Quantum Measurement

We now review the standard formulation of quantum mechanics that we have developed, which is based on the 4 postulates listed below(with some embellishments):

- All physical systems are represented by ket vectors $|\psi\rangle$ normalized to 1, i.e., $\langle\psi | \psi\rangle = 1$. The ket labels represent **everything that we know about the system**.

- Measurable properties of physical systems are represented by linear operators called **observables**.

So restating part of the first postulate, the ket labels represent **the values of all observables of the system that have been measured - we PREPARED the state!!!**

If a vector associated with a particular state $|\psi\rangle$ is an eigenvector, with eigenvalue α, of operator $\hat{A}$ associated with a particular measurable property of the system, i.e., if $\hat{A}\,|\psi\rangle = \alpha\,|\psi\rangle$, then the system in that state **definitely** has the value α of that measurable property.

This implies that if one performs a measurement corresponding to the observable represented by A on a system in the state $|\psi\rangle$, then with **certainty** (probability = 1) the measurement yields the value α for that measurable property.

Observables are represented by **Hermitian** operators (they have real eigenvalues).

Since the eigenvectors of any Hermitian operator form a complete, orthonormal set, they can be used as a **basis** for the Hilbert space of the system

Finally, if the system is in the state $|\psi\rangle$ and one measures an observable $\hat{B}$, where $|\psi\rangle$ is **not** an eigenvector of $\hat{B}$, then the **only possible results** of the measurement are one of the eigenvalues $\{b_k\}$ of $\hat{B}$.

- Dynamics of state vectors

The state vectors of any system change with time via **deterministic** laws (similar to classical rules).

We define the **time evolution or time development operator** that governs how a state vector changes in time by the relationship

$$|A, t + \Delta t\rangle = \hat{U}(\Delta t)\,|A, t\rangle$$

or the state vector at time t + Δt is given by the time evolution operator (Unitary) $\hat{U}$ operating on the state vector at time t.

In general, the ket labels(which contain whatever we know (have measured) about the state) are the only thing that changes.

The time evolution operator is a unitary operator since the state vector must remain normalized to 1, i.e., the vector length cannot change, and this is guaranteed by the use of a unitary operator.

The only changes to state vectors in quantum mechanics are changes in direction (phase).

The time evolution operator is related to the energy operator (this follows from time translation invariance)

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

- Connection with Experiment/Measurements

We have specified above what happens when one measures a certain property of a physical system at the moment when the state vector of the system is an eigenvector of the operator representing the property - probability = 1 that we get the corresponding eigenvalue.

What if one measures a certain property of a physical system at a moment when the state vector of the system does not happen to be an eigenvector of the operator representing that property (which is most of the time) - what are the probabilities?

We needed a new assumption.

Suppose the system is in the state $|\psi\rangle$, and one carries out a measurement of a property (observable) associated with the operator $\hat{B}$.

We assume the eigenvectors of $\hat{B}$ are the vectors (states) $|b_i\rangle$, which means that $\hat{B}\,|b_i\rangle = b_i\,|b_i\rangle$, i=1,2,3,…. where the b_i are the corresponding eigenvalues.

Quantum theory now assumes that the outcome of measurement is strictly a matter of probability.

Quantum theory stipulates that the probability that the outcome of a measurement of $\hat{B}$ on the state $|\psi\rangle$ (not an eigenvector) will yield the result b_i (remember the only possible results of measurement are the eigenvalues of $\hat{B}$ no matter what state the system is in), is equal to $|\langle b_i | \psi\rangle|^2$ (the **Born rule**).

The probability is given by the absolute square of the corresponding component.

remember that —— component = amplitude

The quantum mechanics formalism based on these postulates + embellishments correctly predicts experimental results for all known experiments (over 110 years).

Some ideas implied by these rules are:

These rules imply that one **cannot** say anything definite about the value of the observable represented by $\hat{B}$ when system is in a state $|\psi\rangle$, which is NOT an eigenvector of $\hat{B}$.

One can only make probability statements.

Before one measures the observable represented by $\hat{B}$ when the system is in a state $|\psi\rangle$, which is NOT an eigenvector of $\hat{B}$, the system **does not** have a value of that observable, according to quantum theory!

Our information about any state is only a set of probabilities.

But all of your experience says that objects have values for measured quantities before they are measured, i.e., your experience tells you that the observable represented by $\hat{B}$ has a value even if we do not measure it.

That is your view (the standard classical view) about what is real and what is not real.

Quantum theory implies you are wrong in both cases!!

Where is the "collapse" postulate?

Since the system has a definite value of the observable represented by $\hat{B}$ after the measurement, i.e., a pointer points to a value or a counter clicks or a mark is registered on a piece of paper (note that these are all **irreversible occurrences** which must be the end result of any measurement) and there is no mechanism to produce a single value in the rules as presented so far, how does it happen?

Most presentations add another rule(as we did earlier) at this point called **collapse of the state vector.**

It is usually proposed that the effect of a measurement is to irreversibly change(collapse) the state vector (which was not an eigenvector of $\hat{B}$) into an eigenvector of $\hat{B}$ (corresponding to the eigenvalue just measured) so that it would be observed to have a definite (probability = 1) value for a subsequent measurement of the operator $\hat{B}$.

This extra rule says that state vector changes(discontinuously) during measurement from representing range of possibilities (superposition of all possible states) to definite state or only one possible outcome.

Which particular eigenvector it gets changed into is determined by outcome of measurement and cannot be known until then!

