Entanglement, EPR and Bell

Two threads are running through this part of lecture.

1st stems from Einstein's desire

to discredit quantum physics as fundamental theory of reality

which led to the most impressive and far-reaching confirmations of the quantum view.

2nd derives from Schrodinger desire to destroy QM and

who introduced term entanglement to physics community and showed how

entangled states lead to very bizarre features of quantum world.

Multiple passes through each topic

—> see different derivations, discussions and points of view.

Hopefully this enhances understanding!

Is Quantum theory complete?

Despite being responsible for number of key developments in theory,

Einstein was never happy with final form of QM.

1st he objected to uncertainty principle

—> developed thought experiments and arguments to get around it.

Bohr and Heisenberg made counter arguments that saved the uncertainty principle and QM many times.

Many of Einstein's proposed mechanical devices (some shown on next slide!!!!)

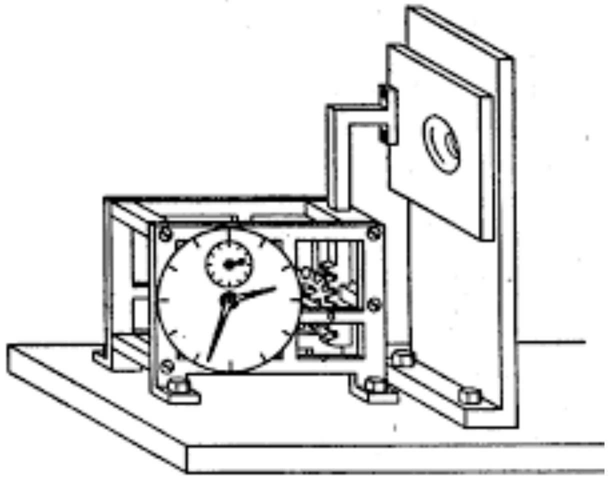
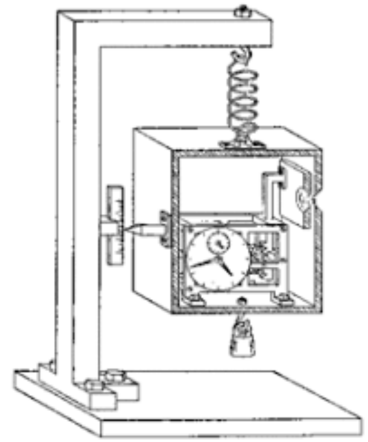
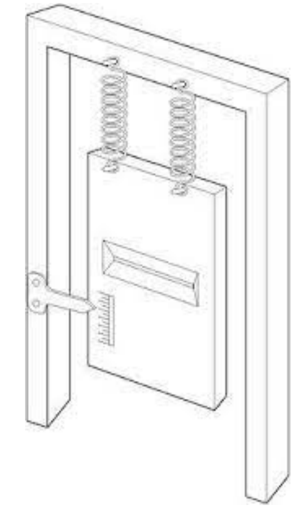


FIG. 6





After losing earlier arguments, Einstein then switched from trying to show QM was inconsistent

(gets different answers in similar cases)

to instead proving QM is incomplete (exist situations not covered by theory).

In 1935 Einstein thought he found way to discredit QM.

With Boris Podolsky and Nathan Rosen (EPR paper),

he constructed an argument that he thought

demonstrated the need for a deeper(more deterministic) theory to replace QM.

Paper titled

''Can a quantum mechanical description of physical reality be considered complete?''

Paper first defines a complete physical theory:

If theory is complete,

then every element of physical reality

must have counterpart in physical theory.

Reasonable, but we need to know what is meant by **element of physical reality**?

It is not straightforward.

EPR —> cannot guess or try to figure out elements of physical reality.

They have to be found from the results of experiments and measurements.

World is too subtle and surprising to guess correctly.

Need experiments as guide.

Still not simple.

Experiments —> huge amounts of data —> useful info + random noise.

Need to separate key features of information from random (experimental) fluctuations.

EPR directly tied theory with experiment and said following condition must be met:

> **if, without in any way disturbing the system, we can predict**
>
> **with certainty the value of a physical quantity**
>
> **then there exists an element of physical reality**
>
> **corresponding to this physical quantity**

We note that the part about **"disturbing the system"** —> clever **trap** for quantum theory.

Then EPR apply their condition for reality to a specific case.

Quantum theory employs state vector $|\psi\rangle$

—> systems and operators

—> physical variables $\hat{O}$.

If |ψ⟩ happens to be eigenstate of $\hat{O}$, then

$$\hat{O}\,|\psi\rangle = a\,|\psi\rangle$$ a = corresponding eigenvalue

Then can say with certainty that measurement of physical variable linked to

$\hat{O}$ —> a (one of our postulates).

Then, EPR condition —> physical variable

represented by $\hat{O}$ = element of reality (not clear if in general or only in that state?).

However, if one chooses to measure another physical variable

linked to operator $\hat{Q}$ and |ψ⟩ is not an eigenstate,

then cannot predict with certainty what value measurement will produce.

Expand |ψ⟩ over basis of eigenstates of $\hat{Q}$

and calculate expectation value for set of results,

or probability of given result.

Only way to predict with 100% certainty what measurement will give

is to disturb system by turning |ψ⟩ into one of eigenstates of $\hat{Q}$.

Thus, EPR condition —> physical variable represented by

$\hat{Q}$ cannot be element of physical reality, or, at least, not at same time as $\hat{O}$.

Specifically, electron in state $|p\rangle$,

eigenstate of momentum operator, $\hat{p}$

—> momentum is part of electron's reality.

However, in this state, position cannot be element of electron's reality

as $|p\rangle$ not eigenstate of $\hat{x}$.

Basis of EPR argument is the philosophical view

that one cannot create reality

by bending the electron to fit words "disturb the system".

Should be able find set of physical variables and theory

-> predict them with certainty,

as reality is out there to be found,

not something created.

Having set up conditions for reality

and illustrated them for simple quantum mechanical case,

EPR spring trap by describing a situation

where it appears quantum theory

fails to describe every element of a system's reality.

E(instein)P(odolsky)R(osen) Paradox(?) Details

Challenge to standard (Copenhagen) view of QM

—> surprising aftermath - resulted in1960s (25 years later!) work of John Bell.

1st discuss EPR and then fundamental work of Bell.

In 1935, EPR produced an argument

—> escape from the standard way of thinking about QM.

1st, they define completeness:

A description of the world is complete (for EPR) if nothing that is true about the world, nothing that is an element of the reality of the world is left out of the description.

EPR **never** actually presented a prescription

for determining what all elements of reality are (**made any challenge difficult**).

Instead, they did something much narrower

(which was **sufficient** for the purposes of their argument).

They wrote down a condition for a measurable property of system at any instant

to be element of reality of system at that instant.

Condition is that:

if, without in any way disturbing a system, we can predict with certainty (with probability = 1) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity.

What does this condition mean?

Consider following question:

If measurement of particular observable $\hat{O}$ of certain physical system S

were carried out at certain future time T,

what would outcome be?

Suppose a method is available so that,

prior to time T,

we can answer question with certainty.

Also, suppose method used —> no disturbance of system S.

Then(via EPR) there must be definite information(**hidden somehow**)

about outcome of a future $\hat{O}$ measurement on S at T.

For instance, suppose we just measured color of electron.

Since measurements of color are repeatable,

can predict with certainty what the outcome of later color measurement will be,

if measurement carried out.

Making prediction need not involve any further interaction with electron.

So EPR reality condition says color must, **at that moment**,

be element of reality of this electron.

—> Identical to standard way of thinking in QM! Just —> Color talk now applicable

Suppose, instead we measured hardness of electron.

To predict with certainty

what the outcome of a future measurement of color of electron would be

(if made measurement),

need to measure color of electron (interact and potentially disturb state).

EPR reality condition does not say color of electron

is element of reality of electron at that moment.

Again agrees completely with standard way of thinking!   Color talk not applicable!

EPR then want to argue the following:

If predictions of QM correct,

then there must exist elements of reality of world

which have no corresponding elements

in Quantum Mechanical description of world

They are attempting to use QM against itself.

They are saying QM is missing something!   It is not complete!

Their Argument: (presented in the hardness-color world by me for our benefit)

Consider system consisting of two electrons.

Electron 1 located at position 1,

and electron 2 located at position 2.

Assume that color-space state of two electrons is (was created somehow)

(note —> is a nonseparable or entangled color state - like two gloves in story)

$$|A\rangle = \frac{1}{\sqrt{2}} |green\rangle_1 |magenta\rangle_2 - \frac{1}{\sqrt{2}} |magenta\rangle_1 |green\rangle_2$$

State $|A\rangle$ is necessarily eigenstate of some observable(always true for all states)

(a Hermitian operator) of this pair of electrons, say $\hat{O}$,

where $\hat{O}$ $|A\rangle$ = +1$|A\rangle$(eigenvalue = +1).

We happen to have written $|A\rangle$ in color basis.

Now convert to hardness basis. Remember

$$|green\rangle = \frac{1}{\sqrt{2}} |hard\rangle + \frac{1}{\sqrt{2}} |soft\rangle$$
$$|magenta\rangle = \frac{1}{\sqrt{2}} |hard\rangle - \frac{1}{\sqrt{2}} |soft\rangle$$

Substituting get

$$|A\rangle = \frac{1}{\sqrt{2}}\,|soft\rangle_1\,|hard\rangle_2 - \frac{1}{\sqrt{2}}\,|hard\rangle_1\,|soft\rangle_2$$

Note it has the exact **same** nonseparable form in both bases.

In fact, it takes **same** nonseparable form in any basis! **—> a maximal entangled state**

Collapse postulate for two-particle entangled states (we develop needed extra theory)

If in state of type |A⟩, then the following rules apply during measurements.

$$|A\rangle = \frac{1}{\sqrt{2}}\,|green\rangle_1\,|magenta\rangle_2 - \frac{1}{\sqrt{2}}\,|magenta\rangle_1\,|green\rangle_2$$

1. If, in state |A⟩, we only measure particle 1,
then if particle 1 is measured to be green,
state collapses to $|green\rangle_1\,|magenta\rangle_2$

i.e., at same time we know 1 is green we **also** know 2 is magenta,

WITHOUT actually measuring 2.

That is what entangled states do!!

Similar result if measured 1 is magenta —> state would be $|magenta\rangle_1\,|green\rangle_2$

i.e., at the same time that we know 1 is magenta we **also** know 2 is green,

WITHOUT actually measuring 2.

2. If, in state $|A\rangle$, the probability of measuring 1 is green is still 1/2.

This is now shown below.

Probability for measurement of some property of particle 1

in a **2-particle entangled state** is calculated as follows:

(a) Step 1:

Since we not measuring particle 2,
must calculate probability for measuring 1
for all possible states of particle 2.

In this case

$$prob(\text{1 is green}) = prob(\text{1 is green AND 2 is magenta}) + prob(\text{1 is green AND 2 is green})$$

(b) Probability (x AND y) is calculated as shown below:

$$prob(\text{1 is green AND 2 is magenta}) = |\,\langle green|_1 \langle magenta|_2 \,|A\rangle\,|^2$$

Carry out measurement of color of electron 1 (state in color basis).

Outcome of measurement either green or magenta with equal probability

(using two-particle state probability rules).

As stated above, in state A

$$prob(1\, is\, green) = prob(1\, is\, green\, and\, 2\, is\, magenta) + prob(1\, is\, green\, and\, 2\, is\, green)$$

since not looking at 2.

Now

$$prob(1\, is\, green\, and\, 2\, is\, magenta) = |\, \langle green|_1 \, \langle magenta|_2 \, |A\rangle \,|^2 = 1/2$$

$$prob(1\, is\, green\, and\, 2\, is\, green) = |\, \langle green|_1 \, \langle green|_2 \, |A\rangle \,|^2 = 0$$

Total probability to be green = 1/2. Similarly for being magenta!

Moreover, QM says(this is experimentally confirmed)

that in event outcome of measurement green,

then outcome of any subsequent measurement of color of electron 2

will necessarily be magenta

and in event outcome of measurement magenta,

then outcome of any subsequent measurement of color of electron 2

will necessarily be green.

—> a **2-particle “collapse”** occurred!

Both statements follow directly from collapse postulate for two-particle states.

Remember the 2-path experiment for color/hardness where similar things happened.

EPR assumed

(the only assumption (called **locality**) they make

on top of basic assumption that predictions of QM are correct)

that things could in principle be set up in such a way

as to guarantee that measurement of color of electron 1

produces no physical disturbance whatsoever in electron 2.

To them —> self-evident statement!

What influence could there be???

Any number of ways to satisfy this condition.

Could separate two electrons by immense distance

(no assumption says any properties of QM changes with electron separation).

Then two measurement events,

according to special relativity, could not influence each other

since c = max info speed(called spacelike separated).

Or could insert impenetrable wall between them

(nothing assumed says any properties of QM

depends on what located between electrons).

Or could set up array of detectors to verify that

no measurable signals pass from one electron to other during experiment

(QM predicts array will never register anything).

Very important point.

Locality assumption says

I cannot punch you in the nose unless my fist gets to the place where your nose is (in space and time)

Of course, something I do with my fist far from where your nose is

can cause some other fist which is near your nose to punch your nose

(i.e., something I do with my fist might signal

somebody else to punch you in the nose).

Their seemingly obvious assumption is just this:

if my fist never gets anywhere near your nose then I cannot punch you in the nose directly. If you got punched in such an arrangement, then it cannot be my fist that punched you

If something I do with my fist far from your nose is cause of your getting punched in nose,

then necessarily some causal sequence of events at contiguous points in space

and at contiguous points in time (propagation of some signal for instance)

stretches all the way without break from whatever it was I did with my fist

to your being punched in the nose.

Important thing about sequence of events

is it necessarily requires some finite time

(exact amount depends on path in space and time)

to completely unfold.

Shortest time possible would occur

if signal(s) travel with speed of light in straight line

(maximum speed of propagation of information).

So summarizing their assumption:

measurement on color 1 has no effect on measurement of color 2

if measurements are spacelike separated(cannot be connected by light)

Returning to color basis entangled state

$$|A\rangle = \frac{1}{\sqrt{2}}\,|green\rangle_1\,|magenta\rangle_2 - \frac{1}{\sqrt{2}}\,|magenta\rangle_1\,|green\rangle_2$$

It is clear that one can predict with certainty,

if locality true,

without disturbing electron 2,

what outcome of subsequent measurement of color 2 will be.

Measure color 1

→ know outcome of measurement of color 2

is opposite of outcome of measurement of color 1

or know color 2 without measuring it!

Reality condition then says that color is element of reality of electron 2.

i.e., color of 2 has definite value when in state $|A\rangle$.

So both 1 and 2 have definite color values in this state!

Switch to hardness basis —> talk about hardness measurements.

$$|A\rangle = \frac{1}{\sqrt{2}}\,|soft\rangle_1\,|hard\rangle_2 - \frac{1}{\sqrt{2}}\,|hard\rangle_1\,|soft\rangle_2$$

Using same arguments in hardness basis

find that both 1 and 2 have definite hardness values!

Since can actually prepare states like $|A\rangle$ in real world,

—> EPR say standard interpretation must be false.   WHY?

EPR concluded both color and hardness are elements of reality of electron 2,

even though the two observables are incompatible according to QM.

Thus, formalism must be incomplete,

since some elements of physical reality of world

have no corresponding elements in QM.

Must exist hidden facts(not in postulates)

about color and hardness of 2, when in state $|A\rangle$.

Is there way out of EPR's proposed dilemma?

Nothing in QM formalism allows both color/hardness of electron 2

to be predicted with certainty simultaneously.

Similar arguments hold for electron 1.

EPR were clearly very clever!

If true, statement that system in state $|A\rangle$

constitutes incomplete description of state of pair of electrons

—> are some hidden variables.

EPR say QM predicts everything correctly, **but is still wrong.**

EPR noticed something odd about the collapse postulate for two-particle systems.

It was nonlocal.

If two particles are initially in a nonseparable state,

then a measurement carried out on one can seemingly cause changes,

instantaneously, in the quantum mechanical description of the other,

no matter how far apart two particles are or what lies in between.

Suppose pair of electrons initially in state $|A\rangle$

and measurement of color 1 carried out.

Outcome of measurement either green or magenta(equal probabilities).

Collapse postulate for two-particle systems

says as soon as measurement over,

state of 2 will be either $|\text{magenta}\rangle$ (if 1 was green)

or $|\text{green}\rangle$ (if 1 was magenta)

depending on what happened in measurement.

EPR said nonlocality is disposable artifact of particular mathematical formalism,

of particular procedure for calculating statistics of outcomes of experiments

—> must be undiscovered procedures,

which give rise to same statistical properties,

but are local (no infinite speeds necessary).

30 years later, Bell showed that their suspicion was **completely wrong**!

Bell's work(discuss later)

—> proof that any attempt to be realistic

about values of observables of the pair of electrons in state $|A\rangle$,

must necessarily be nonlocal.

Result was even more serious than that!!

Bell actually proves(as we will see) that there exists genuine nonlocality in actual workings of nature, however we attempt to describe it.

Nonlocality is feature of QM,

and via Bell's theorem

will necessarily be a feature of **every** possible manner of calculating

which produces the **same** probability predictions

(which are experimentally correct) as QM.

What is this quantum nonlocality?

First, in state |A〉,

statistics(probabilities) of outcomes of measurements on 2

depend nonlocally on outcomes of measurements on 1, and vice versa.

But do statistics of outcomes of measurements on 2,

when system in state |A〉,

depend nonlocally on

whether a measurement is actually carried out on 1 (and vice versa)?

Let us figure it out.

Suppose system in state |A〉,

and measure color 2.

$$|A\rangle = \frac{1}{\sqrt{2}}\,|green\rangle_1\,|magenta\rangle_2 - \frac{1}{\sqrt{2}}\,|magenta\rangle_1\,|green\rangle_2$$

Using |A⟩ in color basis (color-talk)

plus standard quantum-mechanical rules

for calculating probabilities of measurement outcomes

implies outcome of measurement on 2

equally likely to be green or magenta.

