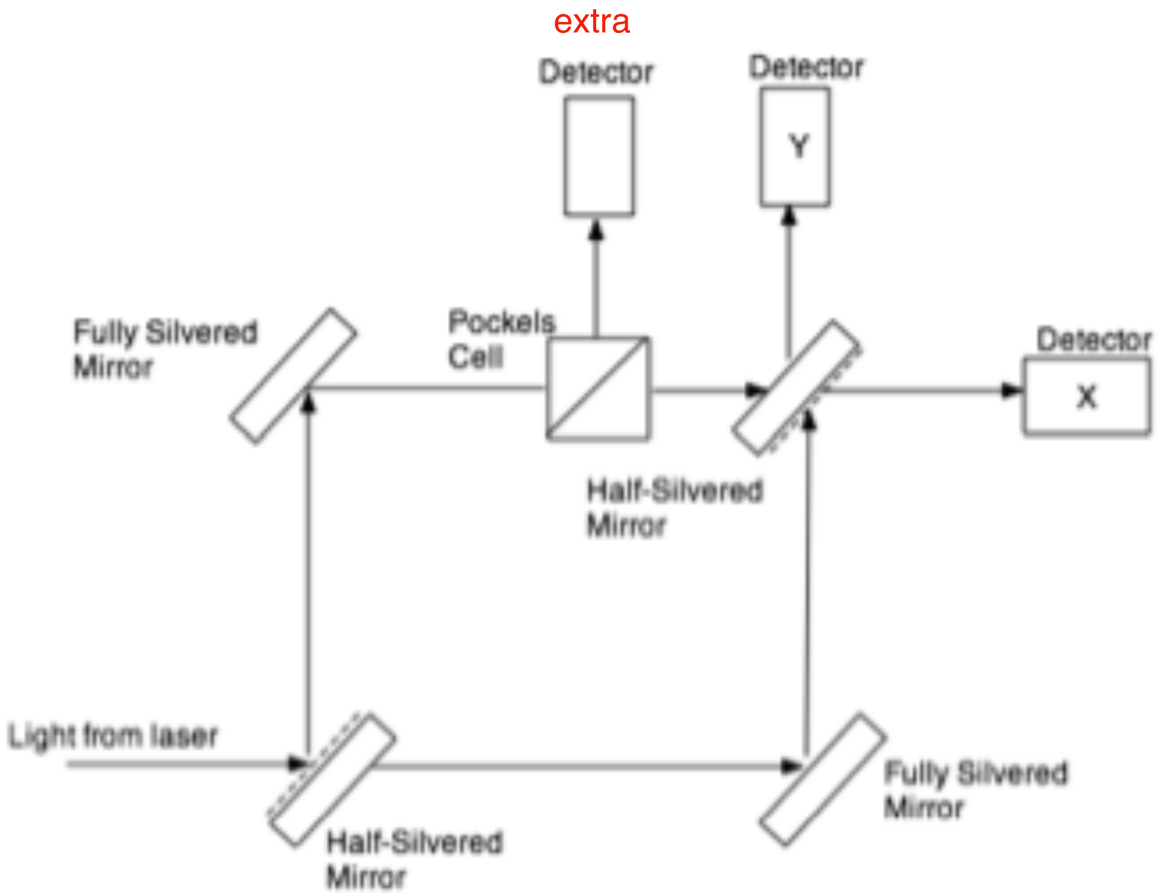
Let us see how.

Introduce Pockels cell(PC) (can divert photons extremely fast) on one route. Details later.

—-> Trying to **find out what photons are doing while in the interferometer.**

Consider setup where PC set to divert photons(to extra detector).

Photon leaves laser and arrives at first half-silvered mirror.



If reflected,

then PC will divert it so don't see at X or Y.

However, if photon transmitted

by first half-silvered mirror,

misses PC, and turns up at either X or Y.

Either case

no interference pattern

(we have gotten which-path information).

If set PC to pass photons,

then changes what happens and get interference pattern.

For extreme case of equal path lengths, no photons ever arrive at Y.

Assume two path lengths exactly same. So have:

(1) If PC set to transmit, then get no photons at Y and all at X.

(2) If PC set to divert, then half of photons detected have equal chance of either X or Y.

This result should make us stop and think.

If photon takes lower route with PC set to divert,

then can get to X or Y.

If takes lower route with PC set to pass,

then photon can never arrive at Y.

But if takes lower route it doesn't go anywhere near PC,

so how can setting of that device affect things?

Is this further hint that somehow or other photon travels both routes at same time?

Again this should sound familiar!

Now experimentalists get very devious. Experimentalists are very clever!!

Set PC to divert photons, but while photon in flight(during experiment),

switch cell over to transmit setting.

Ability to do this very rapidly means can make change

after photon has interacted with first half-silvered mirror.

No magic in doing this. If know when photon left laser,

can estimate how long takes to get to half-silvered mirror.

Provided we switch PC after this time,

but before photon had time to reach detectors X and Y,

then can perform experiment as described.

If setting of PC has(in someway) influenced the photon,

then original setting should have determined that photon takes one path or other

and certainly not both at once.

Now think we have changed setting after decision(at 1st 1/2-silvered mirror) made by photon

NOTE: Of course, word decision is not appropriate here.

Photons do not make decisions.

Clearly, hard not to be anthropomorphic when describing experiments.

In fact, can trigger PC in random manner.

Record setting of PC and match arrival of photons at one detector or another.

Can then run experiment for many photons

and record arrival at different detector positions.

After experiment run for while,

can analyze data.

Have some photons arriving at Pockels detector

and some at far end of experiment.

Latter group sorted out into those that arrived

when PC set to divert,

and those that made it when PC set to transmit.

Remarkably, when data separated,

photons that arrived at far side with PC set to transmit show interference pattern.

Other photons that arrived with PC set to divert

(obviously committed to other path and so missed it) show no interference pattern.

In every case PC set to divert photons and switched after photon left first mirror.

With PC set to divert, photons follow one route or other.

Then switched PC, destroying ability to know which path photons travelled,

and producing interference pattern.

Hard to believe that changing setting of PC can have an influence

that seems to **travel backward in time(retrocausality)** to when photon reaches first mirror.

Last statement, was made using ordinary words, not mathematics(nothing strange is happening) —> our attempt to use words may be generating total nonsense of course!

The mathematics is clear. QM makes the correct predictions in all cases!

From both photon experiments and color/hardness experiments

we have seen that quantum physics is a contextual theory

-> adequate description of behavior of quantum object (light/electrons)

requires an understanding of whole (the context) experimental setup.

Quantum behavior depends on **context**!

Experimental results (answers to questions) **depend** on what questions we are asking!

Another Interference Experiment

Clearly, we get into difficulty with experiments

when we piece together an understanding of the "whole"

by looking at the component parts on their own.

When everything is together, things behave differently. **—> whole > sum of parts!!**

However, results of one experiment are not an accurate guide to another.

If this is case,

then conclude that photons always take only one route,

as indicated in experiments that looked for route followed.

However, know that as soon as do not have such ability

to tell path of photons

they seem to take both routes at once

or something that is equivalent to that,

or maybe nothing we can imagine with our macroworld minds/words!

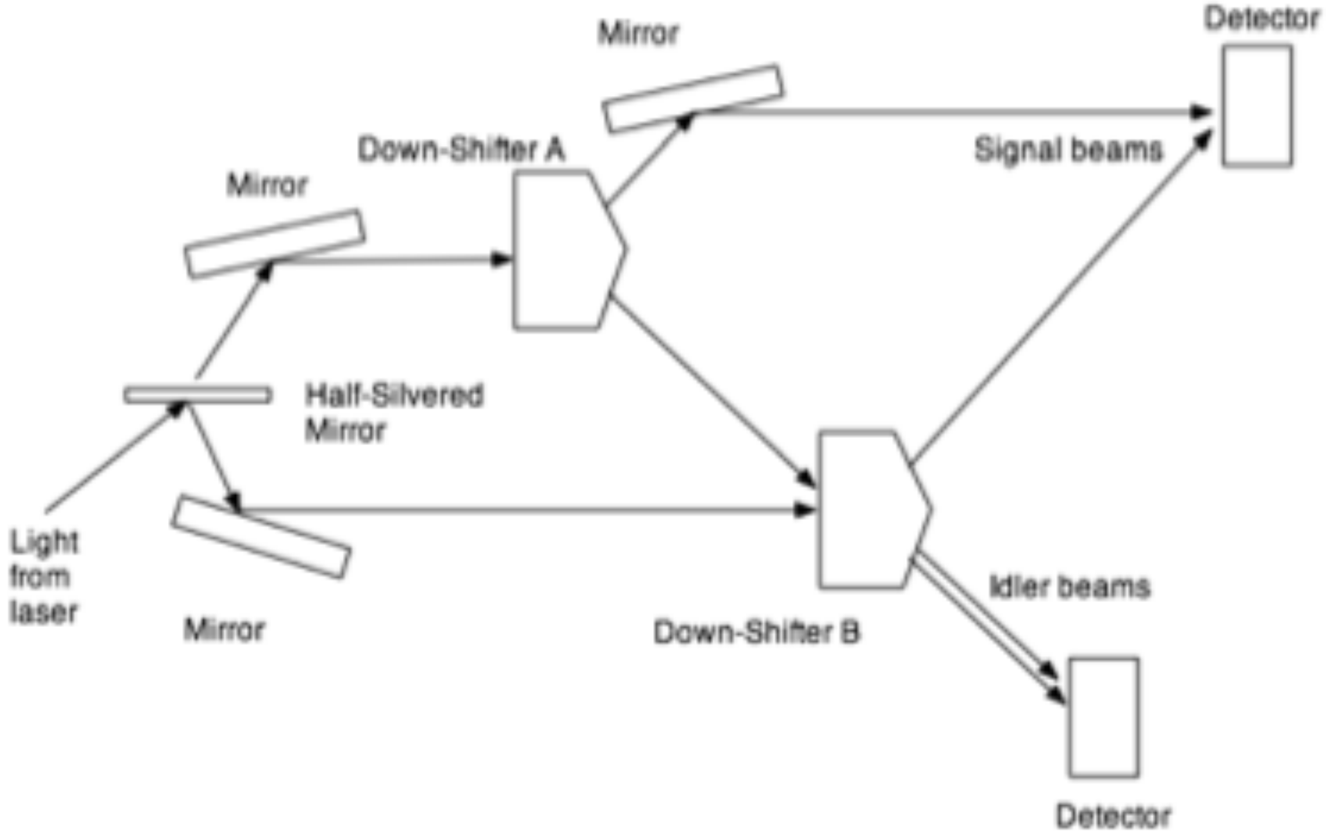
Another experiment we now look at, pushed this notion further

by showing interference pattern can be destroyed

without directly influencing the photons creating it.

Experiment uses crystal known as down-shifter.

Device absorbs 1 photon to produce 2 new photons, each with half the original energy.



Laser light is sent onto 1/2-silvered mirror

and 2 beams separately

directed into down-shifters A or B.

Each down-shifter produces a

signal beam and an idler beam,

The difference between two beams is nothing

more than **subsequent** way they are used.

2 signal beams directed to detector

produce interference pattern (different path lengths)(same as Mach-Zehnder).