It cannot be predicted! It is at this point that randomness enters quantum mechanics.

I believe that this last rule is not needed and should not be added.

First of all, no real mechanism is ever given for “how” this process actually takes place, second no specifications are given as to exactly “when” it occurs and third, the process has NEVER been actually observed in the laboratory.

Also, we don’t actually need the collapse mechanism to make any predictions that have been confirmed by experiments.

That suggests to me that it does not exist! That it is NOT a real process!!

I will now proceed to develop a proposal for “definite outcomes“ **without** using any “collapse” rule.

The path I will follow will not be direct, but it will represent the actual tortuous path I followed to get to the conclusions.

I will meander about, repeating myself often on purpose so I can make subtle changes to the previous ideas as I learned new wrinkles along the way.

I will initially end up with one conclusion, which we will see has flaws and then I will fix the flaws and get what I now think is the correct answer at the end.

So, have fun following my story!

The Measurement Process(some repetition of earlier discussions)

We consider a system consisting of a quantum system (Q-system) and a measurement system (M-system).

If the meter, which we assume is initially OFF (state $|0\rangle_M$) was turned ON when quantum system was in $|+\rangle_Q$ state, then according to the above rules the combined system evolves to

$$|+\rangle_Q\,|0\rangle_M \to |+\rangle_Q\,|+1\rangle_M$$ i.e., meter (assuming a good meter) reads +1.

Similarly, if the meter turned ON when the system is in the $|-\rangle_Q$ state, then combined system evolves to

$$|-\rangle_Q |0\rangle_M \rightarrow |-\rangle_Q |-1\rangle_M$$ i.e., meter (assuming a good meter) reads –1.

This indicates that measurement, within framework of our rules, CORRELATES or ENTANGLES the dynamical variables (Q-system) being measured and the macroscopic (M-system) indicator of the meter, which we assume can be directly (macroscopically) observed (and is **irreversible**).

Let us expand(and repeat) all aspects of this discussion.

We have supposed above that the meter has eigenvectors (labelled by the corresponding eigenvalues), i.e., it is a quantum system also.

$$\begin{aligned} &|+\rangle_M \Rightarrow meter \text{ on}: \ reading \ +1 \\ &|-\rangle_M \Rightarrow meter \text{ on}: \ reading \ -1 \\ &|0\rangle_M \Rightarrow meter \text{ off} \end{aligned}$$

and the quantum system has eigenvectors (labelled by eigenvalues)

$$\begin{aligned} &|+\rangle_Q \Rightarrow value = +1 \\ &|-\rangle_Q \Rightarrow value = -1 \end{aligned}$$

Now suppose that the initial state of the quantum system is a **superposition**

$$|\psi\rangle = a|+\rangle_Q + b|-\rangle_Q$$

and thus the initial state of the combined system is given by

$$|initial\rangle = \left(a|+\rangle_Q + b|-\rangle_Q\right)|0\rangle_M$$

which represents the system in a superposition state and the meter off.

We are interested in the evolution of this state according to QM.

We note as stated above, if, instead of the above initial state, we started with initial states

$$|A\rangle = |+\rangle_Q |0\rangle_M \qquad \textbf{OR} \qquad |B\rangle = |-\rangle_Q |0\rangle_M$$

and then turn on the meter, these states must evolve as

$$|A\rangle = |+\rangle_Q |0\rangle_M \longrightarrow |A'\rangle = |+\rangle_Q |+\rangle_M$$

$$|B\rangle = |-\rangle_Q |0\rangle_M \longrightarrow |B'\rangle = |-\rangle_Q |-\rangle_M$$

respectively, indicating that the meter measured the appropriate value (definition of good meter) since system is in eigenstate and has a definite value with certainty.

If system is in initial state corresponding to a superposition, however, then the assumed **linearity** of quantum mechanics says must it evolve into

$$|initial\rangle = \left(a\,|+\rangle_Q + b\,|-\rangle_Q\right)|0\rangle_M \longrightarrow |final\rangle = a\,|+\rangle_Q\,|+1\rangle_M + b\,|-\rangle_Q\,|-1\rangle_M$$

We note a problem immediately,

i.e., the meter has not ended up in a state with a definite value

- it remains in a superposition of two macroscopically different pointer readings, which is **never** observed in the real world.

Hence, if as most physicists assume,

the state vector represents a complete description of the Q-system,

there seems to be a need for the **"collapse"** rule

to fix the result and obtain **"definite"** values!

Since we **will not** be incorporating the “collapse” rule, we must proceed in a **different** way.

We **cannot use** the state vector as the fundamental object in quantum theory
since it **seems** that it must collapse (or maybe not!) for quantum mechanics to work
and we are **not** including “collapse”.

Introduce new mathematical object called the Density Operator to remedy this situation:

Now we present an **alternate** way of representing quantum states.

Reasons why come later!

It is still an open question whether this alternative is a **simple** mathematical convenience or a more ontologically **true** representation.

Either way, it has a key role to play in modern interpretations of quantum theory and also leads to a possible solution to measurement problem.

This discussion is more mathematical than rest of these notes, but you will benefit if you persevere and work(follow) through material.

It is very important.

The problem, as we will now see, lies with assuming that the state vector is the proper way to represent the Q-system **during** the measurement process.

Now, the expectation value, which is the fundamental measurable quantity, is the **average** of set of measurement results taken from a collection of systems in **same** state.