Suppose system is in state |A⟩

$$|A\rangle = \frac{1}{\sqrt{2}}\,|green\rangle_1\,|magenta\rangle_2 - \frac{1}{\sqrt{2}}\,|magenta\rangle_1\,|green\rangle_2$$

and measure color 1,

and then measure color 2.

Measurement of color 1 equally likely to be green or magenta.

If green,

collapse postulate says subsequent measurement of color 2

will be magenta,

and if magenta,

collapse postulate says that subsequent measurement of color 2

will be green.

So, when system in state $|A\rangle$,

outcome of a measurement of color 2 is equally likely to be green or magenta

whether or not measurement of color 1 carried out first.

Suppose system is state $|A\rangle$ and measure hardness 1,

and then measure color 2.

$$|A\rangle = \frac{1}{\sqrt{2}} |\text{hard}\rangle_1 |\text{soft}\rangle_2 - \frac{1}{\sqrt{2}} |\text{soft}\rangle_1 |\text{hard}\rangle_2$$

Using $|A\rangle$ in hardness basis plus probability rules

says outcome of hardness measurement on 1 equally likely to be hard or soft.

If outcome of first measurement is soft,

collapse postulate plus probability rules say outcome of second measurement (color 2)

is equally likely to be green or magenta because 2 is **now hard**.

Same result true if outcome of first measurement is hard.

So here is where we are:

When system in state $|A\rangle$,

outcome of measurement of color 2

is equally likely to be green or magenta,

whether measurement of color 1 is carried out first

or measurement of hardness 1 is carried out first

or no measurement on 1 is carried out

Probabilities of various outcomes of measurement on 2

do not depend in any way

on whether or not a measurement is made on 1 first.

Since the predictions of QM are correct,

there must be non-local influences in nature

and they must be of particularly subtle kind.

Outcomes of measurements do sometimes

depend non-locally on outcomes of other, distant measurements,

but outcomes of measurements invariably **do not** depend non-locally on

whether any other distant measurements actually get carried out.

Another way (tricky…you must think about this one) to say this:

Non-local influences are so subtle that they cannot be used
to transmit any signal containing information, non-locally,
between two distant points.

They cannot encode information you want to send
on decision to make a measurement or not to make one,
or on a decision about which measurement to make,
since no such decisions can have detectable non-local effects.

NOTE: Nonlocality existence is crucial. As we will see later, the solution to the measurement questions depends on the existence of nonlocality in QM.

Look at simple experiment (1980 with photons —> it took experimentalists 35 years!)
that refutes EPR and agrees with Bell's results (will derive shortly).

Will discuss experiment again later (more detail and mathematical rigor).

Before looking at experiment we must talk about electron spin!

Digression about Electron Spin

Electron spin is real-world measurable property of electrons.

Measuring devices are Stern-Gerlach(SG) boxes, for example.

Operator for electron spin $\hat{\mathbf{S}}$ has following properties.

1. When electron measured by SG device oriented in particular direction (say z)—> SGz box, measured values for electron spin observable in z-direction are only ±1/2. SGz box is represented by observable $\hat{S}_z$.

2. Same result holds for any other direction, in particular, x-direction with an SGx box. SGx box represented by observable $\hat{S}_x$ (only ±1/2 also).

3. Corresponding eigenvectors/eigenvalues given by

$$\hat{S}_z\,|U\rangle = +1/2\,|U\rangle \quad , \quad \hat{S}_z\,|D\rangle = -1/2\,|D\rangle \qquad \hat{S}_x\,|R\rangle = +1/2\,|R\rangle \quad , \quad \hat{S}_x\,|L\rangle = -1/2\,|L\rangle$$

From earlier:
$$|U\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle) \quad , \quad |D\rangle = \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$$

Can see from mathematical formalism,

electron spin in z and x directions are **mathematically** the same as color and hardness.

One very different feature of this real-world system

is that can orient SG boxes at any arbitrary angle θ (only 2 ways in color-hardness world)

where $\theta = 0^\circ$ correspond to the +z direction,

$\theta = 90^\circ$ correspond to the +x direction

and $\theta = 180^\circ$ correspond to the –z direction and so on.

Thus, any arbitrary direction states can be written(from QM studied at junior level) as

$$|U_\theta\rangle = \cos\frac{\theta}{2}|U\rangle + \sin\frac{\theta}{2}|D\rangle \qquad |D_\theta\rangle = -\sin\frac{\theta}{2}|U\rangle + \cos\frac{\theta}{2}|D\rangle$$

EPR experiment with electron spin - this is actual experiment that was done.

Consider pair of electrons in entangled state like $|A\rangle$

where we use the observable **spin(up/down in any direction)**.

$$|A\rangle = \frac{1}{\sqrt{2}}|U\rangle_1|D\rangle_2 - \frac{1}{\sqrt{2}}|D\rangle_1|U\rangle_2 = \frac{1}{\sqrt{2}}|U_0\rangle_1|D_0\rangle_2 - \frac{1}{\sqrt{2}}|D_0\rangle_1|U_0\rangle_2$$

This state arises from the decay of an atom in Spin = 0 state of an atom where spin angular momentum is conserved : 0 —> (1/2) +(-1/2)   The experiment has been done!

Electrons separate in physical space without changing the state vector.

We end up with two separated electron beams.

Each beam has equal numbers of |U⟩ and |D⟩ electrons (probability = 1/2 for each component).

Each electron in one beam

is **correlated** with a partner in the other beam(started from same atom)

since if measure electron in one beam in state |U⟩,

then partner(same magnet orientations) in other beam is in state |D⟩ due to

entanglement/collapse

and vice versa.

This nonseparable state remains nonseparable

no matter what basis (direction we measure spin) we use.

A state in an arbitrary direction basis is

$$|A\rangle = \frac{1}{\sqrt{2}}\,|U_\theta\rangle_1\,|D_\theta\rangle_2 - \frac{1}{\sqrt{2}}\,|D_\theta\rangle_1\,|U_\theta\rangle_2$$

Now suppose we measure beam 1 in direction θ_1 and beam 2 in direction θ_2

and count number of times (note these are not necessarily the same directions)

beam 1 up **and** beam 2 up
or = a MATCH
beam 1 down **and** beam 2 down

beam 1 up **and** beam 2 down
or = a MISS
beam 1 down **and** beam 2 up

QM says (straightforward but messy calculation) that the experimental results depend only on the angle difference

$$\varphi = \theta_2 - \theta_1$$

and **not separately** on θ_1 and θ_2 —-> Non-locality in action!!

That is the sign that a **correlation** exists!

Results are confirmed experimentally

no matter where the two detectors are located………

in same room, or on opposite sides of city or wherever!!!!

******* Experiment has now reached 144 km apart! *******

Suppose we code the results coming from the two detectors as follows.

If $\varphi = 0$, z–axes of 2 detectors are in same direction and get

$$beam\ 1 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow \ldots\ldots\ldots\ldots$$
$$beam\ 2 \rightarrow \downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow \ldots\ldots\ldots\ldots$$

i.e., ALL MISSES (remember if 1 is up, 2 is down in this case).

Get two opposite binary data sequences in two detectors

$$1010010001101001\ldots\ldots$$
$$0101101110010110\ldots\ldots$$

If $\varphi = 180^\circ$, z–axes of 2 detectors are in opposite directions and get ALL MATCHES, i.e.,

$$beam\ 1 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow \ldots\ldots\ldots\ldots$$
$$beam\ 2 \rightarrow \uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow \ldots\ldots\ldots\ldots$$

(up in 1 is in opposite direction to up in 2 or up in 2 is down in 1).

—> get identical(not opposite) binary sequences in this case.

Quantum mechanics predicts number of MATCHES, etc varies like

$$\begin{aligned}
[\#\text{MATCHES}] &= [\#\text{ELECTRONS}]\sin^2\tfrac{\varphi}{2} \\
[\#\text{MISSES}] &= [\#\text{ELECTRONS}]\cos^2\tfrac{\varphi}{2} \\
[\#\text{MATCHES} + \#\text{MISSES}] &= [\#\text{ELECTRONS}][\sin^2\tfrac{\varphi}{2} + \cos^2\tfrac{\varphi}{2}] \\
&= [\#\text{ELECTRONS}]
\end{aligned}$$

as it should!

Now set angle between detector axes to be $\varphi = 120^\circ$.

Quantum mechanics predicts(from our formalism)

$$[\#\text{MATCHES}] = \frac{3}{4}\,\#\text{ELECTRONS}$$

What would EPR say (you must remember EPR believe in locality)?

If $\varphi = 60^\circ$, then $[\#\text{MATCHES}] = \dfrac{1}{4}\,\#\text{ELECTRONS}$

(same as quantum mechanical prediction!)

So now EPR align detectors ($\varphi = 0^\circ$).

Then rotate detector 1 such that $\varphi = 60^\circ$

—> would again get 1/4 MATCHES.

So now EPR re-align detectors ($\varphi = 0^\circ$).

Then rotate detector 2(opposite direction) such that $\varphi = -60^\circ$ and also

—> would get 1/4 MATCHES.

EPR then predict $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}\ \mathrm{MATCHES}\ (\mathrm{not}\ \frac{3}{4}\ )$

i.e., two separate measurements cannot affect/change each other
—> prior results are unchanged
That is what locality means!!

EPR —> two measurements do not influence each other and therefore totals just ADD.

Experiment says answer is 3/4!!. Quantum mechanics is correct!

—> Local view of reality due to EPR cannot be correct.

Somehow information seems to be flowing

between detectors 1 and 2 NO MATTER WHAT WE DO.

Speed of information flow(if assume any signal exists…..) might be infinite

(best measurement so far $\approx 10^7$ c = 3×10^{15} m/ sec - but might not mean anything!!).

Important to emphasize that

have not observed any non-local interactions directly,

but have only indirectly demonstrated need for them.

Each measurement is separately local,

i.e., measure any spin direction in either detector

and always get 50-50 up and down

no matter what other detector is doing.

It is the sequences that differ(in a random way)

as we change the angles.

This means, as before,

that we cannot transmit any instantaneous messages(that would violate SR).

We only find out the sequences(messages) when we bring them together,

which takes some time.

So, it seems in QM, that my QM fist can punch your QM nose without being near your QM nose!!

Bohr and EPR - A Discussion

Bohr produced a reply to EPR within 2 months of paper publication.

His counterargument uses the **contextuality** of quantum theory

and the principle of complementarity to punch holes in EPR’s definition of reality.

Complementarity

Bohr had a unique, deeply thought out, and very subtle view

on nature of science and interpretation of quantum theory.

Defies any easy and compact summary.

Some writings might —> instrumentalist viewpoint —-> oversimplification of his view.

Bohr was convinced that humans could never “picture” the inner mechanisms of atoms

(Feynman will have more to say about this crucial point later).

His conclusion follows from nature of our human language.

Daily experience

—> ideas and concepts we deal with comfortably and express

in language restricted to classical world; but may not extend easily into quantum world,

hence, may need to abandon classical thinking and develop new concepts.

—> need new language as we already have seen

Snag(??)

Experiments rely on gathering together instruments and equipment

very much bigger than atomic scales

—> described by everyday language (including adding technical terms for brevity).

All measurements on atoms and particles

-> numbers read from dials on measuring apparatus (macroworld scale).

Essential to success of science that equipment be described in ordinary terms

—> makes possible for others to understand and reproduce experiment.

Caught in dilemma.

Experiments should not only be capable of being described,

essentially, in everyday language, but must also be so to make science possible.

Yet, when we gather the results of experiments together —> a description of atomic world

where we find that same everyday language and ideas fail to provide explanations.

Photons act as particles in some circumstances and as waves in others or **maybe neither**.

Can find appropriate mathematics to describe situation

but does not help us **“visualize or speak”** about photons.

Bohr hoped for a way out of trap <—> principle of complementarity.

He accepted that the two completely contradictory pictures of photon

could not be tied together into one picture, but he said that does not matter,

since we never do experiment that required the use of both pictures at **same time**.

We never ask questions requiring wave and particle answers at same time.

We never set context implying wave and particle properties at same time

—> Acceptable to employ this sort of "double think"

or using only one of pair of complementary views at any one time,

depending on what was most appropriate.

Actual true nature of photon can never be expressible in any language

which we we use to describe the classical world,

but we could shadow what was going on by keeping complementary pictures in mind.

—> concepts such as position, momentum, frequency, energy, and so on,
can be defined only in context of an experiment to measure them.

If want to talk about frequency of light (wave picture)
then need to do experiment where the wave picture appropriate.

If want to talk about momentum of photon (particle picture)
then do totally different sort of experiment.

By insisting, Bohr ensured that concepts such as momentum, energy, frequency, and so on,
remained rooted in classical world, while at same time acknowledging that
they would have only a limited application in quantum world.

Bohr wrote -

> **"quantum theory forces us to adopt a new mode of description designated as complementary in sense that any given application of classical concepts precludes simultaneous use of other classical concepts which in a different connection are equally necessary for elucidation of phenomena".**

—> Copenhagen interpretation of QM

The emphasis on relating meaning of theoretical terms

to experiment where terms are measured gives Bohr’s thinking an instrumental tinge.

However, careful reading of his writings

confirms that Bohr retained the ambition of a realist tempered by conviction that atomic

world could not be described by limited human experience and language.

Bohr’s Reply to the EPR Argument

Bohr’s main point is that measurement of particle A

may not physically disturb state of particle B,

but measurement on A does **set up the context for any information about B**.

EPR used position and momentum instead of color and hardness (Same math).

$$|A\rangle = \frac{1}{\sqrt{2}}\,|x\rangle_1\,|p\rangle_2 - \frac{1}{\sqrt{2}}\,|x\rangle_2\,|p\rangle_1$$

To measure momentum of A —> must build specific device.

Let it interact with something with pre-determined momentum and see what change it makes.

If want to measure momentum of device, we find the uncertainty principle gets in way.

Accurately measuring momentum of device destroys any information about where exactly it is.

In which case, cannot act as position reference point as well.

This applies to both particles.

Measurement of position needs a fixed reference point to work from.

When set up equipment to measure momentum of particle A,
automatically prevent that equipment from forming
precise reference point for position measurement of A.

Set out to measure position of particle B.

Cannot compare position of B with anything of particle A

since cannot fix position of equipment at B's end relative to A.

Have effectively blurred position of particle B by measuring momentum of particle A.

As Bohr puts it:

"If choose to measure momentum of one of particles, lose through uncontrollable displacement inevitable in such measurement any possibility of deducing from behavior of particle position of apparatus, and thus no basis whatever for predictions regarding location of other particle. "

Similar argument applies if choose to measure position of A instead.

Accurate position measurements rely on fixed objects to act as reference points.

When particle A collides with device designed to record its position,

bound to be exchange of momentum between device and particle

i.e., it interacted with measuring device.

With device fixed in place cannot measure that momentum exchange.

Any attempt to measure momentum of measuring device,

so one can figure out how much it has been changed,

is bound to run into uncertainty problems.

Need an accurate fix of position of device to measure position of A,

so cannot be sure of momentum of device.

Ruins potential momentum measurement on B.

Momentum only known relative to object.

Cup of tea on table may be stationary to us,

but if on train passing through station

—> tea in rapid motion for observer standing on platform.

If set up momentum measurement of B, have no basis to compare with A.

Bohr explains:

By allowing uncontrollable momentum to pass from first particle into support, have cut ourselves off from any future possibility of applying law of conservation of momentum to system consisting of .... two particles, and therefore lost only basis for unambiguous application of idea of momentum in predictions regarding behavior of second particle.

Key difference between philosophical positions of EPR and Bohr

—> statement **“without disturbing the system”**.

To Bohr, a property, such as momentum or position,

has meaning **only in context** of experimental equipment used to measure it.

Particle B may not come in contact with any equipment at all;

in such case any information we have about it is useless - nothing one can do with it.

If want to use information,

say to predict where second particle will be after a while

and check that information experimentally,

then second particle has to come into contact with measuring equipment,

or some other stuff has to be linked to it.

As far as complementarity concerned,

Bohr insisted that concepts of momentum and position

work only within specific experimental context.

Two particles can live either in same context,

in which case limited comparisons of same properties can be made,

or not, in which case have two separate bundles of information

that cannot be related to one another.

Einstein and Bohr

Never able to reach common ground <—-> philosophical viewpoints entirely different.

Einstein - probabilities in QM unacceptable.

Use of statistics when calculating behavior of large numbers of particles different

All particles involved have properties such as position and momentum;

but so many —> calculations could not be done,

hence need to average and use statistics to calculate system probabilities.

Bohr - probability in QM fundamental,

—> not a consequence of inability to measure/calculate.

Einstein —> would be another layer of theory underneath QM

—> hidden variables of theory

—> no probability —> QM incomplete.

Einstein(sick of arguing/losing with Bohr) turns to Schrodinger

who also said that fundamentally statistical character of theory

is simply a consequence of incompleteness of description.

Then Schrodinger, on his own, Introduced idea of Entanglement

Schrodinger argued that state of system of two particles(that have interacted)

cannot be written as simple product of individual states for each particle.

$$|\text{Particle A interacting with B}\rangle \neq |A\rangle\,|B\rangle$$

State where particles have interacted

—> **entanglement** of individual states

where we cannot say with certainty which particle is in which state.

State such as

$$|\text{singlet}\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle\,|D\rangle - |D\rangle\,|U\rangle\right)$$

is an entangled state: first particle either $|U\rangle$ or $|D\rangle$.

Basis of EPR argument(position/momentum), and Bohm variation(spins),

is that system **starts** in entangled state.

Schrodinger saw entanglement as the defining characteristic of QM

it is **what** —> QM is totally different from classical physics.

He emphasized view by discussing the disentanglement that happens when a measurement is made.

—-> **Entanglement disappears and particles then have properties!**

Measuring spin of particle A

—> can deduce spin of B if they are in singlet state.