Idler beam from down-shifter A (now too weak to activate down shifter B) is mixed with that from down-shifter B

and both idler beams arrive at second detector.

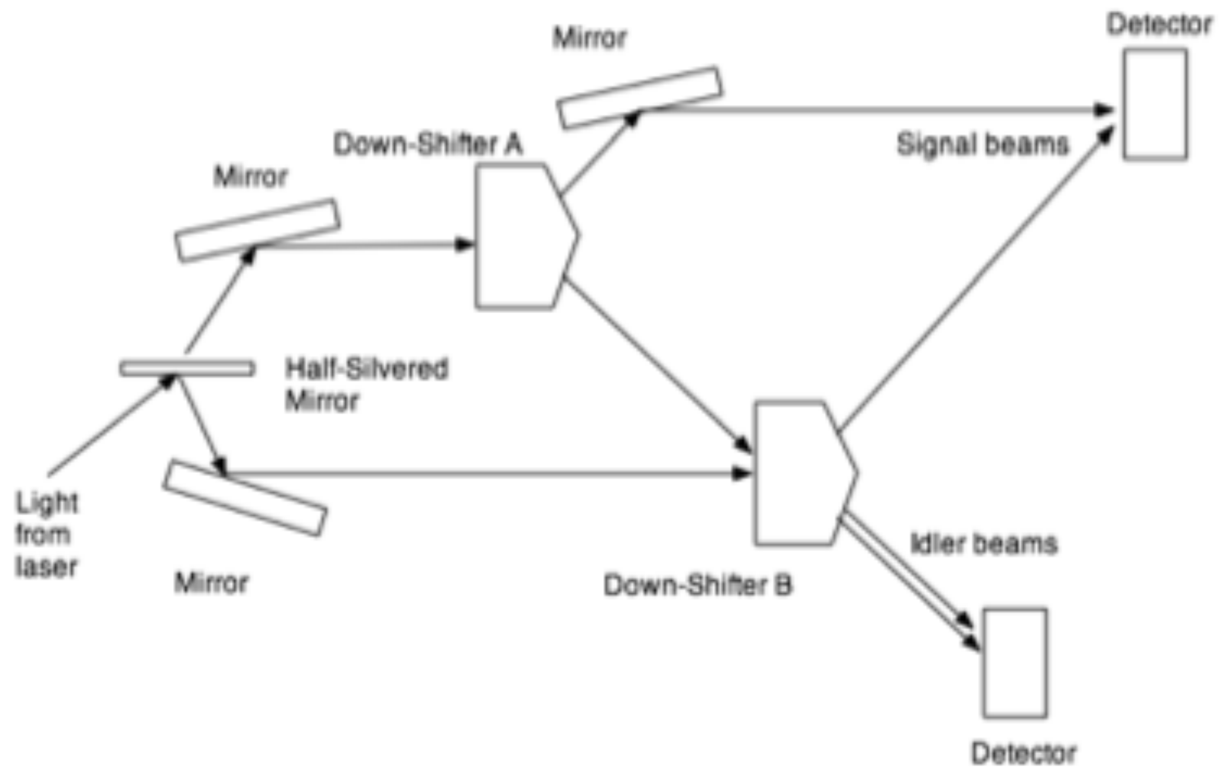
Upshot is that every time photon leaves the laser,

it is observed that a photon of half energy arrives at each detector.

Fact that interference pattern emerges

-> in some manner,

that each photon appears to have travelled along both signal beam paths.



Say photon arrives at half-silvered mirror

and goes on top path (only that path).

Photon arrives at down-shifter A

and produces 2 further photons,

one ends up at signal detector

and other at idler.

No interference pattern

since no information carried to signal detector about other route.

Same true if photon took lower route through experiment.

Only way to get interference pattern at signal detector

is for information to arrive from both routes,

i.e., to have to be two signal beams, one from each down-shifter.

If true,

then down-shifters have to be activated

by something arriving at each one —->

makes it appear that photon from laser went both ways at half-silvered mirror.

Presence of two signal beams

doesn't imply that two photons are arriving at signal detector

i.e., measurement show that there is **only one at a time arriving**.

Most bizarre feature of experiment

is way in which interference pattern can be destroyed

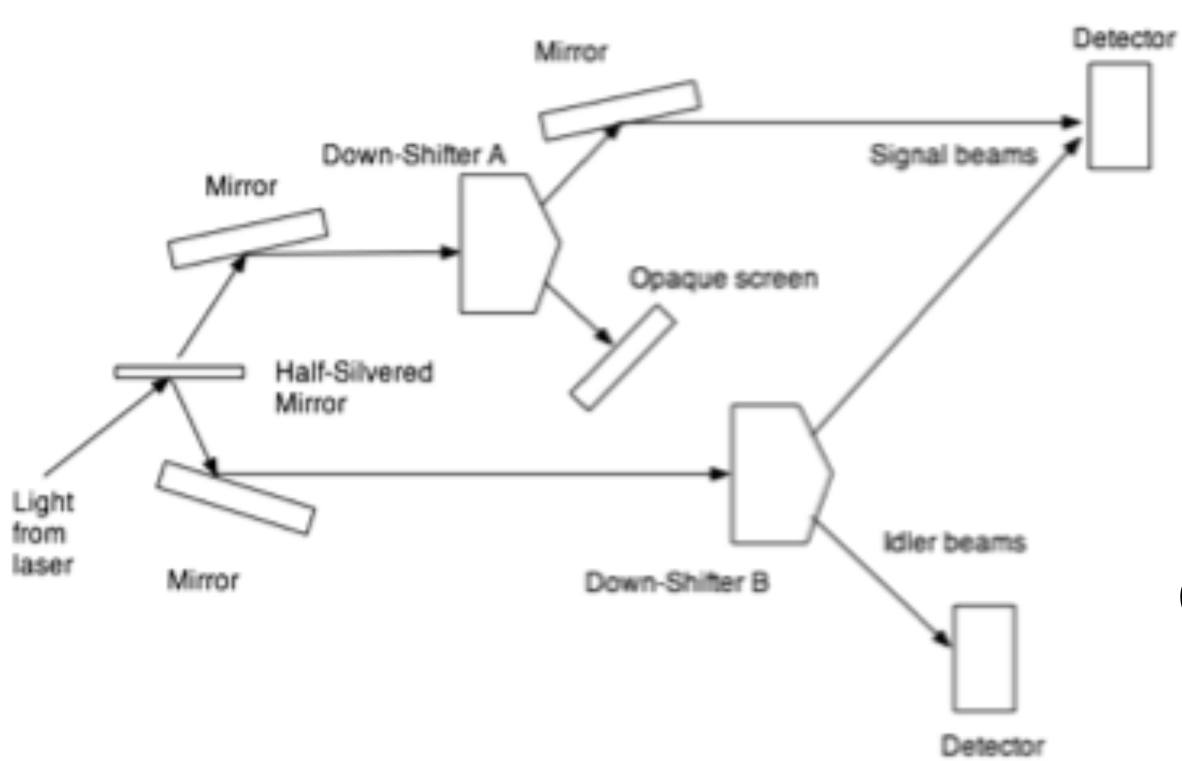
if we have ability to tell path of the photons, even if don't choose to use this information.

Threat of doing this, or rather fact that

experiment(context) is set up to allow possibility, is enough to destroy interference!

Dramatically confirmed by blocking one of idler signals, say from down-shifter A

Logic here remarkable.



Whenever photon picked up

at idler detector we know must have come

from down-shifter B.

Means that photon from half-silvered mirror

must have hit down-shifter to be converted

into photon that arrives at idler detector.

Can further deduce that other photon

from down-shifter B travelled to signal

detector and therefore is photon detected there.

Arrivals correlated!

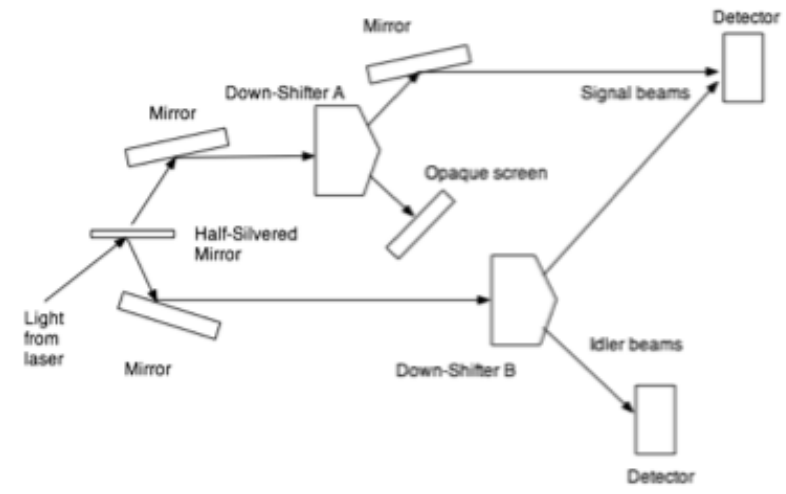
Tracing argument further back,

photon that definitely hits down-shifter B

must have come from half-silvered mirror.

There is no ambiguity about route that this

photon takes from mirror.



Nothing goes along top route; nothing produced from down-shifter A,

so interference pattern disappears.

As long as idler route from down-shifter A open,

have no way of telling which shifter a photon came from.

Ambiguity sufficient to guarantee interference pattern at signal detector.

If don’t know that photon at idler detector came from B (or A),

then don’t know about signal photon either.

Under those circumstances, can’t say which route photon took at half-silvered mirror,

so takes both routes or “something” like that.

Seems that behavior of photon determined by **context** of experiment as whole.

Know no photons coming from down-shifter A,

why does it matter that idler route from A is blocked?

How is information conveyed back to half-silvered mirror

so as to determine what happens there? **Words FAIL us, but QM MATH works!!**

Quantum Erasers (Complementarity and Entanglement)

In earlier discussions, we saw that closing the locality loophole involved switching between different analyzer orientations while emitted photons were still in flight.

The choice between the nature of the measurement was therefore delayed with respect to the transitions that originally created the photons.

Is it possible to make this delayed choice between measuring devices of a more fundamental nature?

For example, in discussion of double-slit measurements, found that if one allows sufficient a number of photons individually to pass through slits, one at a time, interference pattern will be built up.

It **seems** that photon has knowledge of all the paths as though it passes through both slits.

As noted earlier, a skeptical physicist who places a detector over one of slits to show that photon passes through one or other does indeed prove their point - photon detected, or not detected, at one slit.

But then the interference pattern can no longer be observed - because we have which-path information.

Advocates of local hidden variable theories could argue that the photon is somehow affected by the way we choose to set up our measuring device.

It thus adopts a certain set of physical characteristics (owing to existence of hidden variables) if apparatus is set up to show particle-like behavior, and adopts different set of characteristics if apparatus is set up to show wave interference.

However, if we design an apparatus that allows us to choose between these totally different kinds of measuring device, we could delay our choice until photon was (according to local hidden variable theory) **committed** to showing one type of behavior.

Suppose that photon cannot change its mind **after** it has passed through slits, when it discovers which kind of measurement is being made (whatever last sentence given in “words” may actually mean!!)