A straightforward calculation of the expectation value in a specific state takes the following form, with $\hat{O}$ being an operator representing the measurement of the specific physical variable and $|\phi\rangle$ is a state vector of some system in the collection:

$$\left\langle \hat{O} \right\rangle = \langle\phi|\, \hat{O}\, |\phi\rangle$$

If we choose **any set** of basis vectors $\{|i\rangle\}$, i = 1, 2, ... for our vector space, we can expand $|\phi\rangle$ and $\langle\phi|$ as

$$|\phi\rangle = \sum_i a_i\, |i\rangle \quad , \quad \langle\phi| = \sum_j a_j^*\, \langle j|$$

where

$$a_i = \langle i\,|\,\phi\rangle \quad , \quad a_j^* = \langle\phi\,|\,j\rangle$$

Plugging these expansions into the expression for the expectation value:

substitution

rearrangement

$$\langle \hat{O} \rangle = \sum_j a_j^* \langle j | \hat{O} \sum_i a_i | i \rangle = \sum_j \left(\sum_i a_j^* a_i \langle j | \hat{O} | i \rangle \right)$$

$$= \sum_j \left(\sum_i \langle \phi | j \rangle \langle i | \phi \rangle \langle j | \hat{O} | i \rangle \right) = \sum_j \left(\sum_i \left[\langle i | \phi \rangle \langle \phi | j \rangle \langle j | \hat{O} | i \rangle \right] \right)$$

more rearrangement

more rearrangement

Now we define a new operator $\hat{\rho} = |\phi\rangle\langle\phi|$, which, at this point, is just the **projection operator** onto the state $|\phi\rangle$.

We can then write the expectation value as

$$\langle \hat{O} \rangle = \sum_i \left(\sum_j \langle i | \hat{\rho} | j \rangle \langle j | \hat{O} | i \rangle \right)$$

substitution of new definition

Recognizing the relation $\sum_j |j\rangle\langle j| = \hat{I}$ appearing in the expression

one finds that the expectation value can be rewritten in a very interesting form

insert an identity operator

$$\langle \hat{O} \rangle = \sum_i \left(\sum_j \langle i| \hat{\rho} |j\rangle \langle j| \hat{O} |i\rangle \right) = \sum_i \left(\langle i| \hat{\rho} \left(\sum_j |j\rangle \langle j| \right) \hat{O} |i\rangle \right)$$

remove identity operator

$$\langle \hat{O} \rangle = \sum_i \langle i| \hat{\rho}\hat{I}\hat{O} |i\rangle = \sum_i \langle i| \hat{\rho}\hat{O} |i\rangle = \mathrm{Tr}(\hat{\rho}\hat{O})$$

<— the important result!!

sum over diagonal matrix elements

rearrangement

i.e., the expectation value is given by the sum over the **diagonal** matrix elements of the operator product $\hat{\rho}\hat{O}$ (the symbol Tr = Trace is just shorthand for the diagonal sum).

The new operator $\hat{\rho}$ is called the **density operator**.

Why bother to introduce this new operator?

As we will see, the **real power** of density operator approach to QM comes when we have to deal with a situation in which we cannot be sure what state the system is in (just as is the case in the measurement problem!!).

Now, imagine we have a whole collection of identical systems, some in $|\phi_1\rangle$, some in $|\phi_2\rangle$, etc.

We might not know which system in which state, and might not even know how many systems are in any one state.

Example: Think about a beam of electrons that **has not** passed through any Stern-Gerlach (S-G) magnet.

Chances are that the spin states of the electrons are completely random.

Perhaps the best one can know is the probability of finding an electron in any state.

$$P_1 = \mathrm{Prob}(|\phi_1\rangle) \quad , \quad P_2 = \mathrm{Prob}(|\phi_2\rangle) \quad , \quad P_3 = \mathrm{Prob}(|\phi_3\rangle) \quad , \quad ....$$

These probabilities have **nothing to do** with quantum theory.

They simply represent our **ignorance** of the details of what is happening in the laboratory-created beam.

Thus, they are **not related** to any quantum amplitudes.

Given a situation like this, one should still be able to do some useful calculations.

For example, one could work out the expectation value of any measurement as follows.

If one can calculate the expectation value in each individual state, then the overall expectation value is simply given by

$$\left\langle \hat{O} \right\rangle = P_1 \langle \phi_1 | \hat{O} | \phi_1 \rangle + P_2 \langle \phi_2 | \hat{O} | \phi_2 \rangle + P_3 \langle \phi_3 | \hat{O} | \phi_3 \rangle + ... + P_n \langle \phi_n | \hat{O} | \phi_n \rangle$$

that is just the standard definition of the average value!!

Think back to original definition of expectation value which just represents the average value of measurement and this will be clear.

What we have done here is put together the weighted average of the average value for each state, **which, as we said, is just the standard definition of the overall average value**.