—> Have collapsed combined state into one of two entangled possibilities.

In Schrodinger's view disentanglement is of key (possibly **sinister in his mind**) importance

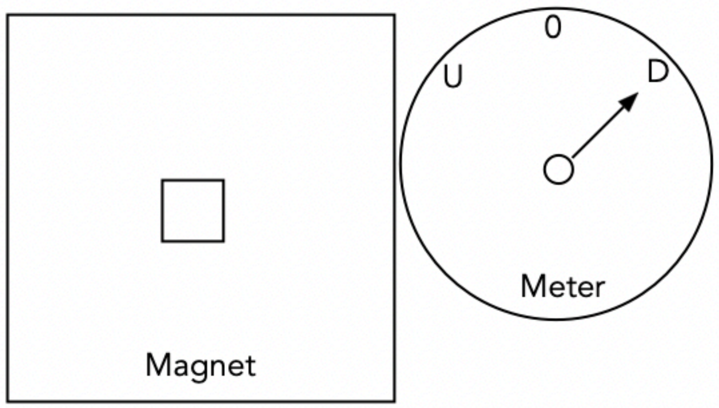
since involved in every act of measurement.

Entanglement and Measurement

Remember earlier argument about measurement/collapse on entangled states.

Possible vertical spin states of electron moving toward S-G magnet

are $|U\rangle$ and $|D\rangle$ in direction appropriate to a particular device.



If use QM to describe S-G magnet(the measuring device),

possible states are going to be $|\phi_n\rangle$ (n for neutral, resting state of apparatus),

$|\phi_U\rangle$ (U for UP showing apparatus registering UP state of electron),

and $|\phi_D\rangle$(D for DOWN registering DOWN state of electron).

Might know electron in $|U\rangle$ before reaches S-G magnet (passed through one beforehand)

—> state of combined system

before interaction of electron and S-G magnet = $|U\rangle|\phi_n\rangle$. **U-beam + pointer at 0**

After interaction, if measuring device any good,

state evolved into $|U\rangle|\phi_U\rangle$ U-beam + pointer to +1

in a process governed entirely by equations of quantum theory

(Schrodinger equation or time-evolution operator $\hat{U}(t)$).

$$|U\rangle\,|\phi_n\rangle \underset{\text{evolves into}}{\Rightarrow} |U\rangle\,|\phi_U\rangle \quad or \quad |U\rangle\,|\phi_U\rangle = \hat{U}(t)\,(|U\rangle\,|\phi_n\rangle)$$

time evolution

Note: no collapse of state has taken place,

just evolution of state of S-G apparatus from $|\phi_n\rangle$ to $|\phi_U\rangle$

during interaction or dial pointer moving from 0 to +1.

i.e., a pointer moved!

i.e., measurement device state changed!

First time we are treating measuring device as described by QM,

rather than incomplete statements(as earlier)

that measuring device collapses state.

If take quantum theory seriously

as an underlying theory of reality, then

have to describe measuring device, (S-G magnets), in quantum way.

—> state **does not collapse** when a measurement takes place;

Quantum states simply entangle with states of equipment!

To see why this happens, consider the following…..

Assume we do not know initial state of electron.

Have to write state as

$$|\psi\rangle = a\,|U\rangle + b\,|D\rangle$$ where $|a|^2 + |b|^2 = 1$

Then, initial joint state of electron and S-G devices is $|\psi\rangle\,|\phi_n\rangle = (a\,|U\rangle + b\,|D\rangle)\,|\phi_n\rangle$

measurement device not entangled!

Based on linearity property of QM, evolves into

$$(a|U\rangle + b|D\rangle)|\phi_n\rangle \Rightarrow a|U\rangle|\phi_U\rangle + b|D\rangle|\phi_D\rangle$$

—> time evolution seems to gives the entangled state that we need to deal with.

Problem is that state has not collapsed in this view!!

Is there anything we can do?

Consider what happens when passing physicist comes to apparatus

to read off measurement results.

If we consider ourselves part of same universe as everyone else

—> passing physicist described by QM as well.

Must have states such as

$$|\text{physicist has not looked yet}\rangle, |\text{physicist sees UP}\rangle, |\text{physicist sees DOWN}\rangle$$

To record results, have to interact with apparatus so that

$$(a\,|U\rangle\,|\phi_U\rangle + b\,|D\rangle\,|\phi_D\rangle)\,|\text{physicist has not looked yet}\rangle \Rightarrow$$
$$a\,|U\rangle\,|\phi_U\rangle\,|\text{physicist sees UP}\rangle + b\,|D\rangle\,|\phi_D\rangle\,|\text{physicist sees DOWN}\rangle$$

and we become part of entanglement as well!

Probably why Schrodinger felt entanglement was **sinister**:

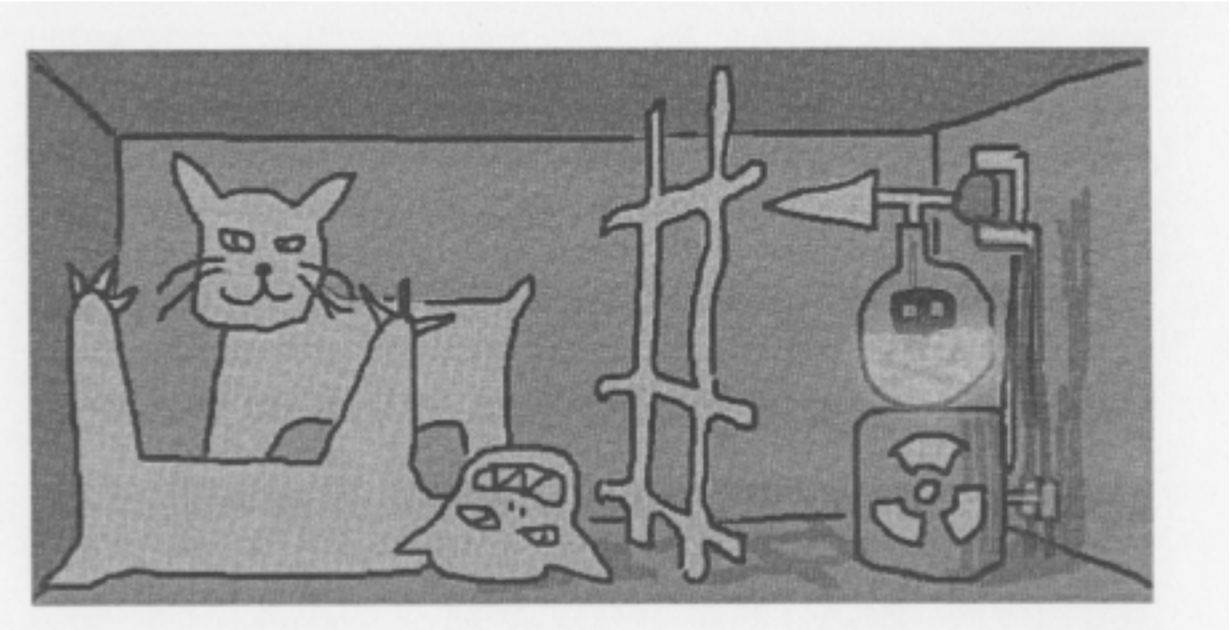
no way to stop it from spreading until

whole universe part of a stupendous entangled state.

—> **infinite regression - which is always a bad thing!**

Schrodinger was just blindly driving down the WRONG road as we will see!!

That Damned Cat - 1st Pass



The sorry tale of Schrodinger's cat
has assumed the status of a
quantum mechanical f**airy tale**.
Amazing how thoroughly this cat
has entered quantum lore,
especially when we consider that 1 quote
is only reference to unfortunate creature in all of Schrodinger's prodigious written output.

In the standard Schrodinger's cat experiment
a small radioactive device is linked to a trigger controlling the release of poison gas.

After a period of time, quantum state of radioactive device is in superposition of states

$$|\text{decayed}\rangle + |\text{not decayed}\rangle$$

some have decayed, some have not!

State of gas and ultimately state of cat are entangled with this state(or thought to be)
—> not possible to tell in quantum description
if cat is alive or dead(more details later).

Schrodinger introduced cat as a means of

illustrating the bizarre nature of entanglement

and superposition of quantum states.

(actually wanted to ridicule the ideas so people would replace them)

Remember when we introduced superposition

—> quantum states can be combinations of states that don't make sense classically

we did so because such things were needed to describe quantum systems.

Entanglement then follows directly once that is allowed!!.

Schrodinger's cat illustrates the associated problem of scale.

At what scale do classically forbidden combinations stop being allowed? Is there a size limit?

Experimentally photon seems to travel in two directions at once,

but surely a cat cannot be alive and dead at same time!

(or easier for cat lovers to deal with —- a cat cannot travel two paths at the same time!!)

If we look inside experiment

then can see if cat alive or dead, but that doesn't improve situation.

Our state just becomes entangled and so on…….**infinite regression…………..**

This discussion which is ultimately probing the measurement problem

—> crucial issue for any interpretation of quantum theory.

Bohr **dissolved the problem (in his mind) by insisting** that at some point the measuring device or cat or person is described classically, **breaking** the chain of entanglement. **(Remember this point!)**

I hope that you have been keeping a list of all, the items I have been telling you to remember for later!!

According to Copenhagen interpretation,

the state vector for 2-particle quantum state cat does not disentangle state as the particles separate in space-time.

Instead of changing into two separate vectors, one associated with each particle,

state vector remains entangled

and, when a measurement made,

state collapses instantaneously **no matter how large** separation distance might be.

That is what our version of the theory presently says!

EPR's view of physical reality

—> if two particles are isolated from each other,

then they are **no longer described** by single state vector when a measurement is made.

Reality EPR referring to

=> local reality

and ability of particles to separate(become space like)

into 2 locally real independent physical entities

=> **Einstein separability**.

In EPR experiment,

Copenhagen interpretation or standard view of quantum theory

denies that 2 particles are Einstein separable

and thus **denies they are locally real**

until a measurement is made on one or other,

at which point both become localized and **real** (the collapse process).

Schrodinger's Cat - Another Presentation

Earlier, the collapse of state vector was presented without reference

to the point in measurement process when collapse occurs.

One might assume the collapse occurs at moment the microscopic quantum system interacts

with macroscopic measuring device.

Is this assumption justified?

Now the macroscopic measuring device is **composed** of microscopic entities.

Interactions on microscopic level are supposed to **use QM** for their description.

Suppose a microscopic quantum system

described by state vector $|\psi\rangle$ interacts with measuring instrument

(= **device which responds to interaction with quantum system**

producing macroscopic results like pointers or dials)

with two possible measurement eigenstates $|\psi_+\rangle$ and $|\psi_-\rangle$.

These eigenstates combine with the macroscopic device

to **reveal** one of 2 possible outcomes of a measurement,

deflection of pointer to left (+ result) or right (- result).

Recognizing that the instrument consists of quantum particles,

we must describe state of instrument (repeating earlier arguments - I will do this many times)

before measurement by state vector $|\phi_0\rangle$,

corresponding to central pointer position.

Total state of quantum system+measuring instrument

before measurement described by $|\phi_0\rangle$, is

$$|\Phi_0\rangle = |\psi\rangle\,|\varphi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle + |\psi_-\rangle\right)|\varphi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle\,|\varphi_0\rangle + |\psi_-\rangle\,|\varphi_0\rangle\right)$$

just multiply out…..

where $|\psi\rangle$ expressed in terms of measurement eigenstates

(assume = orthonormal basis) so that amplitudes are

$$\langle\psi_- \mid \psi\rangle = \langle\psi_+ \mid \psi\rangle = \frac{1}{\sqrt{2}}$$

i.e., —> both final pointer results are equally probable.

Description of what happens if we treat

macroscopic measuring instrument as quantum object(repeat of earlier argument).

First, how does initial state $|\Phi_0\rangle$ evolve in time during act of measurement?

Earlier, the application of time evolution operator $\hat{U}$

allowed us to calculate state vector at later time —> $|\Phi\rangle$, as

$$|\Phi\rangle = \hat{U}\,|\Phi_0\rangle \quad \longrightarrow \quad |\Phi\rangle = \frac{1}{\sqrt{2}}\left(\hat{U}\,|\psi_+\rangle\,|\varphi_0\rangle + \hat{U}\,|\psi_-\rangle\,|\varphi_0\rangle\right)$$

What is effect of $\hat{U}$ on these states?

If instrument interacts with quantum system

already present in one of measurement eigenstates ($|\psi_+\rangle$ say),

then total system (quantum system+instrument)

must evolve into product quantum state $|\psi_+\rangle|\phi_+\rangle$, i.e.,

or, in other words, $$\hat{U}\,|\psi_+\rangle\,|\varphi_0\rangle = |\psi_+\rangle\,|\varphi_+\rangle$$ **pointer must move to +**

if quantum system in state(probability = 1)

corresponding to definite pointer position,

then measuring device state must evolve into state **a postulate!**

where pointer is pointing to proper place.

This actually what happens in the laboratory!!

Similarly, we must have $$\hat{U}\,|\psi_-\rangle\,|\varphi_0\rangle = |\psi_-\rangle\,|\varphi_-\rangle$$ **pointer must move to -**

Using the two special case results, we have for evolution of an arbitrary initial state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle\,|\varphi_+\rangle + |\psi_-\rangle\,|\varphi_-\rangle\right)$$ **using QM linearity**

where measuring device states $|\varphi_+\rangle$ and $|\varphi_-\rangle$

correspond to pointer ending up at + or - ——> A macroscopic event!

Result suggests that measuring instrument

evolves into superposition state in which pointer has equal probability

to point either to the left or right.

—-> **It is not in a definite pointer state**.

Thus, **neither the** quantum system nor the pointer has a **definite value after the measurement**.

This state will remain a superposition (**pointer does not point**)

unless we allow for collapse

so that pointer can point (take on definite value)!

That is why the extra postulate is usually added.

If this were final state of system,

then the pointer should never settle down and point somewhere!

Even been suggested by some that it would **flutter** back and forth

between two macroscopically different pointer positions. **Wow - how dumb!**

Collapsing state vector of system + measuring-device

seems to require a further measurement.

Whole argument repeated ad infinitum(infinite regression again).

Are we therefore always locked into an endless chain of measuring processes?

At what point does chain stop

or at what point does state vector “collapse”

—> pointer actually pointing(**which we know it does!!!!**)?

Problem is created by the inability to obtain the collapse of state vector

using continuous, deterministic equation of motion

i.e., via the time evolution operator.

Schrodinger called state vector $|\Phi\rangle$ entangled because,

once generated,

it was impossible to separate it into constituent parts

except by invoking a nondeterministic collapse

i.e, some discontinuous process

i.e., some non-unitary time evolution.

Such collapse is **not accounted** for in equations of orthodox quantum theory

—-> why we had to add a postulate.

Paradox of Schrodinger’s cat is designed

to show this apparent absurdity

by shifting focus of difficulty from microscopic world to macroscopic world.

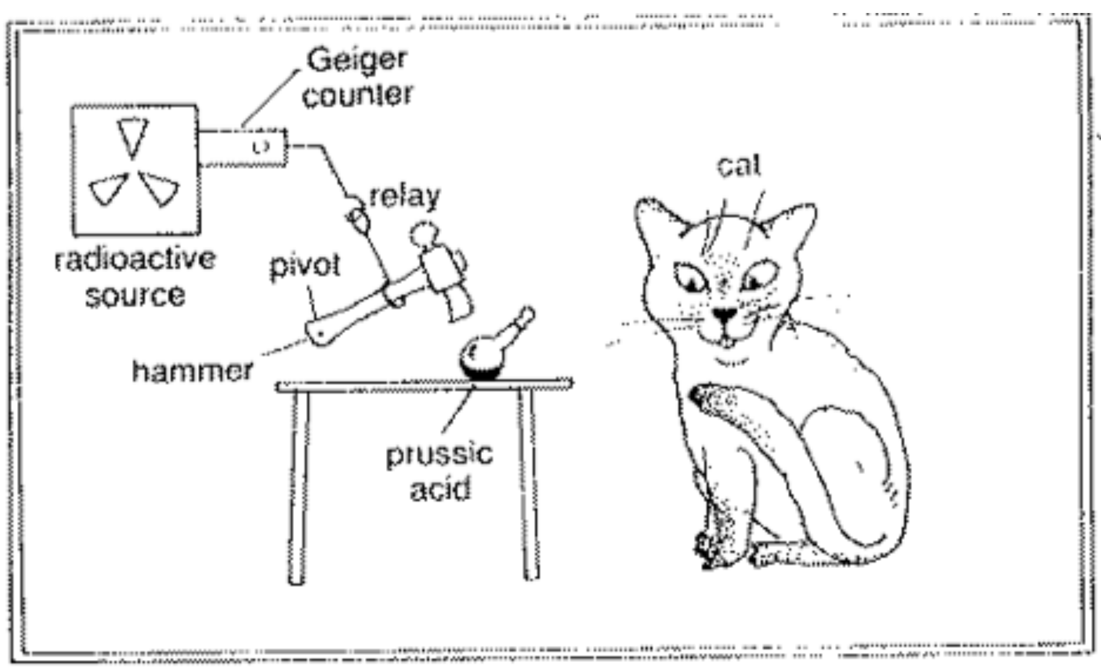
Repeat(more details/ideas): Cat placed in steel chamber together with radioactive source, a detector, a hammer mounted on pivot and bottle of prussic acid.