Ultimate Delayed Choice or Quantum Eraser Experiment

QM says systems can change behavior depending on measurements made on them or in response to decision that has not yet been made.

One part of an entangled pair can affect properties of its partner instantaneously, no matter where in universe its partner happens to be.

A so-called quantum eraser experiment has now been done...it dramatizes several aspects of this quantum strangeness at once.

Experiments dramatically show non-local effects, i.e., ability of experiment in one place to somehow influence outcome of another regardless of time or distance – but without transmitting any signals.

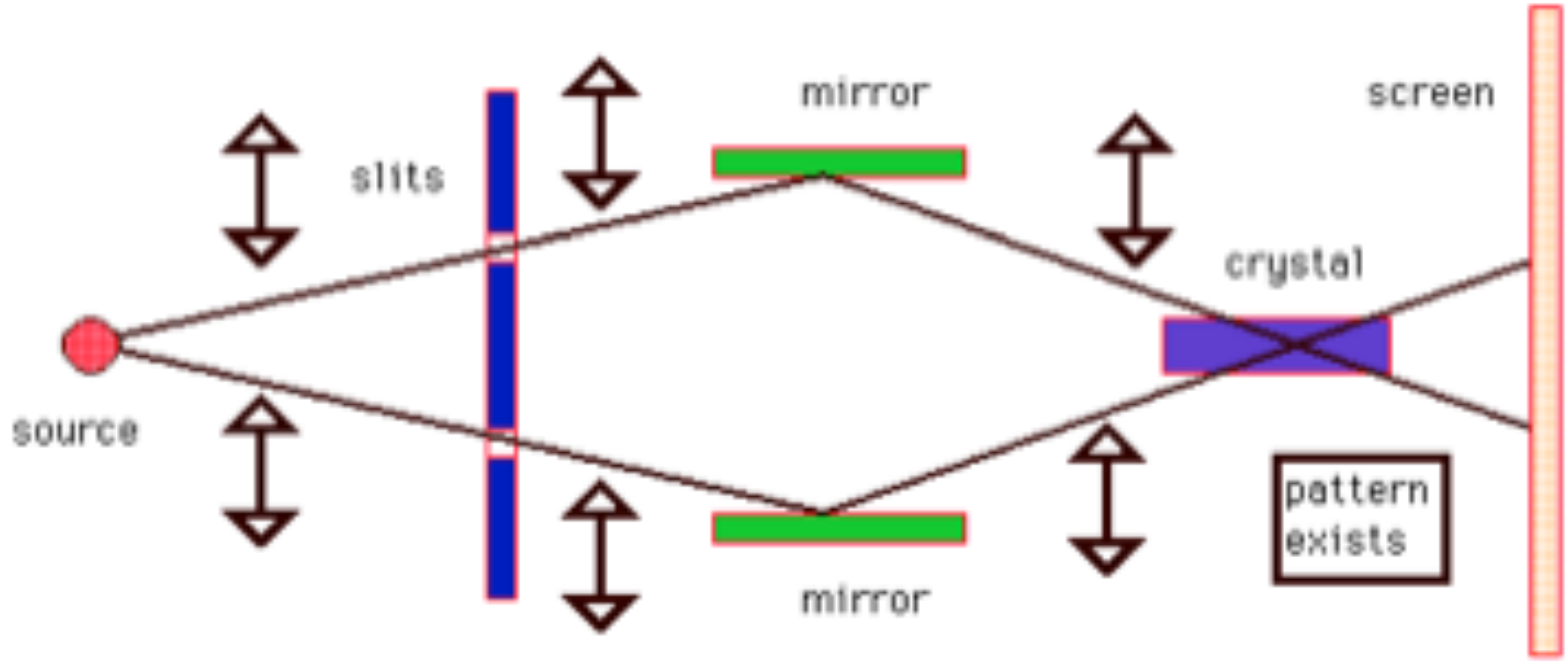
The idea behind the quantum eraser is to make paths(like those in a 2-slit system) distinguishable, which eliminates any interference effects, but then erase the which-path information just before light reaches screen where we actually observe the interference pattern.

QM predicts that interference pattern should then reappear and it does.

Consider a quantum mystery of the following type. **using words always ends too anthropomorphic**

A photon approaching slits will “need to know” whether or not there is an eraser further down path(in its future), so that it can decide whether to pass through slits as a superposition of all possibilities (paths are indistinguishable) and produce an interference pattern later on screen or that should behave as if paths are distinguishable and produce no interference pattern later on screen!!

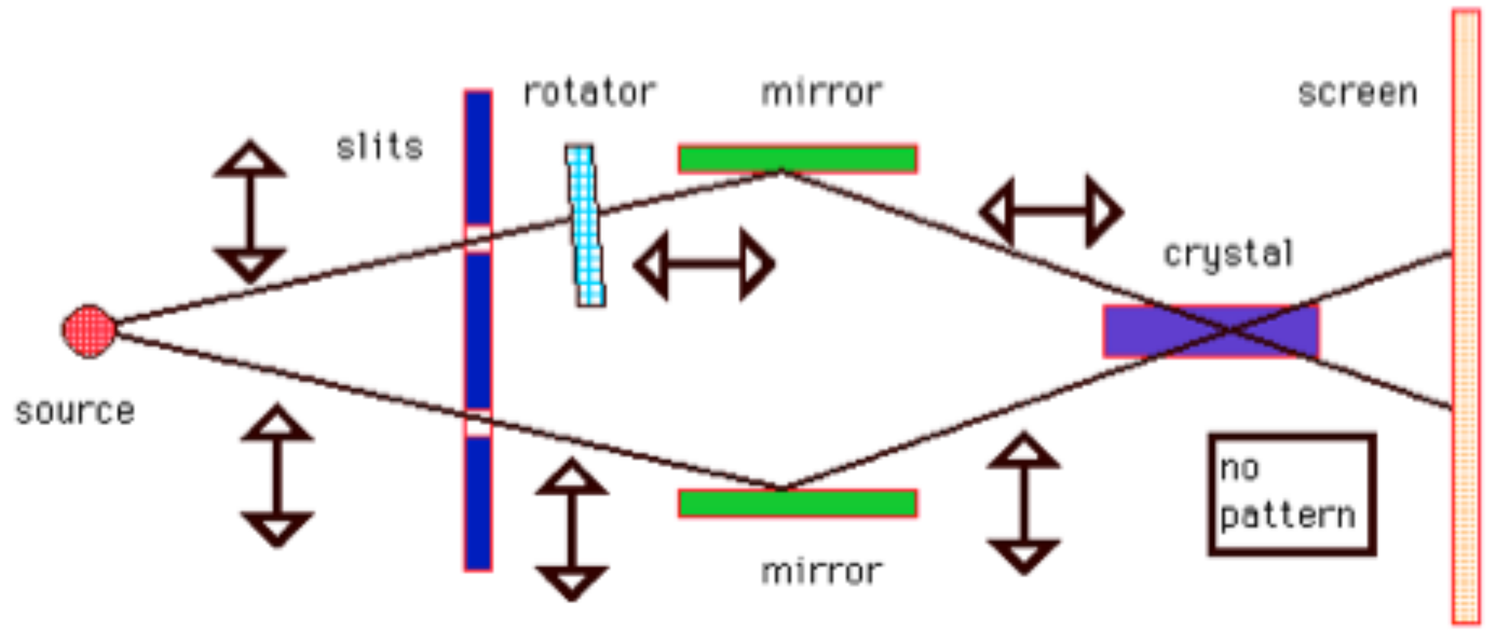
An early eraser experiment can be visualized as the two-slit experiment as shown in diagram below:



Photons passing through double slit are all vertically polarized.

By two paths as shown they can get to a recombination crystal, which remakes(without changing properties) beam that produces an interference pattern on screen, i.e., if paths are indistinguishable, then have superposition of all possible paths(2 paths in this case) and get an interference pattern.

Now insert a polarization rotator on one path only

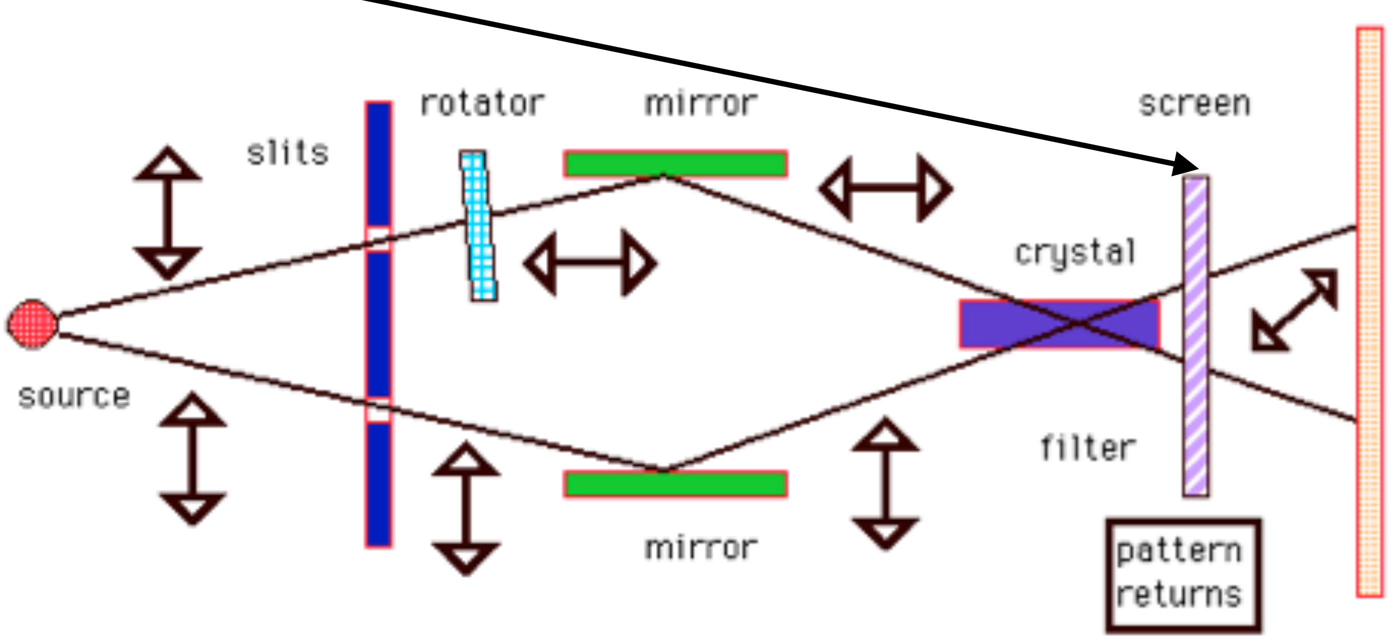


and rotate the polarization of photons to horizontal in that path.

Since the paths now produce distinguishable photons, we get particle like behavior and the interference should disappear.

It does!

Now add the quantum eraser after recombination crystal —> a polaroid at 45°.



The polaroid filter produces equal numbers of photons with both vertical and horizontal polarization (with respect to the new direction).

Half of new vertically polarized photons come from horizontal polarized photons on top path and half of new vertically polarized photons come from vertical polarized photons on bottom path (similarly for new horizontally polarized photons).

It is impossible to tell whether photon was vertically or horizontally polarized before the eraser.