Now, in this case, if one constructs a density operator that is given by the expression

$$\hat{\rho} = P_1 |\phi_1\rangle \langle\phi_1| + P_2 |\phi_2\rangle \langle\phi_2| + P_3 |\phi_3\rangle \langle\phi_3| + \cdots + P_n |\phi_n\rangle \langle\phi_n|$$

sum of probability x projection operators for all possible states(not just 1 state as earlier)

then the expectation value, in this case, is **still** given by

$$\langle\hat{O}\rangle = \mathrm{Tr}(\hat{\rho}\hat{O})$$

Proof(for mathematically inclined):

Now
$$\langle\phi_1|\,\hat{\rho}\,|\phi_1\rangle = \langle\phi_1|\,(P_1\,|\phi_1\rangle\,\langle\phi_1| + P_2\,|\phi_2\rangle\,\langle\phi_2| + \cdots)\,|\phi_1\rangle$$

or
$$\langle\phi_1|\,\hat{\rho}\,|\phi_1\rangle = P_1\,\langle\phi_1\,|\,\phi_1\rangle\,\langle\phi_1\,|\,\phi_1\rangle + P_2\,\langle\phi_1\,|\,\phi_2\rangle\,\langle\phi_2\,|\,\phi_1\rangle + \cdots$$

but
$$\langle\phi_i\,|\,\phi_j\rangle = \delta_{ij}$$

so
$$\langle\phi_1|\,\hat{\rho}\,|\phi_1\rangle = P_1$$

similarly
$$\langle\phi_i|\,\hat{\rho}\,|\phi_j\rangle = P_i\delta_{ij}$$

then
$$\mathrm{Tr}(\hat{\rho}\hat{O}) = \sum_i \langle\phi_i|\,(\hat{\rho}\hat{O})\,|\phi_i\rangle = \sum_i \langle\phi_i|\,(\hat{\rho}\hat{I}\hat{O})\,|\phi_i\rangle = \sum_i \langle\phi_i|\,(\hat{\rho}\sum_j(|\phi_j\rangle\,\langle\phi_j|)\hat{O})\,|\phi_i\rangle$$

or
$$\mathrm{Tr}(\hat{\rho}\hat{O}) = \sum_j\sum_i \langle\phi_i|\,\hat{\rho}\,|\phi_j\rangle\,\langle\phi_j|\,\hat{O}\,|\phi_i\rangle$$

Thus
$$\mathrm{Tr}(\hat{\rho}\hat{O}) = \sum_j\sum_i P_i\delta_{ij}\,\langle\phi_j|\,\hat{O}\,|\phi_i\rangle$$

or
$$\mathrm{Tr}(\hat{\rho}\hat{O}) = \sum_i P_i\,\langle\phi_i|\,\hat{O}\,|\phi_i\rangle = \langle\hat{O}\rangle$$

proof complete!

Some notation: when the density operator takes the form $\hat{\rho} = |\phi\rangle\langle\phi|$ only one term!

it is said to represent a **pure state** and when the density operator takes the form

$$P_1 |\phi_1\rangle\langle\phi_1| + P_2 |\phi_2\rangle\langle\phi_2| + P_3 |\phi_3\rangle\langle\phi_3| + \cdots + P_n |\phi_n\rangle$$

like the beam coming out of source in S-G experiment!!

it is said to represent a **mixed state** (more about mixed states later).

A crucial example of the need for the density operator.

Follow these arguments very carefully.

Consider a box containing a very large number of electrons, each having spin = 1/2.

This means that each electron spin can have a measurable component ±1/2 along **any** chosen measurement direction.

Now, suppose the box has a hole so that the electrons can get out and then go into a Stern-Gerlach device oriented to measure the z-components of spin(**arbitrary choice of direction**).

In order to proceed, we need to know how the box of electrons was **prepared**, i.e., what states the electrons are in or what the original KET labels are…….

Let us consider **two very different cases**:

In the **first** case, we fill the box with electrons that have been prepared in a superposition state

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle) = |\uparrow_x\rangle$$

This preparation can be done by sending electrons through an x-oriented magnet and choosing one of the resulting beams (UP in x in this case).

Thus, in this case, **each** electron is in the the indicated superposition state of z-directions —-> **each** electron is in a superposition of “up” and “down” in the z-direction while **simultaneously** being in a definite state of spin in the x-direction (up).

Hence, the x-value is known and z-value is 50-50 - it was created that way!

We then fill one box with these electrons.

In the **second case**, we send electrons through a z-oriented magnet and collect electrons from both beams "z-up" beam and the "z-down" beam separately.

In this case we **know** that the electrons are either "z-up" OR "z-down", i.e., they each **have** a definite value - they are **not** in a superposition.

We then fill another box with the two collections of electrons of definite z-spin electrons.
Box is now (50-50) up/down in z-direction, **BUT each electron now has a definite value!**.

Remember, the electrons in the box in this case are EITHER in the state $|\uparrow_z\rangle$ OR in the state $|\downarrow_z\rangle$

Now we proceed with the experiments (SGz).

In case (1) we observe "z-up" 50% of the time and "z-down" 50% of the time

and we know that in order to describe this system by a state vector we must say

$$|\psi\rangle_{BOX(1)} = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle) \rightarrow \text{50-50 up-down}$$

i.e., every electron in the box is in this superposition state. which is how we created them

In case (2)) we also observe “z-up” 50% of the time and “z-down” 50% of the time.

But we now have a problem because we are using state vectors as the fundamental object in the theory!

If I did not already know that each electron in the box in this case had a definite value, I would be tempted to describe this system by the **same** state vector as in case (1).

However, we know that is not true!

The electrons in case (2) are not each in a superposition — they all have **definite values**!

So, if I only measure z-components I cannot tell whether I have case (1) or case (2) and I **do not know how to write the state vector** for the box in case (2),

But remember how I created the electrons in case (1).

They all have a **definite** value of the x-component, namely, “x-up’.

So if I subject the electrons coming out of the box in case (1)

to an x-measurement instead of a z-measurement,

I will end up with only **one beam**!