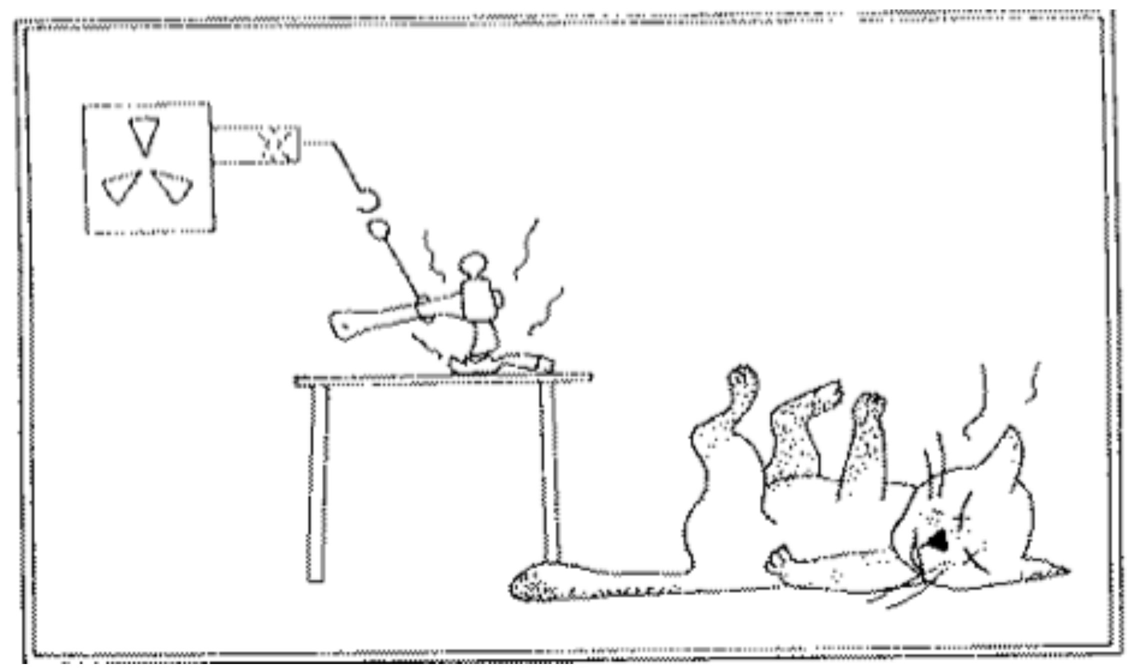
Chamber is closed.

From amount of radioactive material in source and known time for decay(half-life), expected that within one hour probability is 1/2 that one atom has disintegrated (decayed).

If atom does disintegrate, then detector is triggered sending signal to release hammer that smashes bottle releasing prussic acid which kills the cat.

The essential ingredients are shown below





Prior to actually measuring disintegration(decay), state vector of radioactive atom = linear superposition of measurement eigenstates, corresponding to physical states of undecayed atom and decayed atom.

Treating measuring instrument as quantum object and using equations of QM

—> superposition of two possible outcomes of measurement.

But what about cat?

These arguments (now will see trouble we are in…)

—> must express state vector of (system + cat) as linear superposition

of products of state vectors

describing disintegrated atom and dead cat

and describing intact atom and live cat, i.e.,

$$|\Phi\rangle = \frac{1}{\sqrt{2}} \left(|\text{no decay}\rangle \, |\text{live cat}\rangle + |\text{decay}\rangle \, |\text{dead cat}\rangle \right)$$

where, for example, state vector of dead cat

—> state corresponding to

triggered detector, released hammer, smashed bottle, released prussic acid **and** dead cat.

Prior to measurement, physical state of cat must be “blurred” (whatever that word means)

—-> neither alive nor dead but some **unknown** combination of both alive and dead states.

Can perform measurement on (system+cat) by opening chamber

and thus determining the physical state.

Do we suppose(imagine) that at that point (system+cat) collapses

and we record our observation that cat is either alive or dead as appropriate?

What happens?

QM is clear!

Only result that QM predicts is that

if set up N (N large) similar systems,

then when open N boxes after 1 hour

find 1/2 the cats alive and 1/2 the cats dead.

That is what actually happens experimentally.

Millions of measurements and probability always wins!

Although obviously intended to be somewhat tongue-in-cheek,

Schrodinger’s paradox nevertheless brings attention to an important difficulty we must confront.

Copenhagen interpretation

—> elements of empirical reality defined by

nature(context) of experimental apparatus we construct

to perform measurements on quantum system.

Interpretation insists we **resist temptation** to ask

what physical state particle (or cat) actually in prior to measurement

as question is **without any meaning** within this interpretation of QM.

This positivist interpretation sits uncomfortably with some scientists,

particularly those with special fondness for cats.

Thus, some have accepted the EPR argument that quantum theory is incomplete.

They set about searching for alternative theory

that allows attaching physical significance

to properties of particles

without need to specify nature of measuring instrument,

That allows us to define independent reality

and that reintroduces strict causality.

Even though searching for such a theory

might be engaging in meaningless metaphysical speculation,

they believe that it is a search that has to be undertaken.

These are hidden variables people!

Have not succeeded with this approach and as will now see

Bell/EPR arguments say it is a futile search.

John Bell and Bohm's EPR

Einstein believed in **layer** of reality underneath quantum theory

that contains hidden variables (not yet discovered)

—> properties of particles —> return to having definite values before measurement.

When you look at old-fashioned mechanical clock face,

you see hands going around,

but may not be able to see hidden mechanisms driving them.

Careful study of moving hands might

—> can figure out properties of clock,

but would have much better understanding if get inside - see workings.

In Einstein's view quantum mechanics = theory of clock hands;

—> need another theory that looks inside at mechanism

(However, would that process ever end?).

At moment one cannot see mechanism inside quantum systems

<—> have not discovered any type of hidden variables involved.

Since do not have technique for measuring hidden variables (if they exist)

we cannot make definite predictions about outcome of experiments.

When repeat experiments, we are effectively

averaging over various (unknown) possible values of variables

and this ignorance is reason —> why we **end up** using probability in QM.

—> why probability is **not** intrinsic to world

according to Einstein and other hidden variable people!!!

In 1964, John Bell demonstrated to physics community that any hidden variable theory

with EPR-type entangled states would break

another of Einstein's most cherished assumptions

about way world works: **local causality**.

Causality = view that everything happening in world

is due to something else (cause) happening first, and then if we glue it to assumption

that nothing can travel faster than light (Einstein's theory of relativity)

—> **local causality**.

When particles interact, they might influence other's properties,

i.e., measure spin state A,

—> possible that result of measurement influences properties of B:

but —> there must be time delay.

Whatever connection exists between A and B,

information flows between them only at speeds less $< c$.

So, if measure A and then B somewhere far enough away,

fact that A has been measured

will not have reached B at time of measurement

and so can have no influence on measurement make of B

—> **local causality.**

Particle A can only influence other particles that happen to be in its locality.

Inside its light cone!

Bell wondered if a local hidden variable theory

could properly reproduce **all results** of quantum theory,

especially when dealing with EPR-type entangled states.

Now look again at Bohm-type EPR experiment using spin states(some repetition).

Assume existence of a set of hidden variables

that determine results of spin measurements on the particles.

If create two particles in so-called "singlet" state,

$$|A\rangle = \frac{1}{\sqrt{2}} |U\rangle_1 |D\rangle_2 - \frac{1}{\sqrt{2}} |D\rangle_1 |U\rangle_2$$

—> hidden variables must have **opposite** values for each particle.

In experiment, two particles, created in singlet state,

heading toward pair of separated S-G magnets.

Assume right-hand side S-G magnet much closer to place where particles created.

If we could read these hidden values would know particle A was UP (say)

and particle B was DOWN.

Don't know(ignorant) values —> have to mix both possibilities

or say particles in an entangled state.

If we did a series of repeated measurements

and found A **always** UP and B **always** DOWN,

then could avoid using entangled state —> state **IS** only one part of entangled state.

Since we do not see this happening in these experiments

—> we must assume that values of hidden variables

are randomly determined when particles were created.

The spins are always opposite, but which value each one has comes out randomly.

Particle A now reaches S-G 1, magnet set to arbitrary angle.

As result of measurement either get UP along this S-G angle or DOWN.

Probability of each not 50:50 unless S-G angle happens to be vertical(depends on angle).

If want to calculate probability of UP along the S-G angle,

have to look at angle between S-G 1's axis and original vertical.

Standard quantum theoretical calculation works and details not important here.

Point we are making.

While particles are able to interact

—> possible that values of hidden variables in one particle

alter values in other particle.

When A is measured, the interaction between A and S-G 1

might alter hidden variables,

and that information could be communicated to B and

—> influence what happens when measure B.

If set things up —> S-G 1 and S-G 2 far apart,

there is not time enough for any information to travel from S-G 1 to S-G 2

before particle B reaches S-G 2.

So only things that can influence what happens when particle B reaches S-G 2

are the values of its own hidden variables,

supposedly set when particles were **created**.

Compare now to quantum mechanical point of view.

When first measurement takes place at S-G 1,

entangled state of both particles will collapse producing 50:50 UP:DOWN result

no matter what angle of S-G 1 is.

Remember an entangled state has same nonseparable form in any basis.

If at the given **moment when** spin direction of A

manifests along line in direction of magnetic field in S-G 1 ,

then at **same moment** spin direction of B must manifest itself opposite.

If S-G 1 records UP, B arrives at S-G 2 in DOWN state along axis of S-G 1.

If S-G 2 is pointing along same axis, must record DOWN for B, **only same direction fixed!**

if not, -> UP and DOWN along its axis relative to how oriented with respect to S-G 1.

all this can be calculated in QM — it works

Crucial thing is collapse of state is affecting both particles at same time.

If hidden variable theory is going to match the effects of state collapse,

information from one measurement has to get to other

and if set up experiment correctly,

—> must do so faster than speed of light.

By assuming hidden variable theory had to obey rule of local causality,

Bell was able derive formula to compare results of each measurement

over long run of trials.

Experiment can then test Bell's formula

—> either quantum description is correct **or** local hidden variables can work.

Bell's Formula

Build experiment

—> have S-G magnets that can be set to one of three angles 0°, 45° and 90°.

Allow magnets to switch between the 3 angles randomly as experiment goes on.

Then simply sit back and watch singlet states

pass through both magnets and count up results.

If results from two magnets have no influence on one another, then should find that

$$N[\text{S-G1}(0^\circ)UP, \text{S-G2}(45^\circ)DOWN] + N[\text{S-G1}(45^\circ)UP, \text{S-G2}(90^\circ)DOWN]$$

$$\geq N[\text{S-G1}(0^\circ)UP, \text{S-G2}(90^\circ)DOWN]$$

N is number of pairs of particles counted satisfying set of conditions […],

conditions are inside square brackets, i.e., in term like

$$N[\text{S-G1}(0^\circ)UP, \text{S-G2}(45^\circ)DOWN]$$

$$N[\text{S-G1}(0^\circ)UP, \text{S-G2}(45^\circ)DOWN]$$

N is number of times A registered spin UP

in S-G1 when set to 0°

along with particle B registering DOWN

when S-G2 set at 45°.

Although predicted formula looks complicated,

can see how works with aid of figure below, which shows how Bell's formula

$$N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$$

is derived

Note in formula, if A means passes test A, then $\bar{\text{A}}$ means fails test A.

Making things easier, we forget about particles for a moment. and use passing/failing tests - it is same mathematics - that is what is great about mathematics use in physics!

So, replace idea of measuring particle spins

with that of passing or failing 3 different tests - A, B, and C.

i.e., Bell's derived result is a very general result (just a property of numbers and mathematics) that always should be satisfied everywhere if his classical type of reasoning is valid!

The different tests are going to stand for different angles of S-G magnets.

Passing means spin UP and failing means spin DOWN along that angle.

First test set up will be equivalent of measurement of S-G1 and second one S-G2.

Also going to assume that result of one test has no effect on next

—> assuming local causality

Symbol like N(A, $\bar{\mathrm{C}}$) going to stand for number of times pass test A **first**

and **then** fail ($\bar{\mathrm{C}}$) test C,

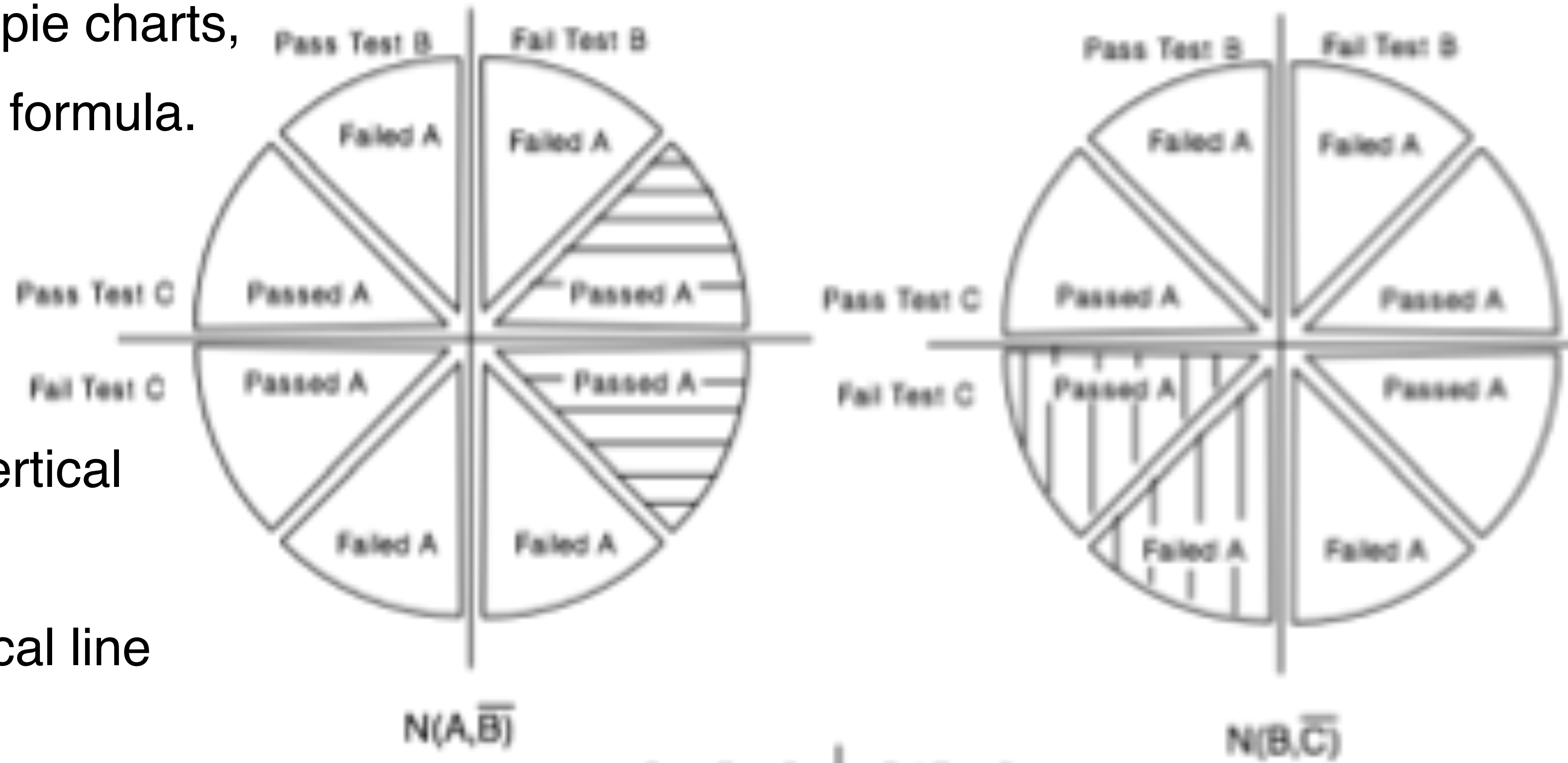
or in experimental terms number of pairs that were spin UP in S-G 1 set to angle A

and spin DOWN in S-G 2 set to angle C.

Now look at figures.

Figure broken up into three pie charts,

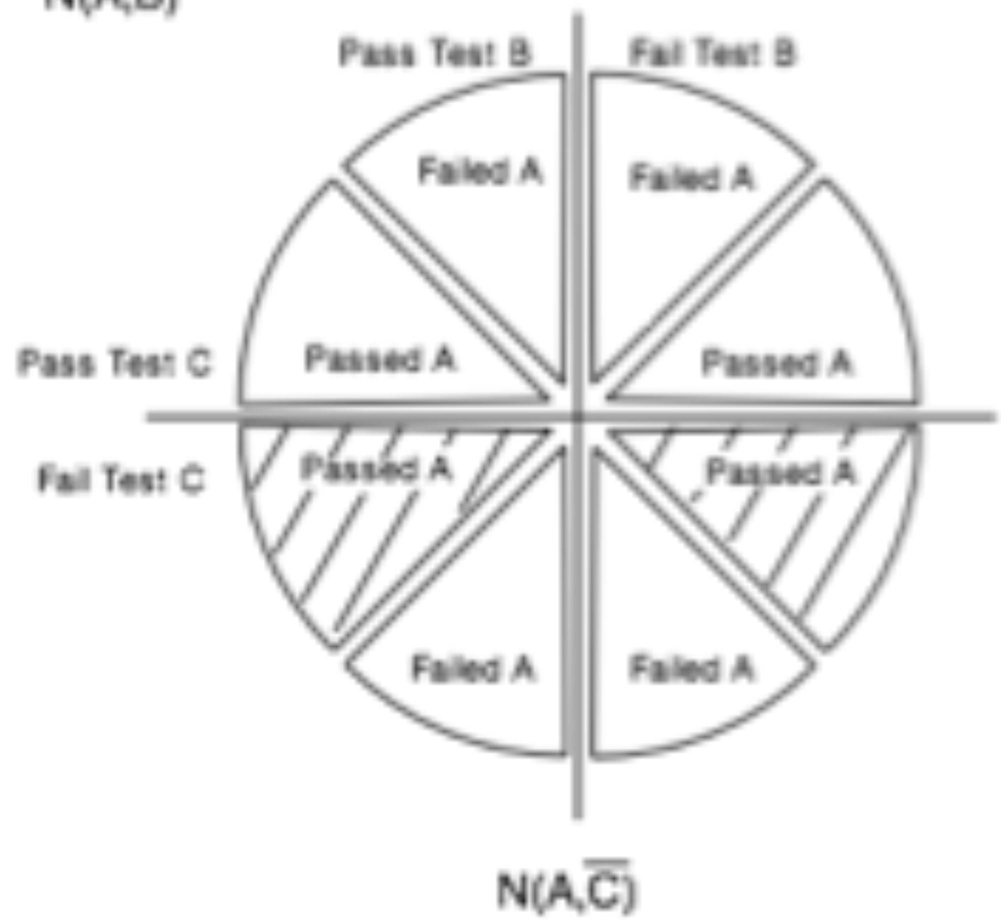
each stands for one part of formula.