Thus, the two paths are once again indistinguishable.

If the interference pattern reappears, then the photon approaching slits somehow needs to know whether or not there is an eraser down line so can decide whether to pass through slits as superposition and produce interference effects or as mixture and produce no interference.

Again, using words here makes things sound silly!! QM mathematics is clear!!

The pattern reappears immediately thus confirming the quantum mechanical prediction.

The filter erases the which-path information caused by the rotator.

A truly astounding result.

What does this say about the classical idea that it is the two-slit system that is the real cause of the interference pattern?

Worry:

Experimenters, knowing the fundamental importance of these results, wanted to leave no possible source of controversy intact.

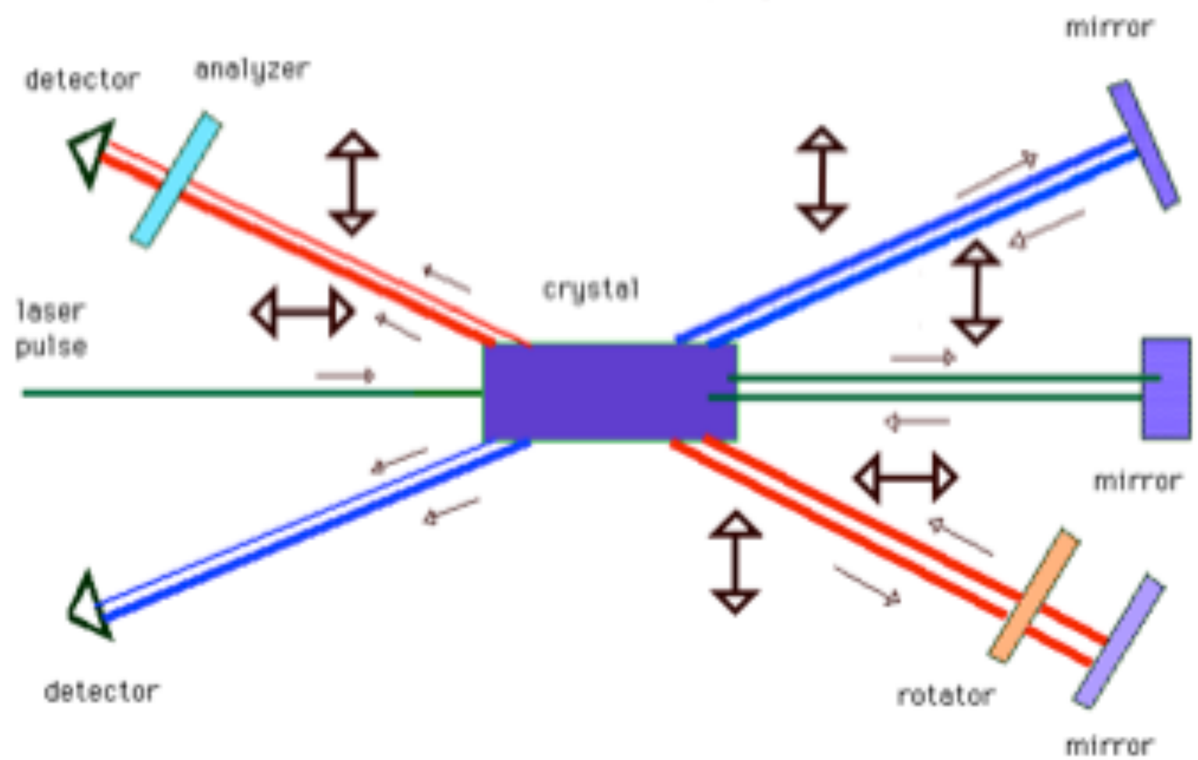
In the above version of experiment, there is a potential problem because the which-path information is carried by same photons that interfere, making experiment difficult to interpret.

New version of experiment...

Here which-path information is not carried by what one would naively call the interfering photon.

Instead, carried by 2nd photon and along the way it also demonstrates the striking non-local effects of QM.

Experiment looks like:



High intensity laser photons are sent into a parametric down-conversion non-linear crystal such as lithium iodate.

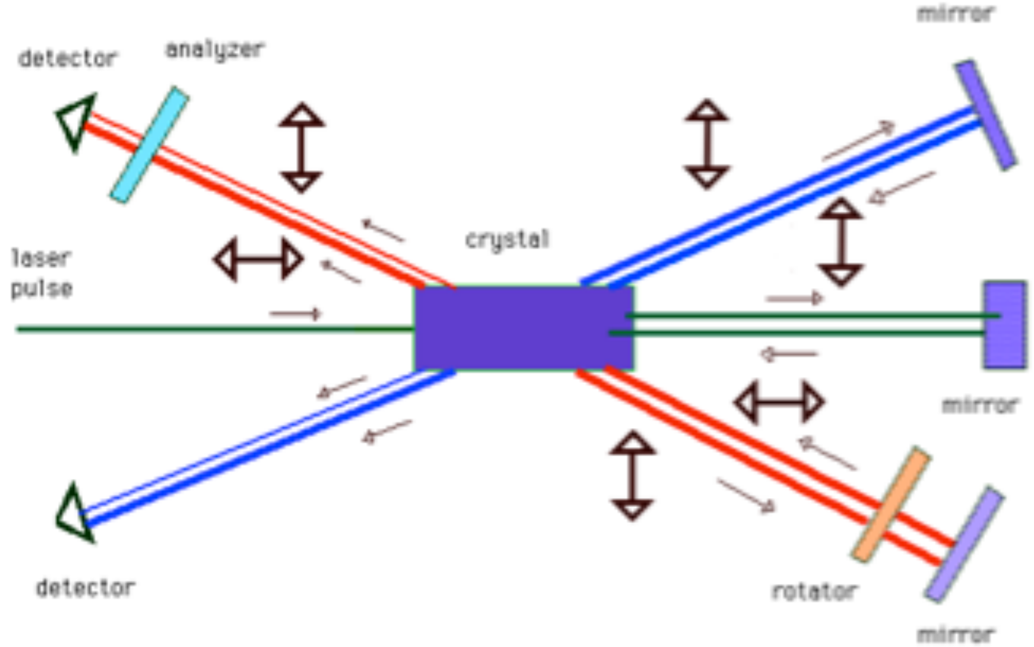
The crystal converts some incoming photons(green) into pairs of identical photons with lower energy and vertical polarization moving at an angle to original direction.

The photons are produced as entangled partners (in a superposition).

Measurement on one photon automatically tells us about the other with no direct measurement on 2nd photon necessary.

Twin beams exit the crystal at angle to the original path.

These are the thick red and blue lines



rotator not in place yet

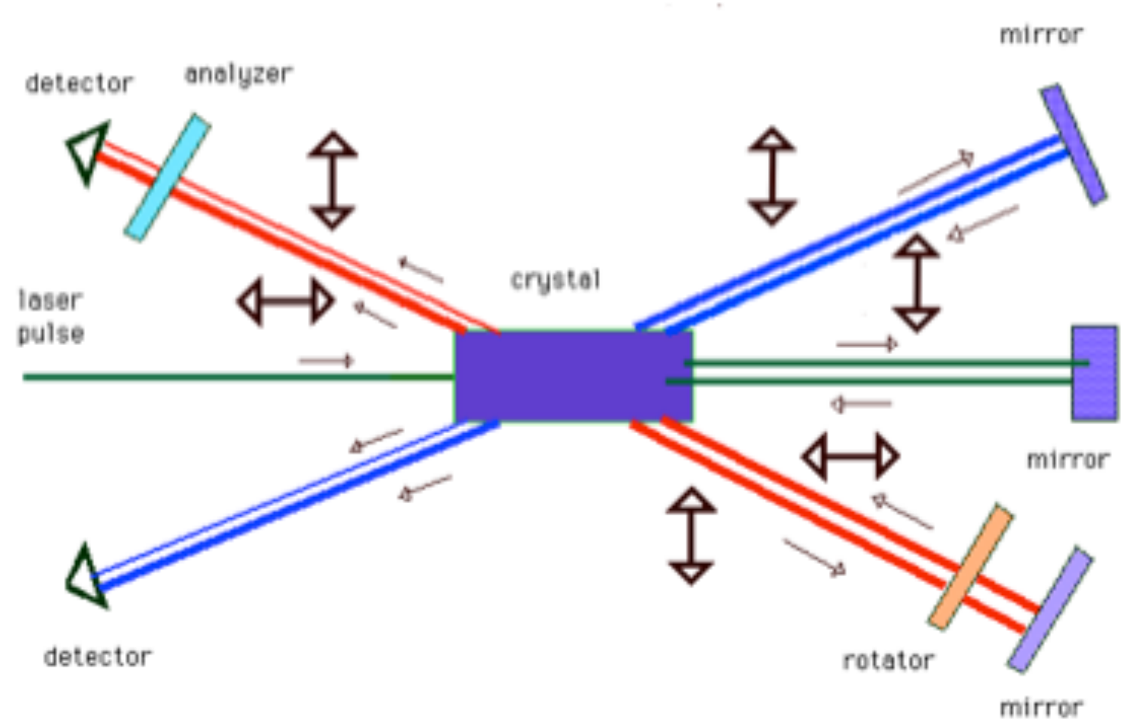
They are reflected back towards the crystal by mirrors and pass straight through it to detectors (the second-pass intensity is now too weak to cause down-conversions).

These are thick red and blue lines.

However, not all laser light gets converted on 1st pass.

Some goes straight through crystal to another mirror(green) and is reflected back into the crystal(still high intensity) where the crystal creates more photon pairs which then follow same path as other beams to the detectors.

These are thin red and blue lines.

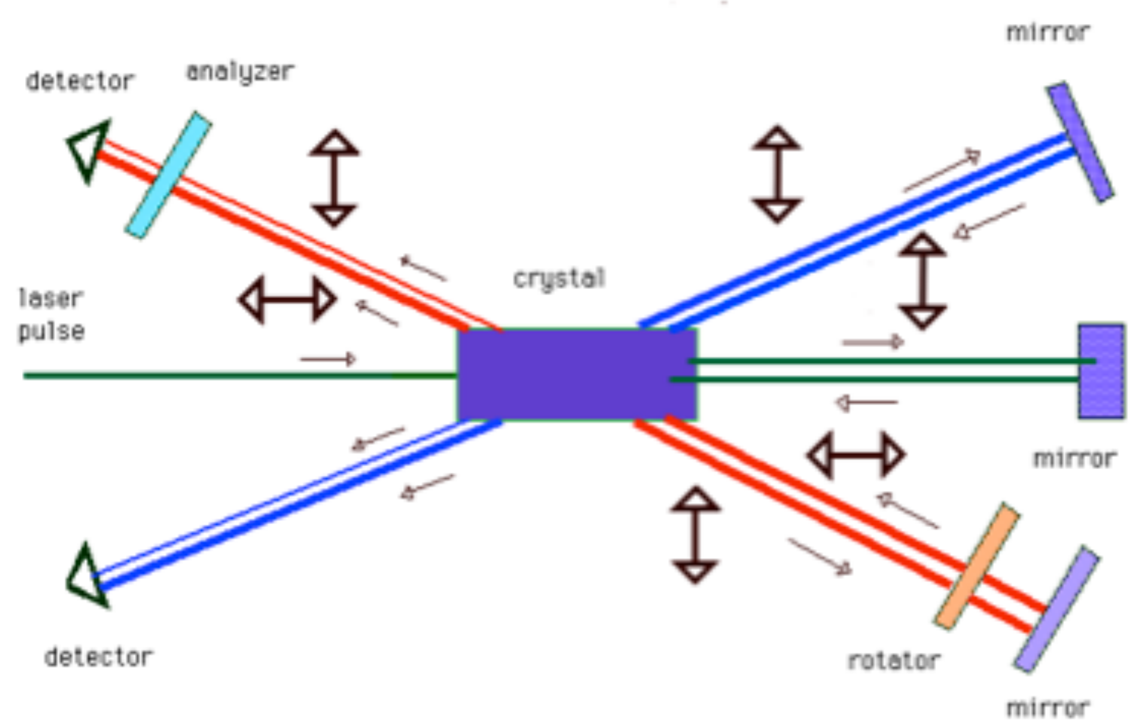


As a result, have two different beams heading towards each detector each having follow different paths.