However, in case (2), the electrons coming out of the box are either “z-up” OR “z-down”

each of which is 50-50 in the x-direction and thus I would end up with two beams after the extra x-measurement!

I get different results for 2nd measurement because they are different states!!

The different results mean that their states must be described differently in QM.

State vectors do not give us the freedom to do this unless we want to monkey around with relative phases between components, i.e., mathematically, we would need to write

$$|\psi\rangle_{BOX(1)} = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle) \quad , \quad |\psi\rangle_{BOX(2)} = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + e^{i\alpha}|\downarrow_z\rangle)$$

where α is a **completely unknown relative phase factor**, which must be averaged over during any calculations since it is different (and random) for each separate measurement (each member of ensemble).

Thus, we would be choosing to allow the two vectors to differ in the relative phase of their components and we are forced to say this is meaningful!

That did not sound to me like the best way to proceed. Let me continue with my journey…

If we use density matrices, we have a very **different story**.

For the pure state a density operator (or matrix) is defined as

$$\hat{\rho} = |\psi\rangle \langle\psi|$$

for some state vector $|\psi\rangle$, i.e., it is the pure state projection operator.

In case (1) this gives

$$\hat{\rho} = |\psi\rangle \langle\psi| = \frac{1}{2}(|1/2\rangle + |-1/2\rangle)(\langle 1/2| + \langle -1/2|)$$

or

$$\hat{\rho} = \frac{1}{2}(|1/2\rangle \langle 1/2| + |-1/2\rangle \langle 1/2| + |1/2\rangle \langle -1/2| + |-1/2\rangle \langle -1/2|)$$

—> 4 terms

projection operator cross-term cross-term projection operator

Derivation of the $\hat{\rho}$ matrix in the (+1/2, −1/2) basis: from our earlier definitions we have

$$\rho = \begin{pmatrix} \langle 1/2|\hat{\rho}|1/2\rangle & \langle 1/2|\hat{\rho}|-1/2\rangle \\ \langle -1/2|\hat{\rho}|1/2\rangle & \langle -1/2|\hat{\rho}|-1/2\rangle \end{pmatrix}$$

where standard calculation says

$$\begin{aligned} &\langle 1/2|\hat{\rho}|1/2\rangle \\ &\quad = \langle 1/2|\frac{1}{2}(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle 1/2| + |1/2\rangle\langle -1/2| + |-1/2\rangle\langle -1/2|)|1/2\rangle \\ &\quad = \frac{1}{2} \end{aligned}$$

and similarly for the other matrix elements and thus we get

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

where the diagonal matrix elements represent probabilities(50-50) and the non-zero off-diagonal matrix elements indicate that one **will observe** quantum interference effects in the system when it is in this state.

It is clear(look at above expression) that any pure state density operator **cannot be written as just the sum** of pure state projection operators (multiplying always generates cross-terms).

In case (2), however, each electron is in a definite state (has a definite value) so that each state is separately represented by a single projection operator

—> only have the sum of two definite terms

$$\hat{\rho} = \frac{1}{2}\left(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle -1/2|\right)$$

we call this form the "ignorance" density operator; it is just a multiple of the Identity operator.

and

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

same algebra

which clearly **is** the sum of pure state projection operators.

This corresponds to what is called a **mixed state**.

Note that the off-diagonals are zero so that this density operator **cannot** lead to any quantum interference effects. The electrons **have** values like classical particles!!

Remember for later, this system(case (2)) is such that electrons have values so that the density operator takes this form (sum of projection operators for each value —> NO cross-terms)!!

Repeating: Note that when electrons DO NOT HAVE VALUES (case (1)) the density operator has interference terms(mixed terms) and cannot be written as a sum of projection operators

A DIFFERENCE that does NOT show up(without extra phases) when using state vectors!!

We note that if we treat case (2) as pure state with the extra relative phase factor we would obtain:

$$\hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & e^{i\alpha} \\ e^{-i\alpha} & 1 \end{pmatrix} \quad \text{which becomes} \quad \hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

when we average over α .

For comparison, let us now digress to see what happens in a **real classical system**.

Consider rolling a standard die which has possible values = 1,2,3,4,5,6 where the probability of occurrence of each value = 1/6.

In this case, the density operator representing the die will be for a mixed state (no interference effects) and the die **has** a value before/after each roll so that we have

$$\rho = \frac{1}{6}(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4| + |5\rangle\langle 5| + |6\rangle\langle 6|)$$

a mixed state

and the expectation value of the $\widehat{\mathrm{ROLL}}$ operator must be

$$\langle \widehat{ROLL} \rangle = \frac{1}{6}(1+2+3+4+5+6) = 3.5$$

standard definition

just sum of values times probabilities!!

Let me show you the mathematical details for completeness.