Each pie chart divided by vertical and horizontal lines.

Left-hand side of each vertical line stands for passing test B, right-hand side for failing test B.

Above horizontal line stands for passing test C and below for failing test C.



Scattered around segments of pie charts are passing and failing tests A.

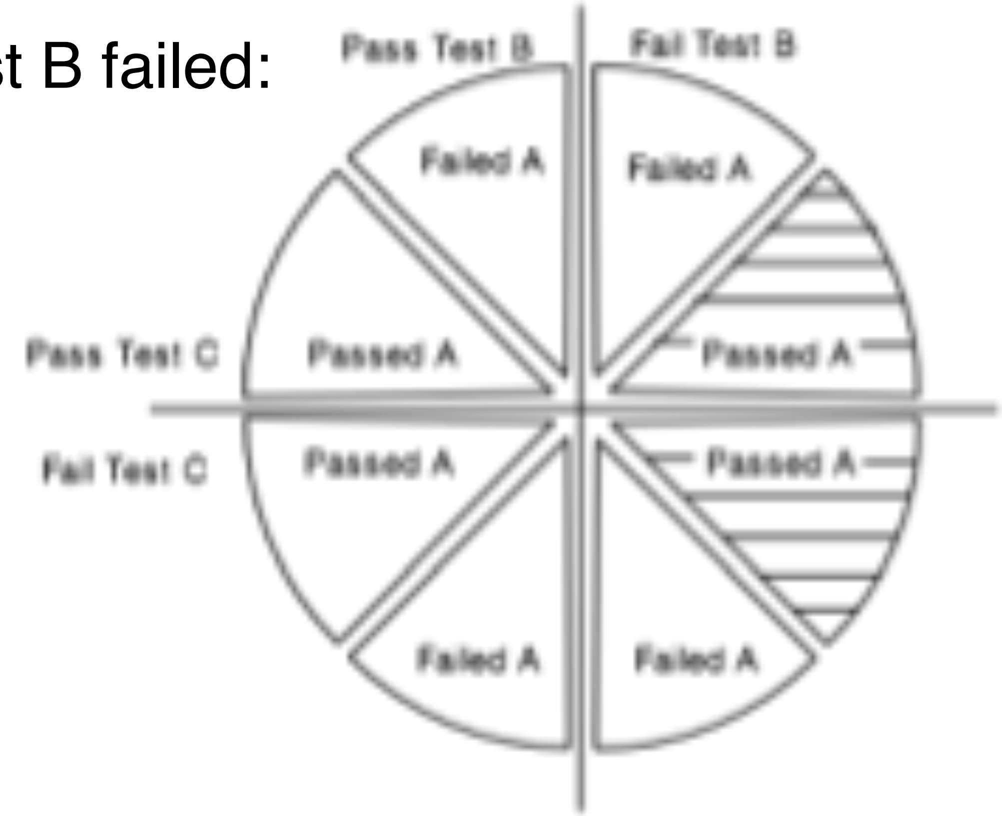
All three possibilities are **equally** represented (same area) in pie charts.

In top left chart,

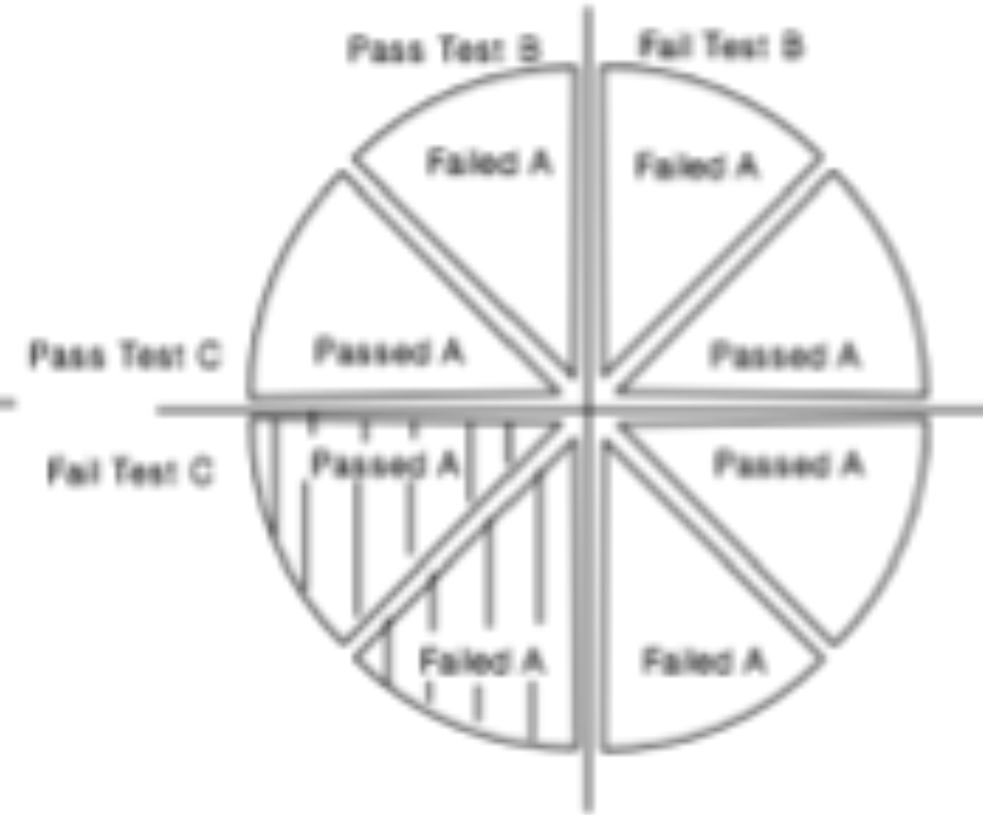
lined segments represent times

that test A passed and test B failed:

N(A,$\bar{\mathrm{B}}$).



N(A,$\overline{\mathrm{B}}$)



N(B,$\overline{\mathrm{C}}$)

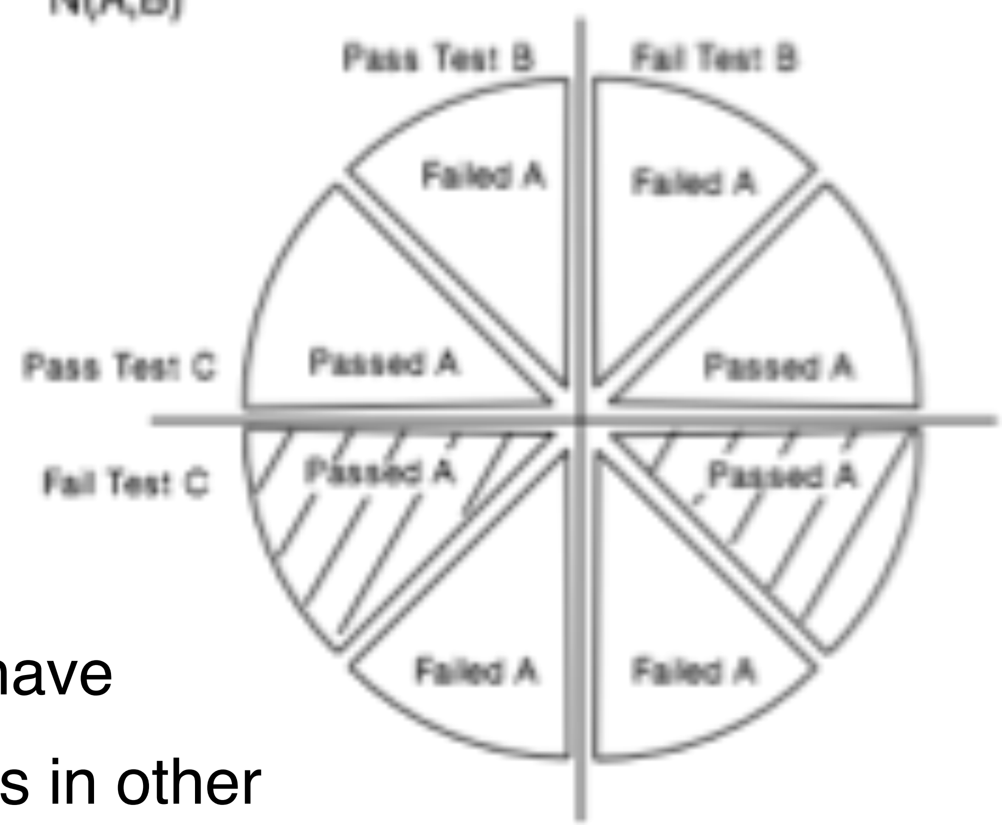
In top right chart, lined segments

are number of occasions that

B was passed and C failed: N(B,$\bar{\mathrm{C}}$).

Finally, in bottom chart lined segments

are passing A but failing C: N(A,$\bar{\mathrm{C}}$).



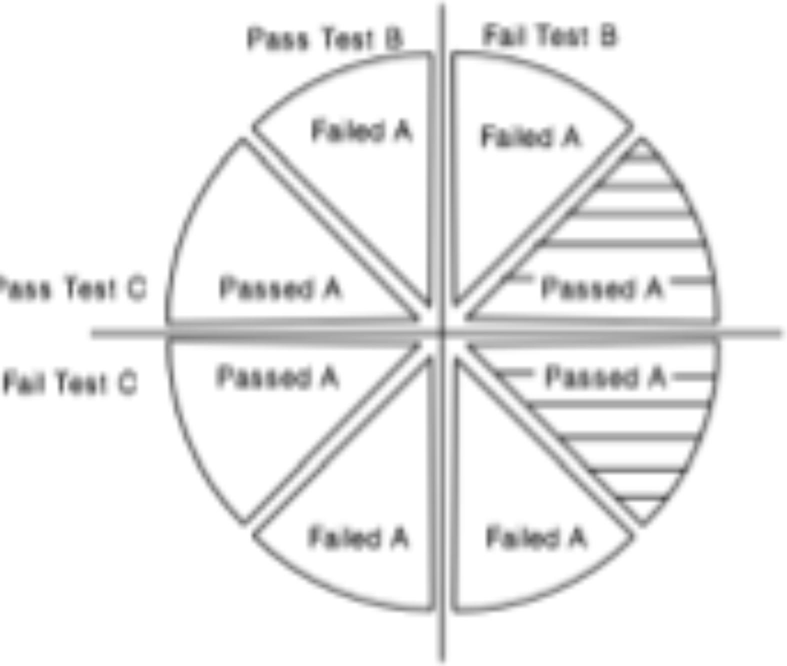
N(A,$\overline{\mathrm{C}}$)

Look closely and compare three charts,

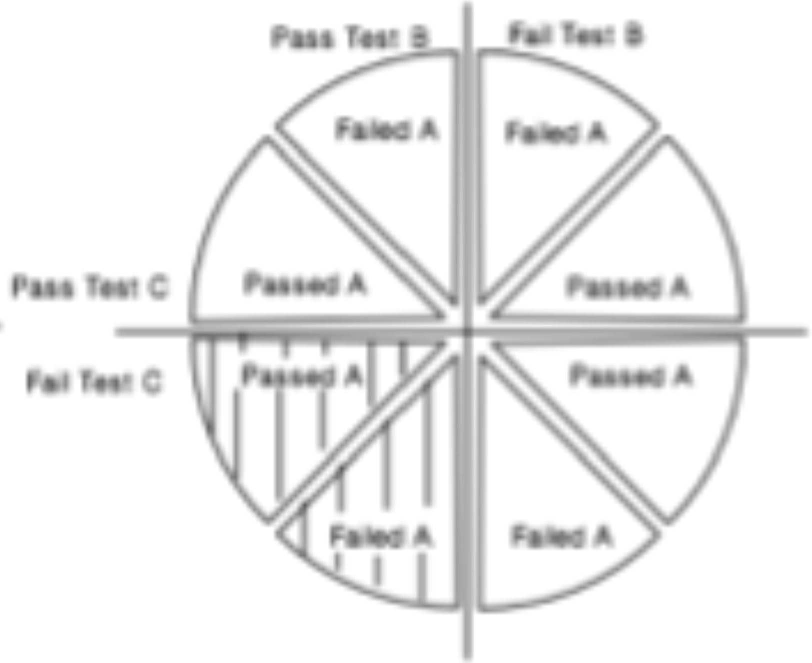
—> marked segments in bottom chart have

already been covered by marked regions in other

two charts - with area to spare!



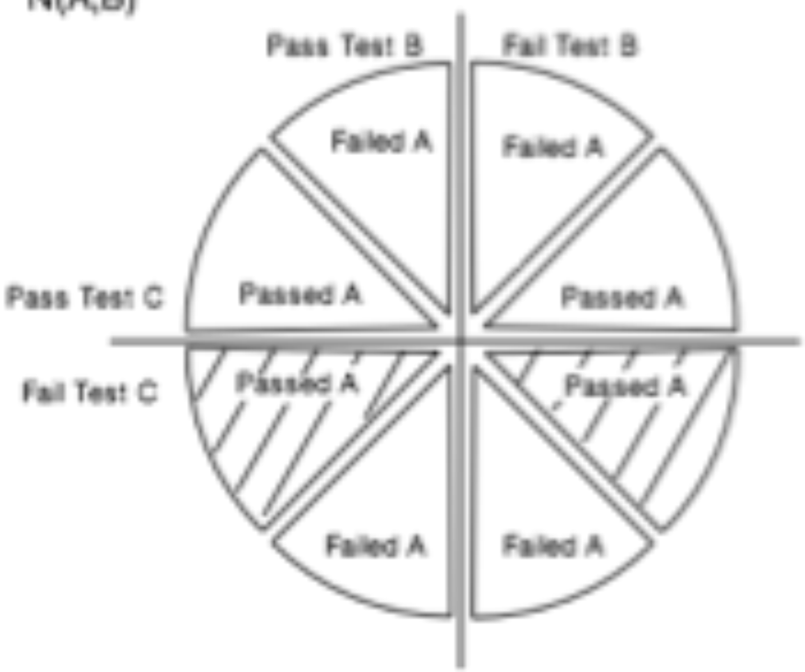
$N(A,\overline{B})$



$N(B,\overline{C})$

So, $N(A,\bar{B}) + N(B,\bar{C}) \geq N(A,\bar{C})$

—> d'Espagnat's version of Bell formula.



$N(A,\overline{C})$

Startling thing about this demonstration is that it shows how few assumptions need to be used in deriving Bell's formula and that it is simple classical property of numbers and standard logic.

Formula itself has nothing to do with quantum theory.

It's simply a relationship between numbers that fall (using standard classical logic) into certain categories.

Classical physics does obey this formula!!

Amazingly, as we will see, quantum physics does not obey this formula!!

That last staement should make you sit up in your chair!!!!

Aspect's Experiment -1st pass

Although d'Espagnat's version of Bell's formula

might be relatively easy to demonstrate mathematically,

it is not easy to test in experiment.

Different version was used for test, quote here which we will not derive(too complicated):

$$|E(a,b) - E(a,b')| + E(a',b) + E(a',b') \leq 2$$

is valid for a local hidden variable theory (with E(a,b) as defined below).

This expression is once again simply a relationship between numbers that fall (using standard classical logic) into certain categories.

Vertical lines in first term simply —> absolute value.

Collection of terms on left-hand side = S parameter, Bell result—> $S \leq 2$.

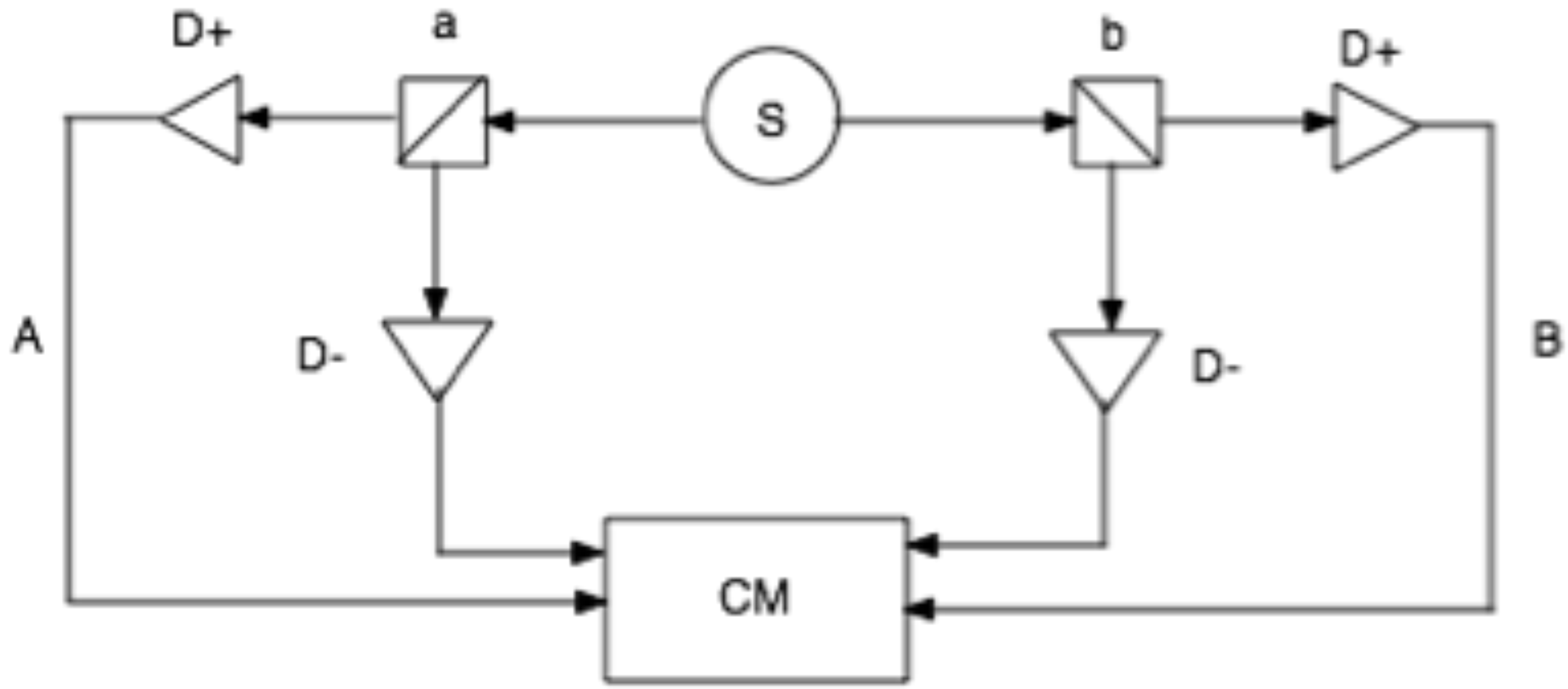
Idea of experiment

—> measure S value

and see if fits quantum theoretical description which —-—> $S > 2$.