Each pair of beams corresponds to a **separate** double slit experiment.

If there is no way to distinguish photons created on first pass through crystal from those created on 2nd pass —> both detectors should have interference patterns and they do!!

Now make one returning beam in one leg distinguishable from other by inserting polarization rotator into red path(as shown below) converting vertical to horizontal polarization in that leg.



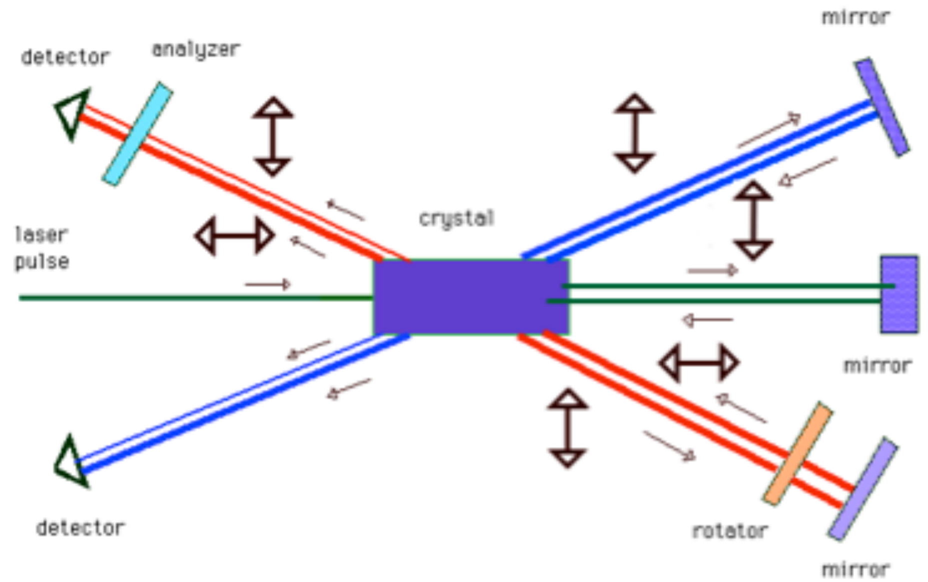
The interference pattern in top detector vanishes instantly as it should since the two (interfering) beams now have distinguishable paths.

Now, however, also find that interference pattern disappears in bottom detector!!!!!!! Why?

We have done nothing to disturb these beams so that beams in the bottom detector still correspond to indistinguishable paths and photons!!!!

Remember, however, that photons are created in entangled pairs, so when red-path photons become labeled with which path info, the same info becomes available to blue-path photons, no matter where they are!!!

This is the concept of non-locality at work.



To erase which-path info, we now add a 45○ polaroid in red path, just in front of detector.

The interference pattern immediately reappears in the top detector as it should (same as previous experiment).

It seemed however, that pattern did not reappear in bottom detector.

One might imagine this is so because erasing red-path photon information does not erase any information from blue path.

In addition, if it did reappear immediately, then we would be able to send signals faster than light with such an eraser.

However, as in EPR(we will see) experiment we find, if we bring the two data sets back together and compare them, the pattern had, in fact, been restored along both paths (need both data sets) to see the **correlations** among individual photons.

It seems like experiment is saying that the **correlations are entangled** and **not the state vectors**!!

That is the key piece of information that we have been looking for as we will see!!!!

We will have more to say about this later when we talk about measurement.

Alternatively, one can do a coincidence measurement, which only looks at those photons counted in each detector simultaneously and one can see interference pattern return directly, that is, which way and interference effects are being recorded for single photons.

Thus, inserting or removing which-path information transforms behavior of light throughout entire system simultaneously demonstrating amazing quantum eraser and dramatic non-local behavior of QM.

This experiment makes it clear that there is direct relationship between tests of complementarity(the concept that two contrasted theories, such as the wave and particle theories of light, may be able to explain a set of phenomena, although each separately only accounts for some aspects) and tests of quantum non-locality.

Interference effects are a direct manifestation of non-local behavior.

These effects can be encoded in the mathematical structure of quantum entanglement - in this case, the entanglement of the states is responsible for the interference with the state used to detect the "which way" information.

These states cannot be disentangled without forcing the system to reveal one type of behavior or other.

They cannot be disentangled to reveal both types of behavior simultaneously.

Though still a subject for debate, consensus is building that complementarity - and hence non-locality and entanglement - is the mechanism for mutual exclusivity in the dual wave-particle nature of quantum objects - what Richard Feynman described as central mystery at heart of quantum mechanics.

Uncertainty Principle

The Heisenberg Uncertainty Principle is at the center of quantum theory —> guarantees the consistency of the QM view of the microscopic world.

Yet there is a puzzle with Heisenberg's discovery:

> **Is it a statement of an epistemological limitation (what we can measure about world) or an ontological limit(fundamental property) written into world itself?**

Let us step back first and get some background ideas.

Expectation is Not Enough

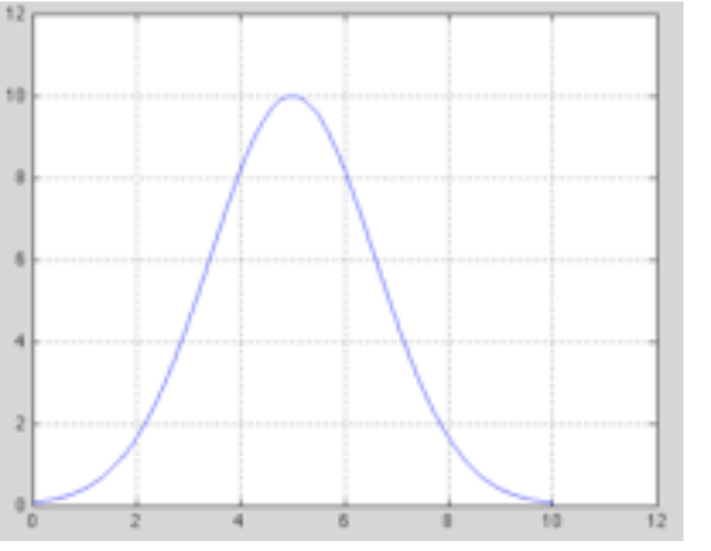
Take a collection of quantum objects, all in same state $|\phi\rangle$ and measure the value of physical property(call it O) for each —> collection of data values $\{o_i\}$, i = 1,2,3,4,……

Find average value of measurements (= to expectation value of operator $\hat{O}$ associated with property O in this state).

Expectation value, however, does not tell whole story.

Also need to know how wide is range of values - how spread out around average.

Collection $\{o_i\}$ could all be very close to average value or some of measurements might produce values considerably different from average, not because of experimental mistake, just due to nature of state being measured.



Whenever physicists think about a perfect set of experimental values —> **normal distribution** curve, as shown.

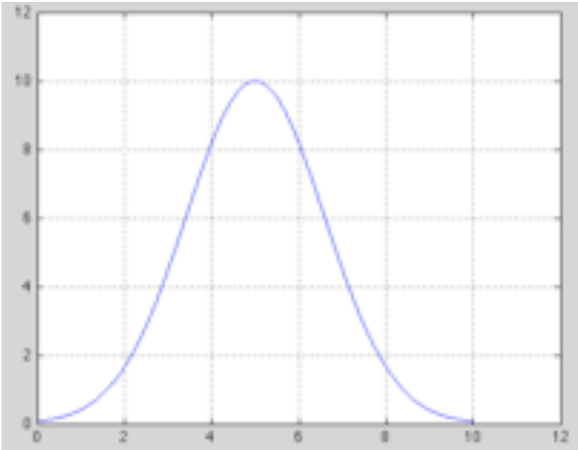


Figure —> results of identical measurements on collection of objects.

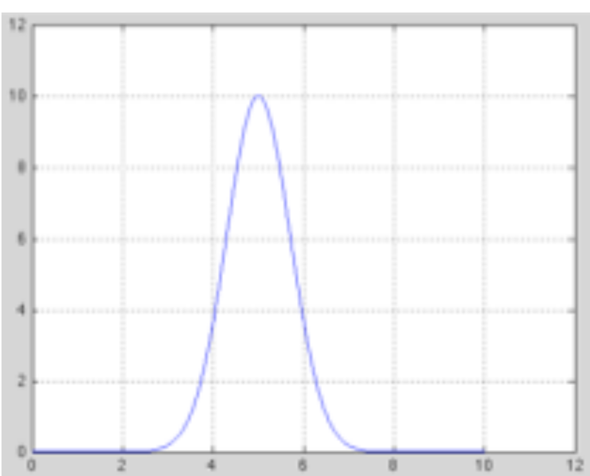
Normal distribution applies to all experimental results..

x–axis —> values of results obtained and y–axis —> number of times value turned up.

—> result 5 obtained 10 times and average value over all readings = 5.

However, exist quite a few times when value significantly different from 5 obtained!

Now consider figure.



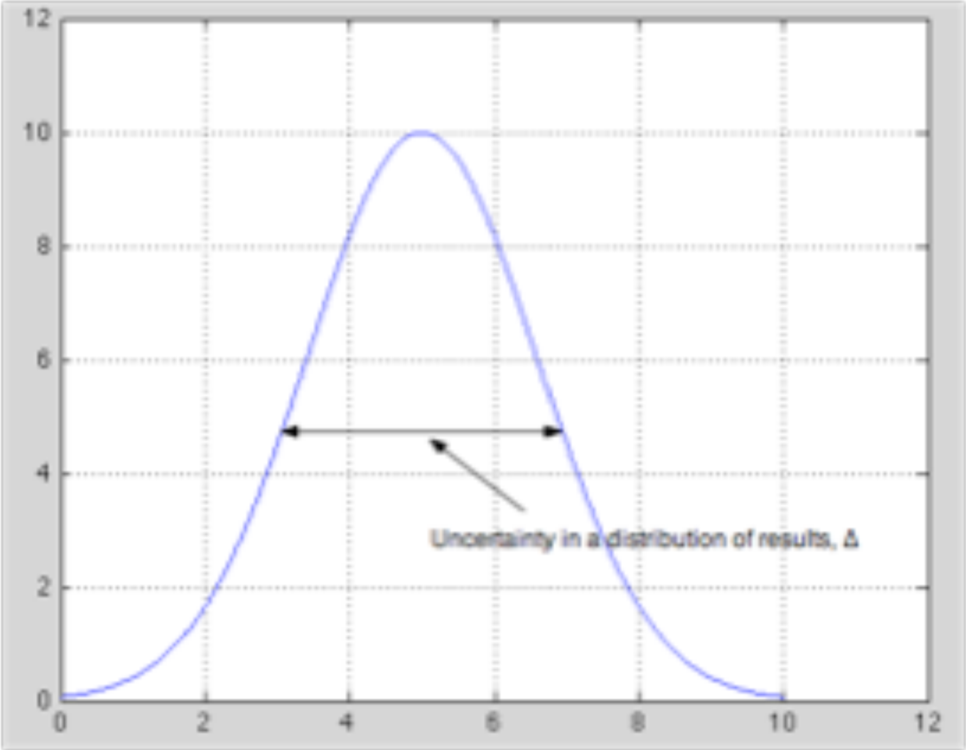
—> normal distribution curve, but much narrower spread of values (peak still at 5, average still = 5).