More formally, we know that any operator can be written as sum of eigenvalues × projections operators, i.e., for the $\hat{B}$ operator introduced earlier we found we can write

$$\hat{B} = \sum_i b_i |b_i\rangle\langle b_i|$$

so that we have for the $\widehat{\mathrm{ROLL}}$ operator

$$\widehat{ROLL} = 1\,|1\rangle\langle 1| + 2\,|2\rangle\langle 2| + 3\,|3\rangle\langle 3| + 4\,|4\rangle\langle 4| + 5\,|5\rangle\langle 5| + 6\,|6\rangle\langle 6|$$

The operator product of the density operator and the $\widehat{\mathrm{ROLL}}$ operator can be written(using the orthonormality of the basis state vectors) as

$$\hat{\rho}\,\widehat{ROLL} = \frac{1}{6}\left(1\,|1\rangle\langle 1| + 2\,|2\rangle\langle 2| + 3\,|3\rangle\langle 3| + 4\,|4\rangle\langle 4| + 5\,|5\rangle\langle 5| + 6\,|6\rangle\langle 6|\right)$$

Thus, the expectation or average value is

$$\langle\widehat{ROLL}\rangle = \mathrm{Tr}(\hat{\rho}\widehat{ROLL}) = \sum_k \langle k|\,\hat{\rho}\widehat{ROLL}\,|k\rangle = \frac{1}{6}(1+2+3+4+5+6) = 3.5$$

as we obtained earlier. I can do all detailed algebra if anyone is interested after class session.

But, in this case, **we know the values are real before the measurement**,

i.e., we are using a macroscopic die with numbers we can **see**!

the important feature

Thus, it **seems** that this **particular** form of a density operator **represents that case**.

The same was true earlier for the electrons in the box when we knew they were either “up” or “down”!

We also had a density operator of the same form!

In both cases the system **had known values before measurements**!

REMEMBER this fact for later.

If we were to add the “collapse” rule it raises a host of questions:

What exactly do we mean, physically and mathematically,

by a “collapse during measurement” of quantum system?

Does collapse occur all at one instant?

What if the state occupies a **finite** volume?

Wouldn’t instantaneous collapse(the **entire** volume changes at once) **contradict** special relativity?

If, instead, collapse occurs during a time interval, then what equation describes its time-evolution during that interval?

Quantum states are presumed to follow the Schrödinger equation,

which prescribes continuous time evolution.

How can instantaneous state collapse be reconciled with smooth evolution?

How can we resolve “problem of outcomes” that appears to arise

when a superposed quantum’s state is measured by “which-state” detector,

creating a so-called entangled state of the quantum and the detector

that appears to be an indefinite superposition

of two macroscopically distinct states of a composite system?

Such questions and more comprise the **original quantum measurement problem**.

There are many alternative interpretations of the quantum physics mathematical formalism,
and several alternative modifications of theory,
have been proposed to resolve problem, with no consensus on a solution.

It is remarkable that, despite the unparalleled
experimental success of quantum theory across a vast range of experiments,
most of the suggested solutions **differ** from standard quantum physics
in one or more significant aspects.

Most involve **new** interpretations of the standard mathematical formalism.

Interpretations such as "human minds collapse the quantum state"
or "all possible collapses occur but only one of them occurs in our particular universe"
or even rejection of physical reality of quantum world
and assumption that quantum probabilities
(and hence changes in those probabilities,
such as quantum state collapse) are mere measures of personal degrees of belief.

yadda, yadda, yadda………sorry if I sound disrespectful!

Other suggestions assume modifications of the standard mathematical formalism,

such as an additional mechanism

that causes quantum states to spontaneously collapse from time to time,

or new "hidden" and hence uncontrollable variables

that create the illusion of quantum randomness. **more yadda, yadda …..**

With all these thoughts/ideas in hand,

we will now continue this discussion of the measurement problem

and finally suggest a resolution of problem of definite outcomes

that lies **entirely** within the framework of standard quantum physics.

As I said, we will follow my tortuous path.

First, we will spend time with one possible solution to the measurement problem.

We will arrive at a point where it “looks” like we found a solution.

But it will not be so!

The reasons will be subtle but clear.

We will then see how to correctly interpret what is happening and find in the end that von Neumann was correct in 1932 even though for 90 years papers were published deriding his ideas!

Let us begin with question of definitions.

What do we mean, physically, by **quantum measurement**?

First, we need a non-anthropomorphic definition of the concept known as "measurement,"

i.e., there were measurements before humans,

so let's broaden its physical definition as follows:

A "quantum measurement" means

any quantum process that results in a macroscopic effect,

regardless of whether humans or laboratories are involved.

Thus not only is an electron striking a laboratory viewing screen
and creating a visible flash a measurement, but also a cosmic-ray muon striking and macroscopically moving a sand grain on a planet in some other galaxy is a measurement.

To analyze a measurement,
we look at a specific experiment: suppose an electron beam passes through a pair of double slits and then impacts a viewing screen.

Just as in Thomas Young's similar double-slit experiment using light, performed in 1801,
a pattern is formed on the viewing screen that **seems** to shows interference between the two portions of the electron beam which are **seemingly**(classically) coming simultaneously through the two slits:

a broad dark-and-bright striped pattern spreads out widely on the screen

- much wider than slits
- indicating regions of destructive (dark) and constructive (bright) interference.

or maybe I should say …

regions of increased electron intensity and decreased electron intensity(including 0)

On closer inspection,

the bright lines are formed by a very large number of tiny individual electron impacts,

each one making a small flash on the screen.

According to our definition, each flash is a measurement

of the position of an electron as hits the screen.

I repeat - Each electron's flash on screen is a measurement!

For the purposes of this analysis, however,

it is more informative to consider a related example of measurement,

still based on the double-slit experiment.

Suppose an electron detector is installed at the slits

and assume that the detector can detect the electron's position

as it passes through slits while disturbing each electron only **minimally** .

As it turns out, measurement,

even by such a minimally-disturbing "which-path detector", changes everything.