Terms in expression directly relate to experimental arrangement

in figure below which shows Aspect's experiment to test Bell's formula.



In 1982(47 years after EPR) Alain Aspect's team at Institute for Optics in Paris

performed significant test of Bell's formula(22 years after - a difficult experiment)

since it was first experiment

that allowed detector angles

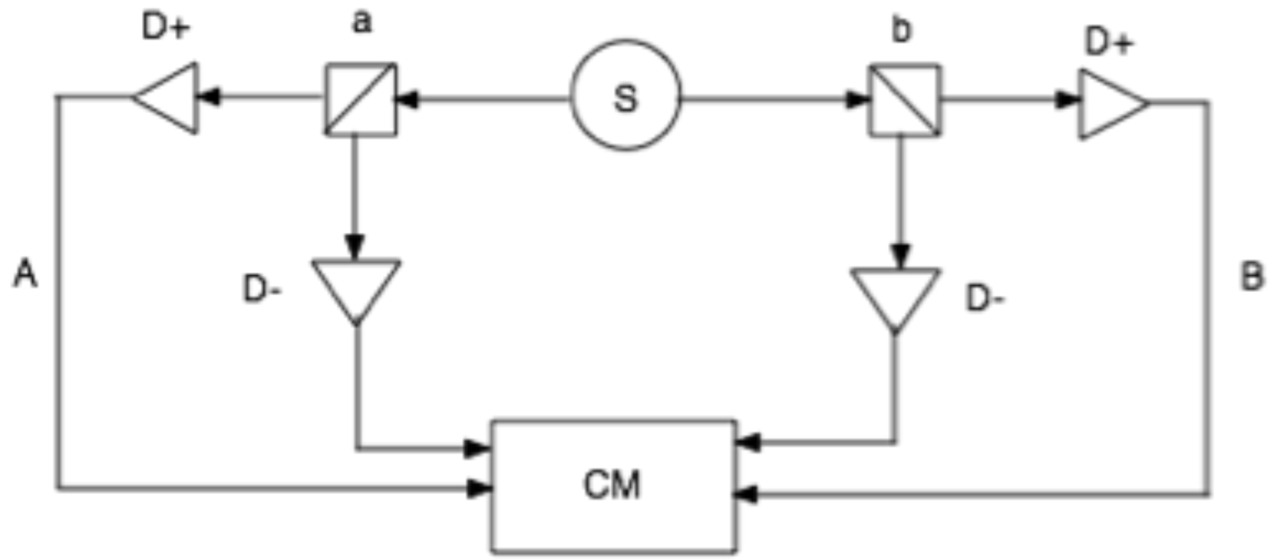
to be changed while particles were in flight (figure).

Describe experiment with more details later……

For now, source S produces pair of photons in combined singlet state.

Either side of source are equivalent of S-G magnets for photons (called polarizers).

These devices, labelled a and b, check polarization of photons (equivalent of spin).



If photon entering device A(left side) is spin UP along device's angles,

detected by D+, otherwise picked up by D−.

Same true on right-hand side in device B.

Devices can be set to different angles:

left-hand one can be set to either angle a or a′, right-hand one to b or b′.

Each of four detectors feeds into coincidence monitor (CM),

which counts number of times get each possibility (in coincidence => simultaneously):

N_{++} which is UP at A and UP at B

N_{--} which is DOWN at A and DOWN at B

N_{+-} which is UP at A and DOWN at B

N_{-+} which is DOWN at A and UP at B

Experiment run while allowing a and b

to randomly switch their angles between a and a′ and b and b′, respectively.

Finally use these counts to construct

$$E(a,b) = \frac{(N_{++} + N_{--}) - (N_{+-} + N_{-+})}{N_{++} + N_{--} + N_{+-} + N_{-+}}$$

Sort of an expectation value for results.

Aspect's experiment conducted with angles a = 0°, a$'$ = 45°, b = 22.5°, and b$'$ = 67.5°;

a combination predicted by quantum mechanical calculation

to break Bell's formula by largest amount.

In all detail,

it is important not to lose sight of the purpose of the experiment

and significance of Bell's formula.

If experimental information fits Bell's formula, so that $S \leq 2$,

then quantum mechanics is **wrong**

and could possibly be replaced by hidden variable theory

subject to condition of local causality.

If $S > 2$ local causality is **broken**

and it seems that results of one measurement can **influence** results of another,

although even a signal traveling at speed of light would

not have chance to reach second measurement in time.

Experiment then represents a **crucial step** in our understanding of quantum reality.

Working with angles in Aspect's experiment, QM predicts $S = 2.828$.

Correcting for detector efficiency (sometimes miss things)

—> $S = 2.70$ = prediction of QM.

Aspect measured $S = 2.697 \pm 0.015$

—> agreement with QM and breaking Bell's formula($S \leq 2$).

Since Aspect's pioneering experiment,

other(more accurate) such experiments —> quantum theory correct always.

In 2001, team from University of Innsbruck used

two detectors (Alice and Bob) 400 m apart to check formula.

(Now, experiments up to 144 kilometers apart)

Their detection efficiency was such that quantum mechanical prediction was $S = 2.74$

and they measured $S = 2.73 \pm 0.02$.

Conditions of 2001 experiment place an even greater restriction on things,

since choice of angle at each detector was truly random,

and angles were set after photons had left source

and results stored at each detector

and compared **only after** experiment had been completed.

All of these totally rule out any communication between two photons,

or even two detectors, that could explain results.

BELL'S THEOREM - An alternative presentation to enhance understanding

Bell demonstrated that under certain conditions

quantum theory and local hidden variable theories

predict different results for same experiment on pairs of correlated particles.

This difference,

which is intrinsic to all local hidden variable theories

and is independent of exact nature of theory,

is summarized in Bell's derived inequalities.

This proof forced questions about hidden variables to immediately change character.

They were no longer academic questions about philosophy
but practical questions of profound importance for quantum theory.

Choice between quantum theory and local hidden variable theories
was no longer matter of **taste**, but matter of **correctness**.

Now for Bertlmann's Socks

Derive Bell's theorem with help of famous Dr Bertlmann
(this whimsical story is by Bell himself invented to show how "classical" it is).

Any philosopher in the street,
who has not suffered through a course in quantum mechanics,
is quite unimpressed by Einstein-Podolsky-Rosen correlations.

She can point to many examples of similar correlations in everyday life.

Bertlmann decides to subject his left socks (socks A from now on)

to 3 different tests:

$$\text{test } a \text{ washing for 1 hour at } 0.0^\circ\, C$$
$$\text{test } b \text{ washing for 1 hour at } 22.5^\circ\, C$$
$$\text{test } c \text{ washing for 1 hour at } 45.0^\circ\, C$$

He is particularly concerned about numbers of socks A that survive intact (a + result)

or are destroyed (a – result)

by prolonged washing at these different temperatures.

He denotes number of socks that pass test a and fail test b as

$$n[a+b-]$$

Being a theoretical physicist,

he knows can discover simple relationships between such numbers

without actually performing tests using real socks and real washing machines.

Makes study inexpensive and more attractive to research sponsors.

He reasons as follows: n[a + b–]

can be written as sum of numbers of socks belonging to two subsets,

one where individual socks pass test a, fail b and pass c

and one where socks pass test a, fail b and fail c, i.e.,

$$n\,[a+b-] = n\,[a+b-c+] + n\,[a+b-c-]$$

This works probabilistically because

$$P(C) + P(not\ C) = 1$$

Also where individual socks pass test b, fail c and pass a

and one where socks pass test b, fail c and fail a and

$$n[b+c-] = n[a+b+c-] + n[a-b+c-]$$

because $P(A) + P(not\ A) = 1$

From first equation follows that

$$n\,[a+b-] \geq n\,[a+b-c-]$$

since all numbers involved are ≥ 0.

Similarly, from second equation follows that

$$n\,[b+c-] \geq n\,[a+b+c-]$$

Adding last two equations gives result

$$n\,[a+b-] + n\,[b+c-] \geq n\,[a+b-c-] + n\,[a+b+c-] = n\,[a+c-]$$

or

$$n\,[a+b-] + n\,[b+c-] \geq n\,[a+c-]$$

derivation only used standard logic and inequalities among numbers

At this stage, Dr. Bertlmann notices a **flaw** in his reasoning,

which all of you will, **of course**, have spotted right at beginning.

Subjecting one of socks A to test a

will necessarily change irreversibly its physical characteristics such that,

even if it survives test,

it may not give result for test b that might be expected of brand new sock.

And, of course, if sock fails test b, it will simply not be available(destroyed) for test c.

Thus, numbers like n[a + b–], etc, have **no** practical (cannot measure in real world) relevance.

Bertlmann now remembers his socks always come in pairs.

He assumes that, apart from differences in color,

physical characteristics of each sock in pair are identical.

Thus, test performed on right sock (sock B)

can be used to predict what result of same test would be

if test had been performed on left sock(sock A),

even though test on A not actually carried out

Remember this is a counterfactual statement - relating to or expressing what has not happened or is not the case!. No test is done on A!

Must further assume that whatever test

he chooses to perform on B sock in no way

affects the outcome of any other test he might perform on A sock,

but this seems so obviously valid that

he does not give it any thought whatsoever!!!!

Can you figure out where we are in the EPR argument? Think causality…….

Bertlmann now devises three different sets of experiments

to be carried out on three samples

each containing same total number of pairs of socks.

In experiment 1, for each pair,

sock A is subjected to test a and sock B is subjected to test b .

If sock B fails test b, this implies that sock A would also have failed test b

had it been performed on sock A.

Thus, number of pairs of socks for which sock A passes test a and sock B fails test b,

which we denote by $N_{+-}(a,b)$

must be equal to (hypothetical) number of socks A which pass test

a and fail test b(same sock), i.e., $N_{+-}(a,b) = n\,[a+b-]$

In experiment 2, for each pair,

sock A is subjected to test b and sock B is subjected to test c.

Same kind of reasoning allows Bertlmann to deduce that $N_{+-}(b,c) = n\,[b+c-]$

where N+–(b,c) denotes number of pairs of socks

for which sock A passes test b and sock B fails test c.

Finally, in experiment 3, for each pair,

sock A is subjected to test a and sock B is subjected to test c.

In a similar manner, Bertlmann thus deduces that $N_{+-}(a,c) = n\,[a+c-]$

where N+–(a,c) denotes number of pairs of socks

for which sock A passes test a and sock B fails test c.

Experimental arrangements summarized below.

Experiment	Sock A Test	Sock B Test
1	a	b
2	b	c
3	a	c

Bertlmann, now using revised calculations, concludes again that we must have inequality

$$N_{+-}(a,b) + N_{+-}(b,c) \geq N_{+-}(a,c)$$

to represent the experiments.

Bertlmann generalizes this result for any batch of pairs of socks

by dividing each number by total number of pairs of socks

(same for each experiment)

to arrive at frequencies with which each joint result was obtained.

He identifies the frequencies with probabilities

for obtaining results for experiments to be performed

on any batch of pairs of socks that, statistically, have same properties.

Thus, he finds that

$$P_{+-}(a,b) + P_{+-}(b,c) \geq P_{+-}(a,c)$$

—> Bell's inequality for this experiment.

Again, just standard classical logic and properties of numbers!

Now follow above arguments again, replacing

1. socks with photons
2. pairs of socks with pairs of entangled photons
3. washing machines with polarization analyzers
4. temperatures with polarizer orientations

we still arrive at the standard Bell's inequality,

i.e., only change words in our description of experiments and not conclusions!

Three tests now refer to polarization analyzers set

with their vertical(optic) axes oriented at

a→0.0°, b→22.5°, and c→45.0°.

Different experimental arrangements summarized as follows:

Experiment	Photon A Angle	Photon B Angle	Angle Difference Δ
1	$a = 0.0°$	$b = 22.5°$	$22.5°$
2	$b = 22.5°$	$c = 45.0°$	$22.5°$
3	$a = 0.0°$	$c = 45.0°$	$45.0°$

Probabilities **predicted** by quantum theory

for given angle **difference** Δ between polaroids in each test given(using QM) by

$$P = \frac{1}{2}\sin^2\Delta$$

i.e., remember matches and misses arguments.

Putting angles above into Bell inequality get

$$\frac{1}{2}\sin^2 22.5° + \frac{1}{2}\sin^2 22.5° \geq \frac{1}{2}\sin^2 45.0°$$

or $0.1464 \geq 0.2500$ which is obviously incorrect!

Thus, for these particular arrangements of polarization analyzers,

the probability formula from quantum theory predicts results

that violate Bell's inequality.

However, all the quantum mechanical probability formula results

agree with all experimental observations!!

Implications:

Einstein could not have seen where EPR argument would lead:

from Bohm's simpler and more practical version,

to Bell's analysis in terms of local hidden variables

to Aspect's experiment and the results that have been produced since.

Quantum theory has survived every test thrown at it and thanks to Bell's argument,

now seems impossible that it will ever be replaced by any local hidden variable theory.

Of course, the option of hidden variable theory

that does not obey local causality is still on the table.

Experimental results showing correlations

between two detectors **could be explained** by communication between two particles

traveling faster than the speed of light,

something that would hardly have made Einstein feel any better about things.

Quantum theory survives Bell's test

because entangled states **collapse** at first measurement.

Remarkably this collapse affects **both** particles in entangled state

no matter how **far apart** they may be.

Bohr simply shrugged this off

as a consequence of having to use the

same experimental context for both particles,

an argument that is logically compelling

but does not seem to do justice to the remarkable properties of entangled states.

Bell's formula is broken by entangled state,

so if want to take realistic view of what's happening,

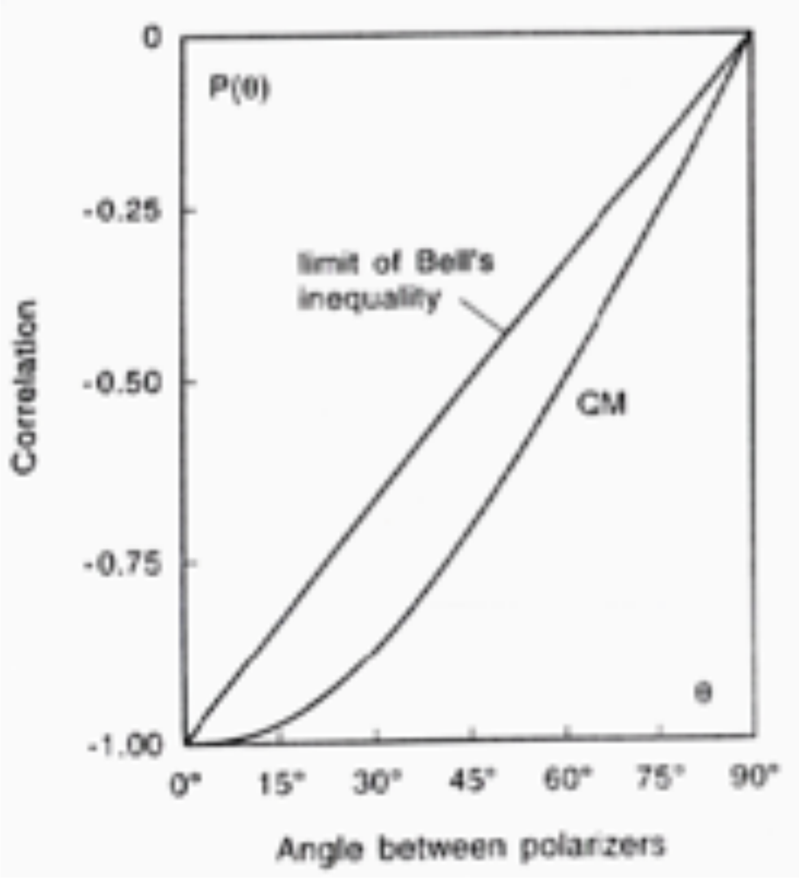
state collapse has to be real physical change

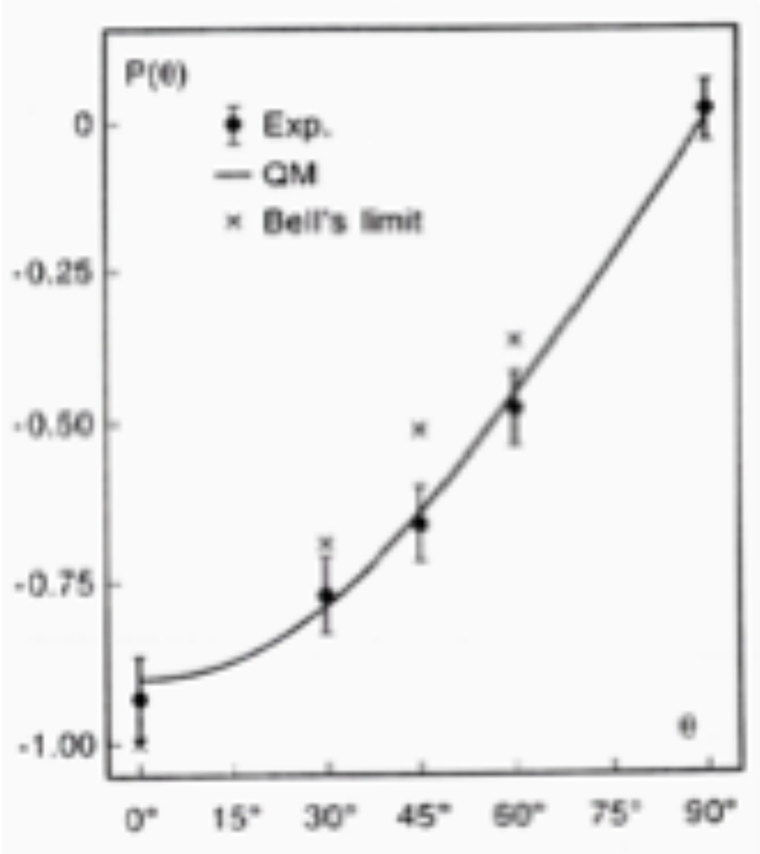
happening everywhere at once.