Much higher proportion of results closer to average than before.

—-> To tell full story of set of measurement results, need more than average - also need width(spread) of distribution as well.

Width obtained by taking each value, working out how far from average and averaging results.

Mathematicians —> **standard deviation**; quantum theory —> **uncertainty**. See figure.



In classical physics, spread, or uncertainty in experimental results generally attributable to normal experimental variations one finds due to small differences in objects being tested.

Also can be attributed to variations in apparatus used or environmental factors.

Experiments in quantum physics suffer from same variations, but in addition have to face what happens when objects being tested are not in eigenstates of variables being measured.

Taking position as an example. Start with state $|\phi\rangle$

$$|\phi\rangle = \sum_x \langle x \,|\, \phi\rangle \,|x\rangle \qquad \phi(x) = \langle x \,|\, \phi\rangle$$

NOTE: I am simplifying by letting x have discrete values!

—> making position measurements on set of particles in this state we will find a range of values, with each possible x value occurring according to probability

$$\phi^*(x)\phi(x) = |\phi(x)|^2$$

$\phi(x)$ = expansion coefficient = component

Plotting results and number for each value —> will draw out $|\phi(x)|^2$ curve with corresponding uncertainty Δx -> measurement distribution

Uncertainty —> direct measure of range of x values over which amplitude $\phi(x)$ is significant —> range of x values highly likely to occur when one makes a measurement.

Calculations then yield the uncertainty.

$$\langle\phi|\,\hat{x}\,|\phi\rangle = \langle x\rangle \qquad \textbf{expectation value}$$

$$\Delta x = \sqrt{\langle\phi|\,\hat{x}^2\,|\phi\rangle - (\langle\phi|\,\hat{x}\,|\phi\rangle)^2} = \sqrt{\langle x^2\rangle - \langle x\rangle^2} \qquad \textbf{uncertainty}$$

Shows that Δx is as much a property of state as $\langle x\rangle$.

Now, same state could be expanded over momentum basis $|p_x\rangle$, and experimental results of momentum measurements plotted —> uncertainty Δp_x.

What about case where we have an eigenstate?

Well, if happen to start with eigenstate, then can predict with absolute certainty what result of measurement of will be.

Plot of results —> no width (aside from unavoidable experimental fluctuations), giving $\Delta p_x = 0$.

However, there is a catch.

Momentum eigenstate cannot also be position eigenstate and vice versa.

When we expand a momentum eigenstate($\Delta p_x = 0$) over position basis —> infinite number of possible positions, leading to $\Delta x = \infty$ —> if $\Delta p_x = 0$, then $\Delta x = \infty$ and vice versa!

Heisenberg's Principle

What about states (majority) that are neither position nor momentum eigenstates?

Given such state $|\phi\rangle$ have

$$|\phi\rangle = \sum_x \phi(x)\,|x\rangle \Rightarrow \langle x\rangle, \Delta x \quad , \quad |\phi\rangle = \sum_p \phi(p)\,|p\rangle \Rightarrow \langle p\rangle, \Delta p$$

i.e., position representation —> $\langle x\rangle$ and Δx and momentum representation —> $\langle p_x\rangle$ and Δp_x.

Since it is **same state** expanded in both representations there must exist a connection between Δx and Δp_x.

Connection was first demonstrated by Heisenberg.

THE HEISENBERG UNCERTAINTY PRINCIPLE $$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

—-> for given state, the smaller the range of probable x values involved in position expansion, the larger range of p_x values involved in momentum expansion, and vice versa.

Key part of expression is ≥ sign —> product $\Delta x \Delta p_x$ **cannot be less than** $\hbar/2$.

Measurement results might be such that $\Delta x \Delta p_x$ is much greater than $\hbar/2$, which is fine; but can never find situation where $\Delta x \Delta p_x$ less than $\hbar/2$.

Quantum theory places a **limit on the precision** of two sets of **related** measurements.

Saying this a different way:

Δx is telling us range of x values where $\phi(x)$ is significantly large if we expand state over the position basis.

Similary, Δp_x is corresponding range of $\Psi(p_x)$ if expand same state over momentum basis.

Uncertainty principle is relating range of $\phi(x)$ to that of $\Psi(p_x)$.

Make position measurement, then most likely get value within Δx of expectation value $\langle x \rangle$.

Make momentum measurement **instead (not after)**, then most likely get value within Δp_x of $\langle p_x \rangle$.

In last statement, the "not after" part is important.

Making position measurement, "**collapses**" state into position eigenstate —> different momentum expansion than original state had.

Uncertainty principle only relates Δx and Δp_x for **same state**.

Here is possible question about the principle.

What about object that is not moving?

Is it not sitting perfectly still (precise momentum, zero) at a precise spot?

Well, ignoring idea that nothing can be perfectly still (orbiting sun, etc), the immediate point is that uncertainty principle **prevents** such state from happening in microworld.

So What?

People drew philosophical conclusions from uncertainty principle immediately.

Heisenberg targeted the principle of causality.

Classical physics —> view that powerful computer, given detailed information about positions, momenta, and forces at work on every particle in universe, could predict future of universe.

Such thinking —> doubts about free will, human mind, morality, etc…

Heisenberg's attack simple.

Uncertainty principle prevents knowing both position and momentum of any particle with sufficient accuracy, ultimately rendering predictions impossible.

Causality or notion that everything that happens is **caused by something happening previously** had been central to physics for hundreds of years.

Experiments in quantum world undermined this thinking with unpredictable outcomes and then came uncertainty principle undermining basis of all physics, namely, ability to measure with infinite accuracy.

—> nature seemed to be built on a set of principles completely contrary to approach science had been using for generations.

It is possible to regard quantum challenge as being **in** microworld and of **no** relevance to macroworld in which we live.

However, study of chaotic systems has shown that small effects do not necessarily lead to small consequences and that **no** measurement (quantum or otherwise) can, even in principle, yield sufficiently accurate results to do job.

Putting aside the philosophical debate, the uncertainty principle is **strange**.

Basic science education —> vital role of measurement and precision.

Measurement has to be a faithful rendering of what really out there, or no point in exercise.

Realism is natural state of physicist.

When run into quantum physics —> begin to doubt instincts - surely it cannot really be like that!

Some say — I'm Not Sure What Uncertainty Means...

Serious point here.

Much is written about meaning and significance of uncertainty principle.

All depends on what side of realist/instrumentalist divide one stands and on exactly what type of interpretation one favors.

Most important aspect of uncertainty principle, from a philosophical point of view, is

whether to take it to be limitation on what we can know about world

or as an inherent limitation on the world itself.

In philosophical speak, is it an epistemological(potential) statement, or is it ontological(actual)?

Whatever Heisenberg thought: first arguments used to demonstrate uncertainty principle had epistemological tinge —> famous gamma ray microscope.

Heisenberg envisaged microscope constructed to use gamma rays rather than light waves.

Microscope could be used to determine position of something as small as electron by detecting gamma rays that scattered off electron.

Path of gamma rays pinpoints location of electron.

Snag is wavelength of gamma rays.

Well known in optics —> limitation on detail that can be seen by microscope.

Limitation depends on wavelength of electromagnetic waves used.

To view level of detail sufficient to locate something as small as electron, quite high-energy gamma rays (very short wavelength) are needed.

Such gamma ray scattering off an electron would transfer a significant amount of energy

causing electron to recoil in an unspecified direction, affecting its momentum.

No way of determining energy transferred and thus, subsequent disturbance to electron's momentum.

If wanted to do that, would have to measure the gamma ray before it hit and that process would then disturb gamma ray.

Key point:

Gamma ray microscope argument assumes electron has position **and** momentum;

Problem: interaction with measuring device —> cannot accurately determine these quantities.

Every time measuring device makes measurement, it interacts with thing it is measuring.

In classical physics, scale of interactions either too small to worry us or can be calculated and accounted for in sensible way.

This cannot be said in quantum physics.

Scale of interaction too large and causes major changes to other properties of object being measured.

Also interaction is inherently unpredictable.

Any attempt to stick another measuring device in to judge scale of interaction will not work as second instrument interacts in manner that cannot be detected.

Infinite regression follows, which like most infinite regressions gets us nowhere.

An important argument which cuts once again to nature of quantum state.

If electron happens to be in an eigenstate of position (or momentum) can say with certainty what result of measuring that position (or momentum) will be.

If electron is not in eigenstate then what can we say?

In QM, one can predict various values to be revealed by measurement, and relative probabilities, but what does that tell us?

What is position (or momentum) of electron in this situation?

If take view that states refer to collections of objects, rather than individual objects, then way forward quite clear —> use statistics!

Inside collection, electrons identical, within limitations laid down by nature of quantum state.

However, could be hidden variables - do not know how to measure - which take different values in collection.

When measurement made, variables determine outcome, but because do not know values, all looks random.

Electrons have perfectly well-determined positions and momenta, but cannot know in advance what they will be since cannot access physics that determines values.

Gamma ray microscope example runs contrary to this way of thinking.

Argument is clearly dealing with single electrons, not collections.

However, does assume that positions and momenta are really there, just partially hidden.

Such hidden variable type view —> uncertainty principle expressing epistemological limits of current science.

Heisenberg himself took view that uncertainty principle was an ontological thing.

To him uncertainty principle expressed limits to which classical ideas of momentum, or position, can be applied in given situation.

Harks back to earlier argument about how classical definition of momentum cannot be applied in quantum physics.

If take this view, then have to say that object that is not in eigenstate of position simply **does not have a position**.

Concept has no meaning in such a state.

Heisenberg would have said that position of particle was a **latent or potential property**.

See supplementary readings — Taking Heisenberg's Potentia Seriously - Epperson

When measurement occurs, latency becomes actuality and classical ways of thinking can now be applied.

—> realistic view, but acknowledges that assumptions about what should be classified as real, may have to be enlarged.

To Heisenberg latent properties should be thought of as real.

Gamma ray microscope was used by Heisenberg as way of introducing uncertainty principle.

However, he held back from stating that was an ontologically valid description of what would actually happen.

Yet More Uncertainty

Have tried to show how uncertainty principle arises as natural consequence of ability to expand state in various bases.