Exactly when the detector turns on,

the subsequent pattern of impacts on screen changes from a striped interference pattern

to a smoothly-spread-out sum of two single-slit patterns,

each showing diffraction but no interference, i.e., the probability function governing

the pattern of new flashes changes after that instant!

The electron impacts no longer look like the old interference pattern —the interference pattern abruptly vanishes, i.e., the probability has changed.

An analogous experiment has been done using light (photons) instead of electrons, and using an interferometer rather than double-slit interference setup as we discussed earlier.

A which-path detector was randomly switched on or off as each photon passed through this experiment;

Photons for which the detector was "off" formed an interference pattern while photons for which the detector was "on" formed the expected no-interference pattern.

Reminder:

If the system is in a “pure” superposition state, then the density operator takes the form

$$\hat{\rho} = \frac{1}{2}\left(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle 1/2| + |1/2\rangle\langle -1/2| + |-1/2\rangle\langle -1/2|\right)$$

i.e., sum of projection operators and “cross-terms” (interference terms) or the matrix form

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

where the diagonal matrix elements represent probabilities

and the off-diagonal matrix elements imply

that one will observe quantum interference effects in this system.

Clearly, any pure state density operator cannot be written

as the sum of pure state projection operators (because of the cross-terms).

If in a “mixed state”, then the density operator takes the form

$$\hat{\rho} = \frac{1}{2}\left(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle -1/2|\right)$$

or the matrix form

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

i.e., sum of projection operators (no interference terms).

As we found in the dice example:

$$\hat{\rho} = \frac{1}{6}\left(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4| + |5\rangle\langle 5| + |6\rangle\langle 6|\right) \quad \text{mixed state}$$

We now state our first attempt at describing what happens during a measurement.

When we observe a quantum system evolve into such a “mixed state” density operator,

then the quantum system can be interpreted “classically”,

i.e., a measurement has taken place

and it has been **irreversibly recorded somewhere**.

That is the important point!!

A full “collapse” is not necessary!!

We only need to arrive at (evolve to) a particular form of the density operator!!

Before continuing let me quote Richard Feynman about what has happened.

"Somewhere in the measuring apparatus changes have occurred and the concept of a quantum amplitude is not valid anymore, i.e., if we are throwing our quantum die, then the quantum amplitude only remains a valid idea until the die comes to rest on the floor.

comments in "red" added by me

And that point you can't do an experiment which distinguishes interfering alternatives from just plain odds (like with classical dice).

—> we are at the "ignorance" density operator stage!

There is no point where you can say a "reduction" has taken place, i.e., there is just a **murky unseen phasing out** of the quantum amplitudes.

Possibly what is happening is that in the macroscopic measuring device, quantum phase information gets so "smeared out" that it is no longer definite and thus the system behaves classically.

It registered an irreversible result!

Since we are not quantum objects we may never be able to see the details of this process, but it clearly happens."

IMPORTANT!!

Feynman hits the nail on the head!

Now let us continue our discussion!

Possible ways to think about getting to the irreversible recording point must be found.

Two digressions —> possible ways to get to the irreversible recording —

(1) The so-called Gambler's Ruin problem (from the mathematicians)

We start by investigating the so-called **Mathematical Problem of the Points** (seems unrelated but it is not! —- that is the funny way mathematics works!)

A sequence of fair coins is flipped.

Player A gets a point for every head and player B gets a point for every tail.

Player A wins if there are a heads before b tails, otherwise B wins.

Find the probability that A wins.

Let α(a,b) be the probability that A wins and

β(a,b) the probability that B wins —> $\alpha(a, b) + \beta(a, b) = 1$ someone wins!

Solution (due to Pascal and Huygens -1500's)

$$\alpha(a,b) = \frac{1}{2^{a+b-1}} \sum_{k=0}^{b-1} \binom{a+b-1}{k} \quad \text{where} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

binomial coefficient

$$k! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot k$$

An example is the **Gambler's Ruin problem**.

Just as in the problem of points, suppose that at some stage A has a counters,

and B has m + n – a counters so that total number of counters = m+n,

and let A's chances of victory at that point be v(a).

The solution when A starts with m counters is given by

$$p_A = v(m) = \frac{1-(\beta/\alpha)^m}{1-(\beta/\alpha)^{m+n}}$$

This works for all cases except $\alpha = \beta = 1/2$ (the 50-50 probability case).

For that special case, the solution is

$$v(a) = \frac{a}{m+n}$$

This implies that there is **always a winner** (hence the the name **Gambler's Ruin** for the loser)

NOTE: Even when $\alpha = \beta = 1/2$ there is still always a winner!!!!

Also note that this relation is **linear** in **a** in this special case!

Pearle's Theorem

What does this have to do with quantum collapse?

Here is a short note from Philip Pearle (a really good physicist and friend): I quote from the note:

I soon found a charming **analogy** for collapse dynamics,

useful for providing an intuitive and non-technical **explanation of how it works**.

I happened to be browsing in Feller's book on probability

(a favorite textbook, from an undergraduate course taught by Stanislaus Ulam)

when I encountered the **gambler's ruin game**.

Two gamblers, initially possessing, respectively, a **fraction** $x_1(0)$, $x_2(0)$ of their combined wealth

(so $x_1(0) + x_2(0) = 1$)

repeatedly toss a fair coin, and the result, heads or tails,

determines which one gives one dollar to the other.

They play until one gambler loses all his money, and the game ends.

The **analogy to QM** arises if the amount of money possessed by one gambler at any time

is proportional to the squared amplitude of one of two states

whose sum is the state vector representing the physical system undergoing collapse.