Perhaps we ought to stop thinking of

two entangled particles as being separate at all.

Comparison of quantum mechanics and Bell inequality from 1st experiments is shown in figures below:





Clearly, quantum mechanics is correct. Even **better** data exists now!

Most important assumption made in reasoning
which led to inequality was **Einstein separability**
or **local reality** of photons(or socks).

Inequality is quite independent of nature of any local hidden variable theory
that could be devised.

Conclusion is inescapable.

Quantum theory is incompatible with any local hidden variable theory
and hence **incompatible** with any form of **local reality**.

Means that any theory that has **same** probabilistic predictions as quantum mechanics

must be nonlocal.

Should not be too surprised by this result.

Predictions of quantum theory are based

on properties of 2-particle state vector which,

before collapsing into one of measurement eigenstates,

is delocalized or entangled over **whole** experimental arrangement.

2 particles are, in effect,

always in contact prior to measurement

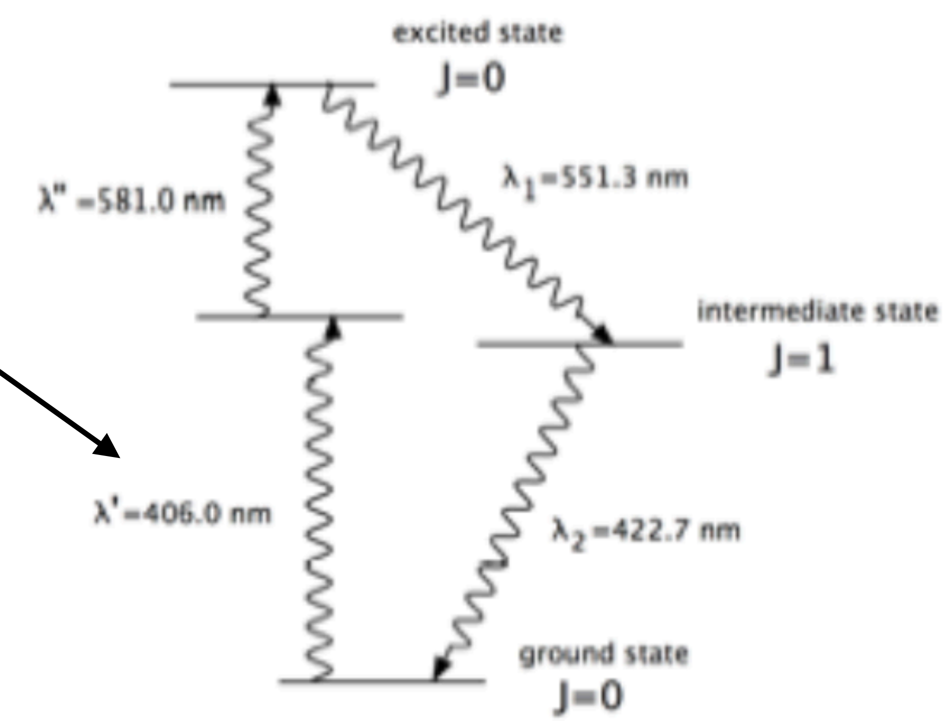
and **can** therefore exhibit a degree of **correlation**

impossible for 2 Einstein separable (locally realistic) particles.

Now for some details of the Aspect experiments.

The Details of the Aspect Experiment at Orsay - Alain Aspect -Institute of Optics - Paris 1980

Light source is beam of calcium atoms, excited by two focused laser beams having wavelengths $\lambda' = 406$ nm and $\lambda'' = 581$ nm respectively.



Calcium based light source - Energy Levels

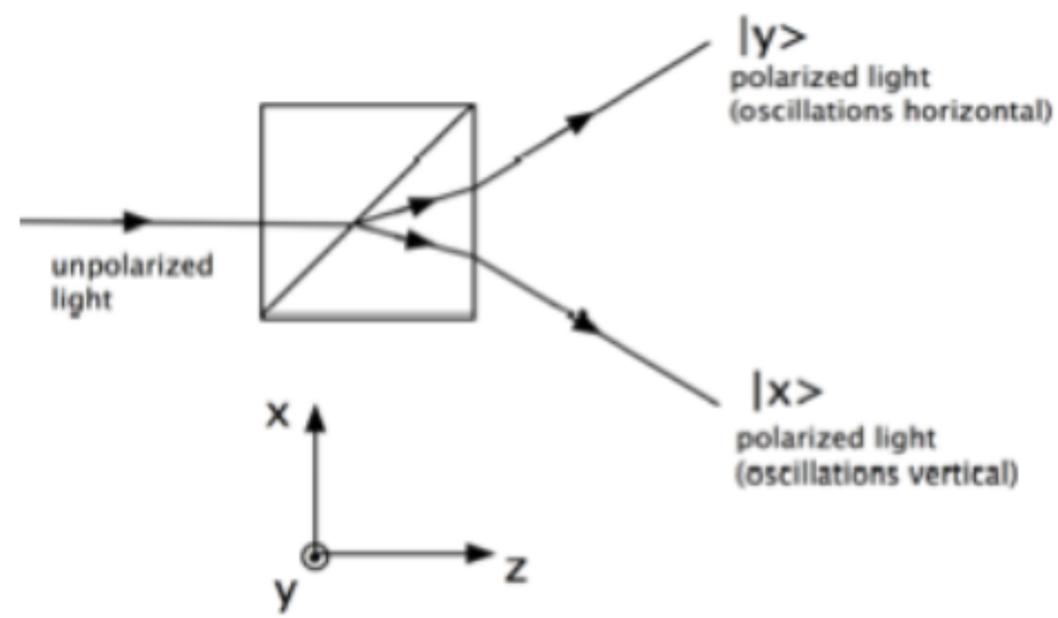
Two-photon excitation produces state having the quantum number J = 0 (angular momentum).

When it decays, this state emits two monochromatic photons having wavelengths $\lambda_1 = 551.3$nm and $\lambda_2 = 422.7$nm respectively, in a cascade of two electronic transitions from the initial J = 0 level to the final J = 0 state, passing through an intermediate J = 1 state, as shown in figure which shows excitation and decay of calcium atom.

Mean lifetime of intermediate state is 4.7ns.

Simplify terminology, call $\lambda_1 = 551.3$ nm light green, and $\lambda_2 = 422.7$ nm light violet.

Polarizers in experiment = Wollaston prisms is shown in figure where we can see its two-valued response.



Wollaston prism is made of quartz or of calcite.

It splits incident beam of natural (unpolarized) light into two beams of equal intensity, polarized at 90° to each other.

If only single unpolarized photon incident, it emerges either in state $|x\rangle$, with probability 1/2,

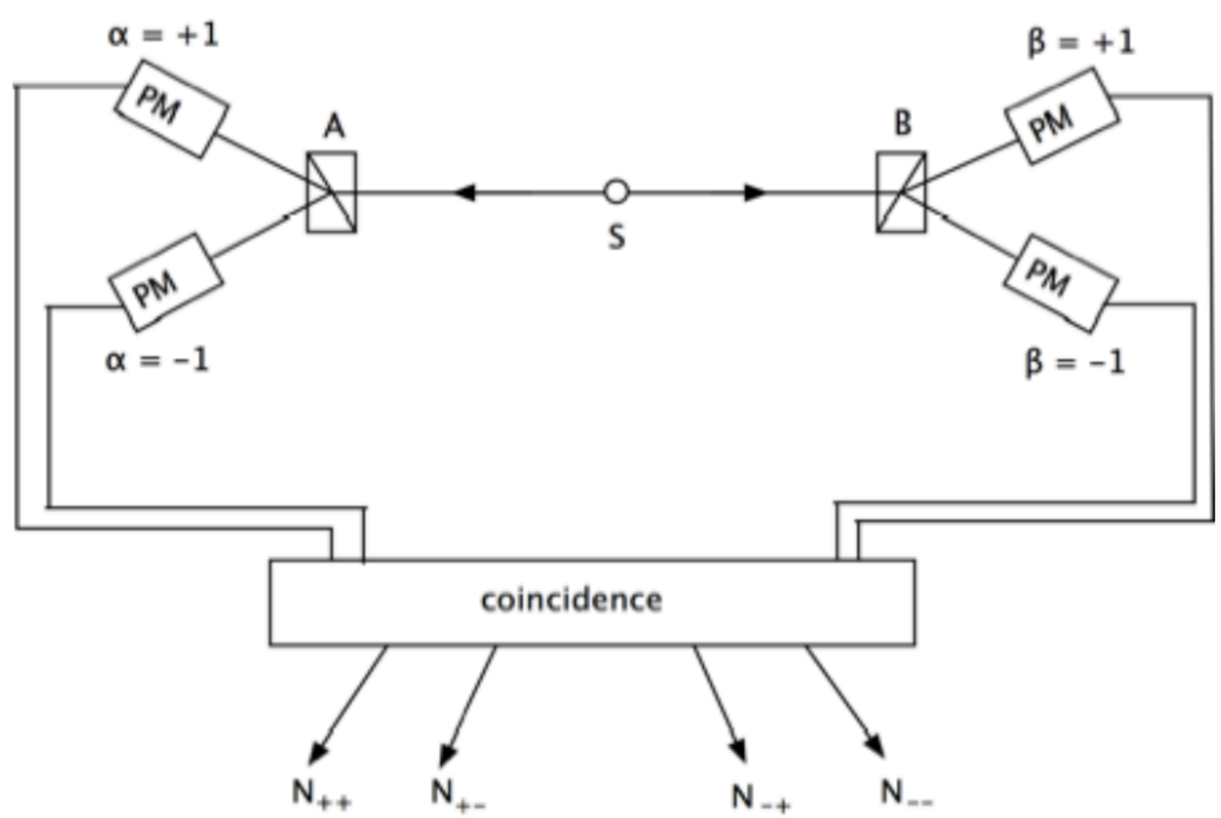
or in state $|y\rangle$, with probability 1/2 (50-50).

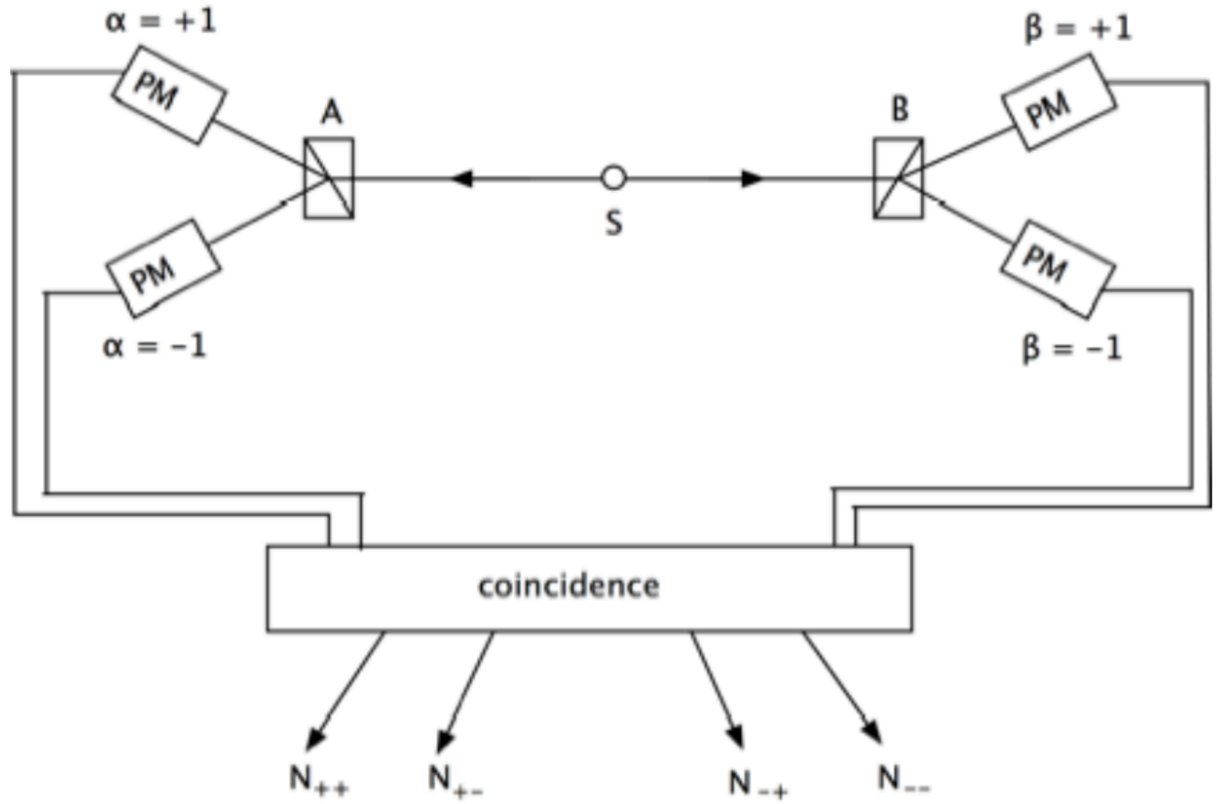
Thus, response of system is **two-valued**.

Photons are detected by photomultiplier tubes (PM) downstream from prism.

Every electric pulse from detectors corresponds to passage of a photon, allowing photons to be counted.

Experimental layout sketched in figure, which shows first Orsay experiment.





It uses coincidence circuit which registers an event whenever two photons are detected simultaneously.

In this way four separate counts are recorded simultaneously, over some given period of time.

In simpler EPR experiment(Bohm), only 2 possible responses are (+1,−1) or (−1, +1).

In situation realized by Aspect four different responses are possible.

N_{++} the number of coincidences corresponding to $\alpha = 1$ and $\beta = 1$, that is, to $\alpha\beta = 1$

N_{+-} the number of coincidences corresponding to $\alpha = 1$ and $\beta = -1$, that is, to $\alpha\beta = -1$

N_{-+} the number of coincidences corresponding to $\alpha = -1$ and $\beta = 1$, that is, to $\alpha\beta = -1$

N_{--} the number of coincidences corresponding to $\alpha = -1$ and $\beta = -1$, that is, to $\alpha\beta = 1$

Resolving time of coincidence circuit is 10ns, meaning that says two photons coincident if they are separated in time by no more than 10ns. Mean lifetime of the intermediate state of the calcium atom is 4.7ns —-> on average only one photon per mean lifetime in the experiment.

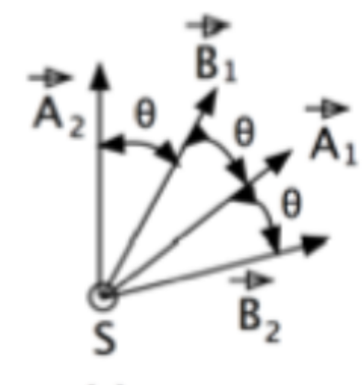
Therefore, after lapse of 10ns = more than twice mean lifetime, almost all atoms have decayed (actually 88%). In other words, efficiency of coincidence counter is very high. Even better now!

Experiment consists in counting, over some given time interval, the four kinds of coincidence:

N_{++}, N_{+-}, N_{-+} and N_{--}. The total number of events is $N = N_{++} + N_{+-} + N_{-+} + N_{--}$.