Although illustrated with position and momentum, are many other physical variables with basis sets that can be used to expand a state.

Unless position and momentum have special significance, must be versions of uncertainty principle that apply to other physical variables also.

Basic rule:

if choose to expand state over pair of bases,

and physical variables involved

happen to be **”conjugate” or “incompatible”** variables,

then going to end up with an uncertainty principle.

State cannot be an eigenstate of two conjugate variables at same time

—> cannot get a definite value from measuring either variable,

and then uncertainty relationship must apply.

Non-Compatibility

If operator acts on its eigenstate, result is that state remains same,

but now multiplied by eigenvalue.

$$\hat{p}\,|p_1\rangle = p_1\,|p_1\rangle \quad , \quad \hat{x}\,|x_1\rangle = x_1\,|x_1\rangle$$

If state not an eigenstate, then strange things happen.

Consider two physical variables represented by operators $\hat{O}_1$ and $\hat{O}_2$,

which happen to be **compatible ==> simultaneously measurable**.

Then state $|\phi\rangle$ can be an eigenstate of both.

What happens when apply both operators on this state, one **after** other?

$$\hat{O}_1 |\phi\rangle = o_1 |\phi\rangle$$
$$\hat{O}_2 o_1 |\phi\rangle = o_1 \hat{O}_2 |\phi\rangle = o_1 o_2 |\phi\rangle$$

In addition, could also do things other way around

$$\hat{O}_2 |\phi\rangle = o_2 |\phi\rangle$$
$$\hat{O}_1 o_2 |\phi\rangle = o_2 \hat{O}_1 |\phi\rangle = o_2 o_1 |\phi\rangle$$

which says that

$$\hat{O}_2 \hat{O}_1 |\phi\rangle = \hat{O}_1 \hat{O}_2 |\phi\rangle$$

or

$$\left[\hat{O}_2 \hat{O}_1 - \hat{O}_1 \hat{O}_2\right] |\phi\rangle = 0$$

Bracket is regarded as being so important in quantum theory that given own name:

THE COMMUTATOR

$$Commutator\left(\hat{O}_2, \hat{O}_1\right) = \left[\hat{O}_2, \hat{O}_1\right] = \left[\hat{O}_2\hat{O}_1 - \hat{O}_1\hat{O}_2\right]$$

If two operator commute, then $\left[\hat{O}_2, \hat{O}_1\right] = 0$, and **can** (not must) have simultaneous eigenstates.

If don't commute, then simultaneous eigenstates not possible and uncertainty relationship follows.

RULE:

Non-compatible physical variables represented by operators that do not commute and so are linked by uncertainty principle.

$$\Delta O_1 \Delta O_2 \geq \frac{1}{2}\left\langle i\left[\hat{O}_2, \hat{O}_1\right]\right\rangle$$

Derive later

Consider Time Again....

Exists uncertainty relationship that does not follow directly from these arguments, namely,

ENERGY/TIME UNCERTAINTY $\Delta E \Delta \tau \geq \frac{\hbar}{2}$ **Derive later**

where τ related to characteristic development time of system, i.e., given by $\tau = \frac{\langle A \rangle}{d\langle A \rangle / dt}$ for any physical variable of system.

Let us see how this works.

Rule 7 states a link between operators and measurement of physical variables.

Have seen several examples of such operators and learned to calculate expectation values.

Where things start to go weird is over a possible $\hat{t}$ operator.

Can measure position, momentum, energy, etc, but how do you measure time?

Let us be clear about this: we can measure duration (e.g., with stopwatch) but what physical measurement tells you actual time?

Issues involved are very subtle, and not every quantum physicist would agree with what I am about to say, but here goes.

Conventional quantum mechanics is acted out on stage formed from three directions in space and one universal time (x,y,z,t).

Think of space arena as giant spider's web; with threads so closely woven that web appears to be continuous disk.

Spider able to crawl around and occupies position at any moment on web.

Position of spider is physical variable -> property of spider's state that can be measured. In quantum mechanics, position represented by operator $\hat{x}$.

In some accounts, spider position operator denoted as $\hat{q}$ to make clear **not same** as the x-value, which is part of web, not part of spider's state.

Time rather different than this.

If can picture it at all, think of it as line stretching out to infinity in both directions.

Spider on line would not be stationary, but sliding along line as time passes.

In most instances cannot conceive of experiment that would enable us to to measure spider's position on time line.

—-> time is not property of spider's state; **hence there is no time operator**.

But what about a system that is changing with time - a falling ball, for example?

Here the momentum of ball increases as time passes, suggesting that it is "time variable".

However, a physical variable that is changing does not necessarily change at a uniform rate (momentum increases at increasing rate),

and when it hits the ground, time as measured by momentum would stop,

although t continues.

There are some systems in which a physical variable corresponds in a more satisfactory way with the passage of time —> clocks.

The second hand of analog clock has angular position θ, which changes with time.

Measuring the angle gives a measure of time for system.

In a real clock there will also be an energy, which will tend to depend on θ so that we can define an energy operator $\hat{E}$ and a time operator $\hat{\theta}$, which will not commute and so lead to uncertainty relationship.

However, θ is only a physical variable of a clock and cannot be applied as time operator to any other system.

Think about Energy/Time Uncertainty another way……

Instead, ask this question: how can we tell if something has changed?

A physical system will have various properties that can be measured.

If suspect something changed system state, can go through new set of measurements to see what's going on.

How to do it in quantum physics?

Every measurement made will “collapse state” into an eigenstate, so can’t necessarily repeat measurements on same state.

If do lots of measurements on collection of identical systems in same state,

how do we tell if values are different

because of some change affecting all systems in collection,

or is just normal spread of measurements

that comes with state not an eigenstate to begin with?

Suppose we are trying to distinguish between range of values caused by inherent

uncertainty in state itself and range caused by some

causal evolution ($\hat{U}(t)$) taking place in systems.

Way to tell is to compare expectation value $\langle O \rangle$ with uncertainty ΔO.

If remeasure collection of systems

—> difference between new expectation value $\langle O' \rangle$ and old greater than ΔO

(i.e., $(\langle O' \rangle - \langle O \rangle) \geq \Delta O$),

then can be sure that system state has actually changed.

Then becomes question of how long have to leave system to evolve, in given situation,

before change can be detected in this manner.

Can work it out if employ rate of change with time (a derivative)operator, d/dt (a derivative)

In elementary physics, distance traveled by moving object is rate at which traveling (speed) multiplied by time travels for ($d = s \times t$).

Equally well , if know rate at which any quantity changing and multiply by time period, that will produce amount by which quantity has changed in that time.

If quantity happens to be an expectation value then $d\langle O \rangle/dt\ \Delta t$ is amount by which expectation value has changed in time duration Δt.

The condition for a measurable change then becomes

$$\frac{d}{dt}\langle O\rangle \Delta t \geq \Delta O \quad \text{or} \quad \Delta t \geq \frac{\Delta O}{\frac{d}{dt}\langle O\rangle}$$

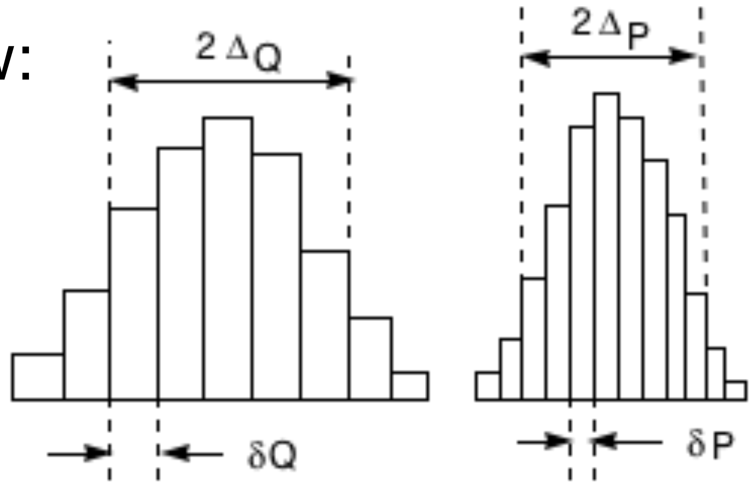
—> how we should interpret Δt in energy-time uncertainty relationship.

It represents duration of time that we have to wait to be sure that energy has changed by amount at least equal to ΔE.

The Meaning of the Indeterminacy Relations

What is significance of indeterminacy relations in the world of experimental physics?

Consider experimental results shown below:



—> frequency distributions for results of independent measurements of Q and P on ensemble of similarly prepared systems,

i.e., on each of large number of similarly prepared systems one performs a single measurement (either Q **or** P).

Histograms are statistical distribution of results.

Standard deviations (variances) shown must satisfy (according to theory) relation

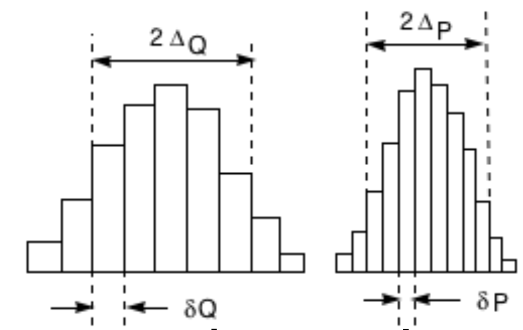
$$\Delta_Q \Delta_P \geq \frac{\hbar}{2}$$

They must be distinguished from resolution of individual measurements, δQ and δP.
Let me emphasize these points:

(1) Quantities Δ_Q and Δ_P are not errors of measurement.

Errors or preferably the resolutions of the Q and P measuring instruments are δQ and δP .

They are logically unrelated to Δ_Q and Δ_P and to uncertainty relations except for practical requirement that if



$$\delta Q > \Delta_Q \quad (\text{ or } \delta P > \Delta_P)$$

then will not be possible to determine Δ_Q (or Δ_P) in an experiment and experiment cannot test any uncertainty relation.

(2) An experimental test of indeterminacy relation does not involve simultaneous measurements of Q and P, but involves measurement of one or other of these dynamical variables on each independently prepared representative of particular state being studied.

Why am I being so picky here?

Quantities Δ_Q and Δ_P are often misinterpreted as errors of individual measurements.

Probably arises because Heisenberg's original paper on this subject, published in 1927, was based on early version of quantum mechanics that predates systematic formulation and statistical interpretation of quantum mechanics as it exists now.

Derivation carried out in this lecture now was not possible in 1927!

Derivation of the Uncertainty Relations in General

Very mathematical - just show how it can be done……

Given two Hermitian operators $\hat{A}$ and $\hat{B}$ and state vector $|\psi\rangle$, **define** two new operators

$$\hat{D}_A = \hat{A} - \left\langle \hat{A} \right\rangle \quad , \quad \hat{D}_B = \hat{B} - \left\langle \hat{B} \right\rangle$$

where $\langle ... \rangle$ equals average or expectation value in a state $|\psi\rangle$.

In statistical analysis of data, one uses quantity called standard or mean-square deviation as measure of uncertainty of observed quantity.

This is defined, for set of N measurements of quantity q by

$$(\Delta q)^2 = (\text{standard deviation})^2 = \frac{1}{N} \sum_{i=1}^{N} (q_i - q_{average})^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (q_i)^2 - \frac{1}{N} \sum_{i=1}^{N} (q_i q_{average}) - \frac{1}{N} \sum_{i=1}^{N} (q_{average} q_i) + \frac{1}{N} \sum_{i=1}^{N} (q_{average})^2$$

$$= (q^2)_{average} - (q_{average})^2$$

where have used

$$q_{average} = \frac{1}{N} \sum_{i=1}^{N} q_i$$

In analogy, define mean-square deviations for $\hat{A}$ and $\hat{B}$ as

$$(\Delta\hat{A})^2 = \left\langle \hat{A}^2 \right\rangle - \left\langle \hat{A} \right\rangle^2 = \left\langle \left(\hat{A} - \left\langle \hat{A} \right\rangle\right)^2 \right\rangle = \left\langle \hat{D}_A^2 \right\rangle$$
$$(\Delta\hat{B})^2 = \left\langle \hat{B}^2 \right\rangle - \left\langle \hat{B} \right\rangle^2 = \left\langle \left(\hat{B} - \left\langle \hat{B} \right\rangle\right)^2 \right\rangle = \left\langle \hat{D}_B^2 \right\rangle$$

Then we have

$$(\Delta\hat{A})^2(\Delta\hat{B})^2 = \left\langle \hat{D}_A^2 \right\rangle \left\langle \hat{D}_B^2 \right\rangle$$

Now we assume that

$$\left[\hat{D}_A, \hat{B}\right] = \left[\hat{A}, \hat{B}\right] = \left[\hat{D}_A, \hat{D}_B\right] = i\hat{C}$$

they do not commute

where $\hat{C}$ is also Hermitian operator and let

$$|\alpha\rangle = \hat{D}_A |\psi\rangle = \left(\hat{A} - \left\langle \hat{A} \right\rangle\right) |\psi\rangle \quad , \quad |\beta\rangle = \hat{D}_B |\psi\rangle = \left(\hat{B} - \left\langle \hat{B} \right\rangle\right) |\psi\rangle$$

Then we have

$$\left(\Delta\hat{A}\right)^2 = \left\langle \hat{D}_A^2 \right\rangle = \langle\psi|\,\hat{D}_A^2\,|\psi\rangle = \left(\langle\psi|\,\hat{D}_A\right)\left(\hat{D}_A\,|\psi\rangle\right) = \langle\alpha \mid \alpha\rangle$$
$$\left(\Delta\hat{B}\right)^2 = \left\langle \hat{D}_B^2 \right\rangle = \langle\psi|\,\hat{D}_B^2\,|\psi\rangle = \left(\langle\psi|\,\hat{D}_B\right)\left(\hat{D}_B\,|\psi\rangle\right) = \langle\beta \mid \beta\rangle$$

Schwarz inequality says that for any two vectors we must have the relation

$$\langle\alpha \mid \alpha\rangle\,\langle\beta \mid \beta\rangle \geq |\langle\alpha \mid \beta\rangle|^2$$

Therefore we have

$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 = \left\langle \hat{D}_A^2 \right\rangle\left\langle \hat{D}_B^2 \right\rangle = \langle\alpha \mid \alpha\rangle\,\langle\beta \mid \beta\rangle \geq |\langle\alpha \mid \beta\rangle|^2 = \left|\langle\psi|\,\hat{D}_A\hat{D}_B\,|\psi\rangle\right|^2 = \left|\left\langle \hat{D}_A\hat{D}_B \right\rangle\right|^2$$

which gives an uncertainty relation of the form

$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 \geq \left|\left\langle \hat{D}_A\hat{D}_B \right\rangle\right|^2$$

Now

$$\left|\langle \hat{D}_A \hat{D}_B \rangle\right|^2 = \left|\langle \frac{1}{2}[\hat{D}_A, \hat{D}_B] + \frac{1}{2}\{\hat{D}_A, \hat{D}_B\}\rangle\right|^2$$

$$[\hat{D}_A, \hat{D}_B] = [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$\{\hat{D}_A, \hat{D}_B\} = \{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$$

$$\left|\langle \hat{D}_A \hat{D}_B \rangle\right|^2 = \left|\langle \frac{1}{2}[\hat{A}, \hat{B}] + \frac{1}{2}\{\hat{A}, \hat{B}\}\rangle\right|^2$$

and

$$\left[\hat{A}, \hat{B}\right]^\dagger = -\left[\hat{A}, \hat{B}\right] \to anti - hermitian \to expectation\ value\ is\ imaginary$$

commutator

and

$$\left\{\hat{A}, \hat{B}\right\}^\dagger = \left\{\hat{A}, \hat{B}\right\} \to hermitian \to expectation\ value\ is\ real$$

anticommutator

Therefore,

$$\left(\Delta\hat{A}\right)^2 \left(\Delta\hat{B}\right)^2 \geq \left|\left\langle \tfrac{1}{2}\left[\hat{A}, \hat{B}\right] + \tfrac{1}{2}\left\{\hat{A}, \hat{B}\right\}\right\rangle\right|^2$$
$$\left(\Delta\hat{A}\right)^2 \left(\Delta\hat{B}\right)^2 \geq \tfrac{1}{4}\left|\left\langle i\hat{C}\right\rangle + a\right|^2$$

where a = real number.

We then have

$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 \geq \tfrac{1}{4}\left|\left\langle i\hat{C}\right\rangle + a\right|^2 = \tfrac{1}{4}\left|a\right|^2 + \tfrac{1}{4}\left|\left\langle\hat{C}\right\rangle\right|^2$$
$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 \geq \tfrac{1}{4}\left|\left\langle\hat{C}\right\rangle\right|^2$$

since $|a|^2/4 \geq 0$.

If $\left[\hat{A},\hat{B}\right] = i\hat{C} = 0$ then get

$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 \geq 0$$

and have no uncertainty relation between observables. They are **compatible**!

On other hand, if $\hat{A} = \hat{x}$ and $\hat{B} = \hat{p}_x$, have

$$[\hat{x},\hat{p}_x] = i\hat{C} = i\hbar\hat{I}$$

which means

$$\left(\Delta\hat{A}\right)^2\left(\Delta\hat{B}\right)^2 \geq \frac{\hbar^2}{4} \qquad \text{or} \qquad \Delta\hat{x}\Delta\hat{p}_x \geq \frac{\hbar}{2}$$

which is Heisenberg uncertainty principle. **It is simply the Schwarz inequality!!** which every sophomore math student understands! It is in QM because we assumed QM is governed by Linear Algebra!

Hopefully, that removes any mysticism associated with the principle!

Finally we derive the Time-Energy Uncertainty Relations

Use of time-energy uncertainty relations in most textbooks is simply incorrect.

Now show (very mathematical) the most one can say about such relations.

Need to derive time-dependence of expectation values. We have

$$\left\langle \hat{Q} \right\rangle = \langle\psi(t)|\, \hat{Q}\, |\psi(t)\rangle \qquad |\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\, |\psi(0)\rangle \rightarrow \langle\psi(t)| = \langle\psi(0)|\, e^{-i\hat{H}t/\hbar}$$

so that

$$\frac{d\left\langle \hat{Q} \right\rangle}{dt} = \frac{d}{dt}\langle\psi(t)|\, \hat{Q}\, |\psi(t)\rangle = \frac{d}{dt}\langle\psi(0)|\, e^{i\hat{H}t/\hbar}\hat{Q}e^{-i\hat{H}t/\hbar}\, |\psi(0)\rangle$$

$$= \langle\psi(0)|\, \frac{d}{dt}\left(e^{i\hat{H}t/\hbar}\hat{Q}e^{-i\hat{H}t/\hbar}\right)|\psi(0)\rangle$$

$$= \langle\psi(0)|\left(\frac{i\hat{H}}{\hbar}e^{i\hat{H}t/\hbar}\hat{Q}e^{-i\hat{H}t/\hbar}\right)|\psi(0)\rangle - \langle\psi(0)|\left(e^{i\hat{H}t/\hbar}\hat{Q}\frac{i\hat{H}}{\hbar}e^{-i\hat{H}t/\hbar}\right)|\psi(0)\rangle$$

$$= \langle\psi(0)|\, e^{i\hat{H}t/\hbar}\frac{i}{\hbar}\left[\hat{H},\hat{Q}\right]e^{-i\hat{H}t/\hbar}\, |\psi(0)\rangle = \frac{i}{\hbar}\langle\psi(t)|\left[\hat{H},\hat{Q}\right]|\psi(t)\rangle$$

or

$$\frac{d\left\langle \hat{Q} \right\rangle}{dt} = \frac{1}{i\hbar}\left\langle\left[\hat{Q},\hat{H}\right]\right\rangle$$

Consider dynamical state of system at time t.

Let |ψ⟩ be vector representing that state. Call ΔQ, ΔE root-mean-square deviations of $\hat{Q}$ and $\hat{H}$ respectively. Applying Schwarz inequality (as above) to vectors

$$(\hat{Q} - \langle Q \rangle) \left| \psi \right\rangle \quad , \quad (\hat{H} - \langle H \rangle) \left| \psi \right\rangle$$

and carrying out same manipulations as (as earlier), find after some calculations

$$\Delta Q \Delta E \geq \frac{1}{2} \left| \left\langle \left[\hat{Q}, \hat{H} \right] \right\rangle \right|$$

with equality being realized when |ψ⟩ satisfies the equation

$$(\hat{Q} - \alpha) \left| \psi \right\rangle = i\gamma(\hat{H} - \varepsilon) \left| \psi \right\rangle$$

where α, γ and ε are arbitrary real constants (as earlier).

Putting everything together we get

$$\frac{\Delta Q}{\left| \frac{d\left\langle \hat{Q} \right\rangle}{dt} \right|} \Delta E \geq \frac{\hbar}{2} \qquad \text{or} \qquad \tau_Q \Delta E \geq \frac{\hbar}{2}$$

where we have defined

$$\tau_Q = \frac{\Delta Q}{\left| \frac{d\left\langle \hat{Q} \right\rangle}{dt} \right|}$$

τ_Q appears as the time characteristic of evolution of expectation value of $\hat{Q}$.

It is time required for center $\langle \hat{Q} \rangle$ of statistical distribution of $\hat{Q}$ to be displaced by amount equal to width ΔQ.

In other words, the time necessary for this statistical distribution to be appreciably modified.

In this way can define characteristic evolution time for each dynamical variable of system.

Let τ be shortest of times thus defined.

τ may be considered as characteristic time of evolution of system itself,

that is, whatever measurement carried out on system at an instant of time t′,

the statistical distribution of results is essentially same as would be obtained at instant t,

as long as difference |t–t′| is less than τ.

According to derivation, the time τ and energy spread ΔE satisfy time-energy uncertainty relation

$$\tau \Delta E \geq \frac{\hbar}{2}$$

If system in stationary state where $\frac{d\left\langle \hat{Q} \right\rangle}{dt} = 0$ no matter what $\hat{Q}$, and consequently τ is infinite, then ΔE = 0. That is meaning of a stationary state.

Ordinary time t is just a parameter in non-relativistic QM and not an operator!

Our derivation **does not** say that

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

which is an equation that has no meaning!

Now on to EPR and Bell