Accordingly, the different kinds of coincidence have probabilities

$$P_{++} = N_{++}/N \text{ corresponding to } \alpha\beta = 1$$
$$P_{+-} = N_{+-}/N \text{ corresponding to } \alpha\beta = -1$$
$$P_{-+} = N_{-+}/N \text{ corresponding to } \alpha\beta = -1$$
$$P_{--} = N_{--}/N \text{ corresponding to } \alpha\beta = 1$$



and the measured average of $\alpha\beta$ is

$$\langle\alpha\beta\rangle = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N}$$

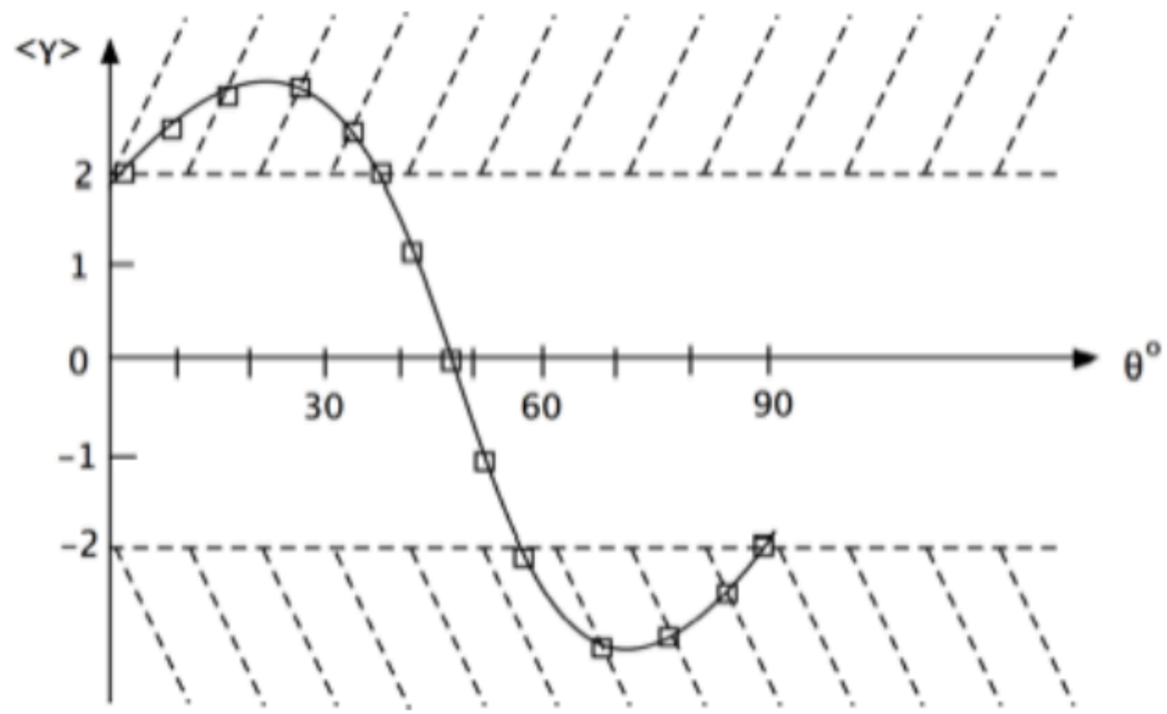
Each set of four coincidence counts corresponds to one particular relative angular setting of (A, B) prisms, and yields a mean value $\langle\alpha\beta\rangle$. But in order to determine the correlation function $\langle\gamma\rangle$ (equivalent to S parameter) used in the BCHSH form of Bell inequality, we need four mean values $\langle\alpha\beta\rangle$.

Therefore, we choose, in succession four different settings(explain) as shown in figure above four counting runs then yield the four mean values $\langle\alpha_1\beta_1\rangle$, $\langle\alpha_1\beta_2\rangle$, $\langle\alpha_2\beta_1\rangle$, $\langle\alpha_2\beta_2\rangle$ which then determine the value of $\langle\gamma\rangle$ via

$$\langle\gamma\rangle = \langle\alpha_1\beta_1\rangle + \langle\alpha_1\beta_2\rangle + \langle\alpha_2\beta_1\rangle - \langle\alpha_2\beta_2\rangle$$

The Results of the First Experiment at Orsay

Results of first Orsay experiment shown in figure. Angle θ which specifies relative setting of polarizers plotted horizontally, and mean value $\langle\gamma\rangle$ vertically.



Correlation function predicted by mathematics of quantum mechanics is

$$\langle\gamma\rangle = 3\cos 2\theta - \cos 6\theta$$

It is drawn as solid curve on graph. According to Bell-Clauser, Horne, Shimony and Holt (BCHSH) version of inequality

$$-2 \leq \langle\gamma\rangle \leq 2$$

so that any hidden-variable theories exclude cross-hatched regions of plane, which correspond to $\langle\gamma\rangle > 2$ or $\langle\gamma\rangle < -2$. Notice experiment goes into cross-hatched regions violating bounds!!

Clearly, can be no doubt that BCHSH inequality is violated; many of experimental points fall outside interval [−2, 2]. At point where violation is maximal ($\theta = 22.5^{\circ}$), one finds

$$\langle\gamma\rangle = 2.70 \pm 0.015$$

which represents departure of over **40 standard deviations** from extreme value of 2. What is even more convincing is precision with which experimental points lie on curve predicted by quantum mechanics - latest experiments are even more dramatic (curve and data exactly same!).

Evidently, for EPR scenario, must conclude not only that hidden-variable theories fail, but also that quantum mechanics is positively the right theory for describing observations.

The Relativistic Test

EPR experiment just described shows that measurements in A and B are **correlated**.
What is origin of correlations?

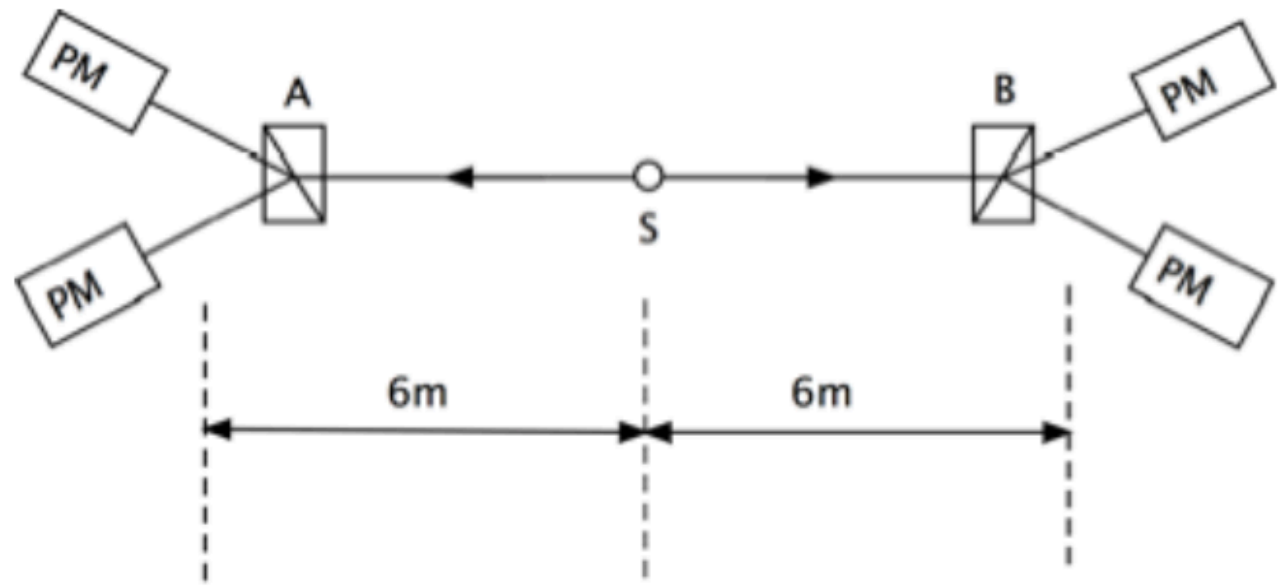
According to quantum theory, before measurement each particle pair constitutes single system extending from A to B, whose two parts are **non-separable and correlated**. This interpretation corresponds to a violation of Bell's inequality and agreement with experiment.

According to hidden-variables theories, particle pair characterized, at instant of decay, by its hidden variable, which determines correlation between polarizations measured in A and B. This interpretation satisfies Bell's inequality but disagrees with experiment.

Accordingly, Orsay experiment supports quantum interpretation (in terms of correlation between two parts A and B of a single system).

However, to reinforce conclusion, must ensure that no influence exerted in ordinary classical sense through some interaction propagated between two detectors A and B, that is, no influence which might take effect after decay at S, and which might be responsible for correlation actually observed.

Let us therefore examine Orsay apparatus in more detail as in figure below where we attempt to test **Einsteinian non-separability**.



When detectors at A and B record a coincidence, means that both have been triggered within a time interval of at most 10 ns, the resolving time of circuit.

Could it happen that, within this interval, A sends to B a signal capable of influencing the response of B?

In most favorable case, such a signal would travel with speed of light in vacuum, which according to relativity theory is upper limit on propagation speed of information, and thereby of energy.

To cover distance AB, which is 12 m in figure, such a signal would need 40 ns.

This is too long by at least 30 ns, and rules out any causal links between A and B in the sense of classical physics.

One says that interval between A and B is **space-like**.

One of advantages of Orsay experiment is that it uses a very strong light-source, allowing sufficient distance between detectors A and B while still preserving reasonable counting rates.

By increasing distance AB step by step, Aspect could check that correlation persists, even when interval between A and B becomes space-like.

This is check that guarantees that two-photon system is non-separable irrespective of distance AB.

It has become customary to speak of principle of **Einsteinian separability** in order to denote absence of correlations between two events separated by a space-like interval.

This is principle that Orsay experiment invites us to reconsider, even though our minds, used to the world at the macroscopic level, find it difficult to conceive of two microscopic photons 12m apart as a single indivisible object (**remember this point for later!!!**).

The Final Stage of the Experiment at Orsay

Though results of first Orsay experiment are unarguable and clear-cut, conclusion they invite is so startling that one should not be surprised at appearance of a last-ditch objection, which as it happens gave experimenters a great deal of trouble.

In the preceding section we discussed possible role of interactions between A and B operating after decay at S, and duly eliminated objection.

But one can also ask whether correlations might be introduced through an interaction operating **before** decay - before experiment even starts!!.

We could imagine that the decay itself is preconditioned by setting of detectors A and B, such influences taking effect through exchange of signals between detectors and source.

No such mechanism is known a priori, but we do know that, if there is one, then Einsteinian non-separability would cease to be a problem, because the mechanism could come into action long before the decay, removing any reason for expecting a minimum 30 ns delay.

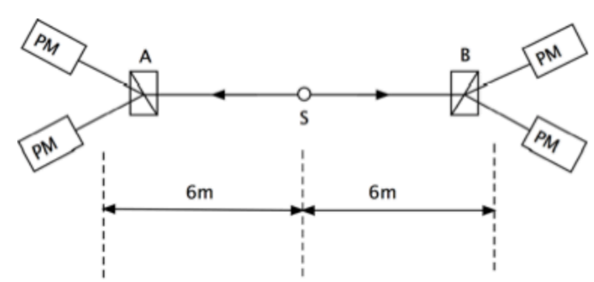
Though such a scenario is very unlikely, objection is a serious one and must be taken into account;

To get around it, experimenter must be able to choose orientation of detectors A and B at random after decay has happened at S.

In more picturesque language,

we would say that two photons must leave source without knowing orientations of polarizers A and B.

Briefly put, this means that it must be possible to change detector orientations during 20 ns transits over paths SA and SB.



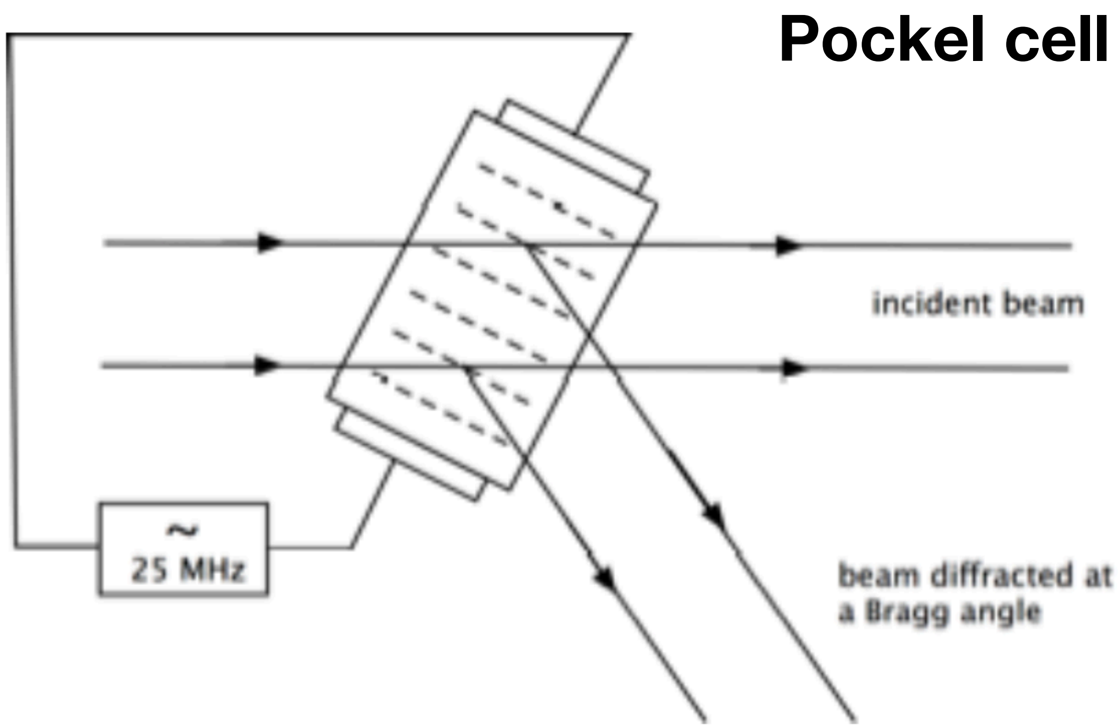
Solution adopted at Orsay employs periodic switching every 10ns.

These changes are governed by two independent oscillators, one for channel A and one for channel B.

The oscillators are stabilized, but however good the stabilization it cannot eliminate small random drifts that are different in the two channels, seeing that the oscillators are independent.

This ensures that the changes of orientation are random even though the oscillations are periodic, provided the experiment lasts long enough (1 to 3 hours).

The key element of the second Orsay experiment is the optical switch shown in figure below.



In a water tank,

a system of standing waves is produced by electro-acoustic excitation

at a frequency of 25 MHz

(corresponds to 10 ns between switchings).

now even better!!

Fluid keeps changing from a state of perfect rest to one of maximum agitation and back again.

In state of rest, light beam is simply transmitted.

In state of maximum agitation, fluid arranges itself into a structure of parallel and equidistant plane layers, alternately stationary (nodal planes) or agitated (antinodal planes).

Thus, one sets up a lattice of net-like diffracting planes.

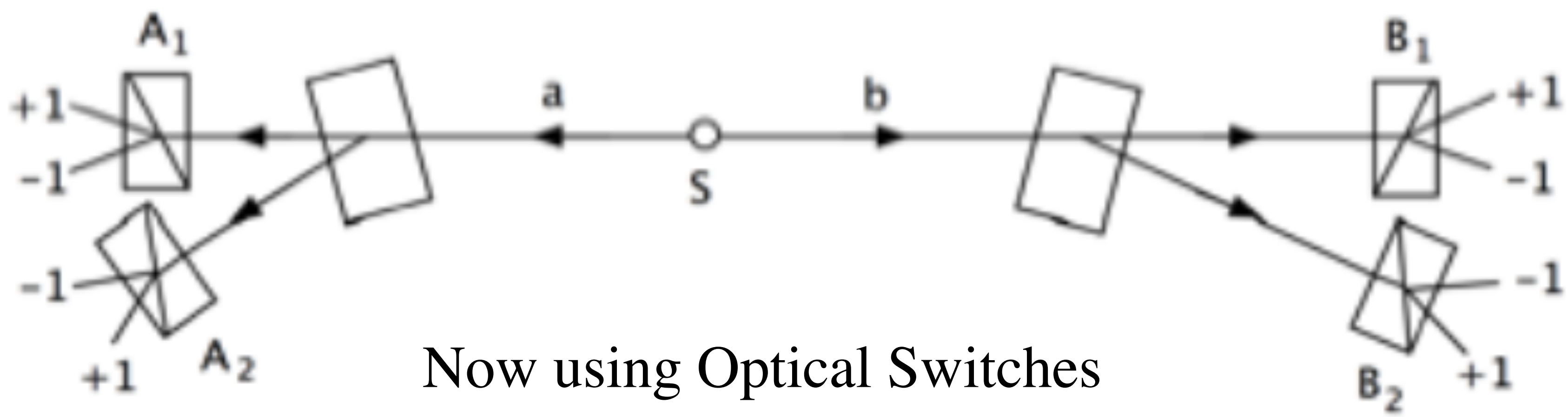
Diffracted intensity is maximum at so-called Bragg angles, just as in scattering from a crystal lattice.

Here, light beam is deviated through 10^{-2} radians (angles in figure are exaggerated for effect).

The two numerical values, 25 MHz and 10^{-2} radians, suffice to show magnitude of the technical achievement.

With the acoustic power of 1 watt, the system functions as an ideally efficient switch.

The second Orsay experiment (using optical switches) is sketched in Figure below.



Now using Optical Switches

In this set-up, photons a and b leave S without knowing whether they will go - the first to A_1 or A_2 and the second to B_1 or B_2.

Second experiment is less precise than first, because light beams must be very highly collimated in order to ensure efficient switching.

Nevertheless, its results exhibit an unambiguous violation of Bell's inequality, reaching 5 standard deviations at the peak;

Moreover the results are entirely compatible with the predictions of quantum mechanics.

Some final thoughts about all these ideas before we proceed to the quantum measurement discussion.

Bell's theorem follows from classical rules for probability.

QM does not agree with the theorem.

The way QM probabilities work is different.

For example: Quantum mechanics make us wonder:

Is FALSE the SAME as NOT TRUE?

The statement

"there is a 70% chance that the proposition - if A is measured, then the result will be a - is true"

is quite different from

"if A is measured, then there is a 70% chance that the result will be a".

It is the latter that is intended in quantum mechanics by the statement:

Prob(A=a|Wave function) =0.7

QM works and NOTHING ELSE works!

Statements from the Pioneers

Bohr once declared when asked whether quantum mechanical algorithms could be considered as somehow mirroring an underlying quantum reality: He said :

> **There is no quantum world. There is only an abstract quantum mechanical description. It is wrong to think that the task of physics is to find out how Nature is. Physics is concerned only with what we can say about Nature.**

Heisenberg said:

> **In the experiments about atomic events we have to deal with things and facts, with phenomena that are just as real as any phenomena in daily life. But the atoms or the elementary particles are not as real; they form a world of potentialities or possibilities rather than one of real things or facts.**

Jordan declared:

> **That observations not only disturb what has to be measured, they produce it. In a measurement of position of an electron, the electron is forced to a decision.**
>
> **We compel it to assume a definite position; previously it was, in general, neither here nor there; it had not yet made its decision about a definite position.**

Clearly, since measurement is so crucial to QM, let us now look at various aspects of measurement theory.