Just as one gambler loses all his money, **so one of the states loses all its amplitude**,

and as the other gambler wins all the money,

so the state vector ends up as totally described by the other state (—> collapse!!).

collapse has occurred here without any interference by gamblers!

Now continuing our discussion…….

Let Q(x) be the conditional probability that a gambler wins the game,

given that he has the fraction x of the total wealth.

If Δ is the fraction of the total wealth they exchange at each toss (i.e., Δ = \$1/total dollars),

the difference equation

$$Q(x) = \frac{1}{2}Q(x - \Delta) + \frac{1}{2}Q(x + \Delta)$$

expresses that there are two routes to win if one has fractional wealth x,

namely lose the next toss and drop to $x - \Delta$ but win thereafter,

or win the next toss and rise to $x + \Delta$ and win thereafter.

The solution of the difference equation is $Q(x) = Ax + B$, where A and B are constants.

Boundary conditions:

Since $Q(0) = 0$ (because you can't win if you have no money) and $Q(1) = 1$
(because you have won if you have all the money), then $Q(x) = x$, i.e.,

$$Q(1) = 1 = A+B \qquad Q(0) = B = 0 \quad \longrightarrow A = 1, B = 0 \quad \longrightarrow Q(x) = x$$

That is, if one starts with the fraction $x = x(0)$ of the money,
one has the probability $x(0)$ of attaining all the money —> you get to $x = 1$,

which is exactly collapse behavior!!

The game can be modified to have many players,
to have Δ change as the game progresses
(e.g., to get smaller as one gambler gets closer to losing), etc.

So, one may think of quantum collapse

(getting to the point where an irreversible measurement has been made)

as a gambler's ruin competition among the states in a superposition,

to see which final state wins the game (remains at the end).

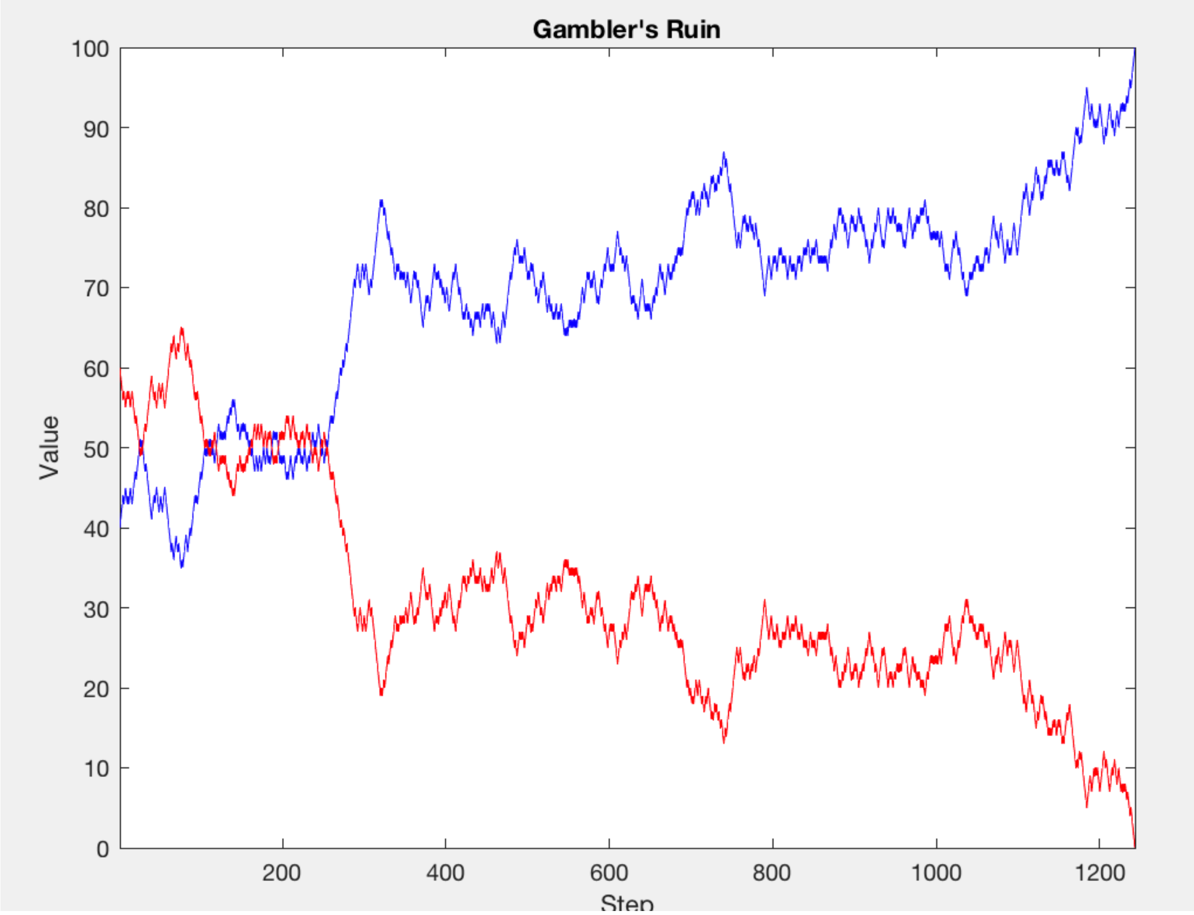
I ran a simulation of the Gambler's Ruin problem using OCTAVE on a computer.

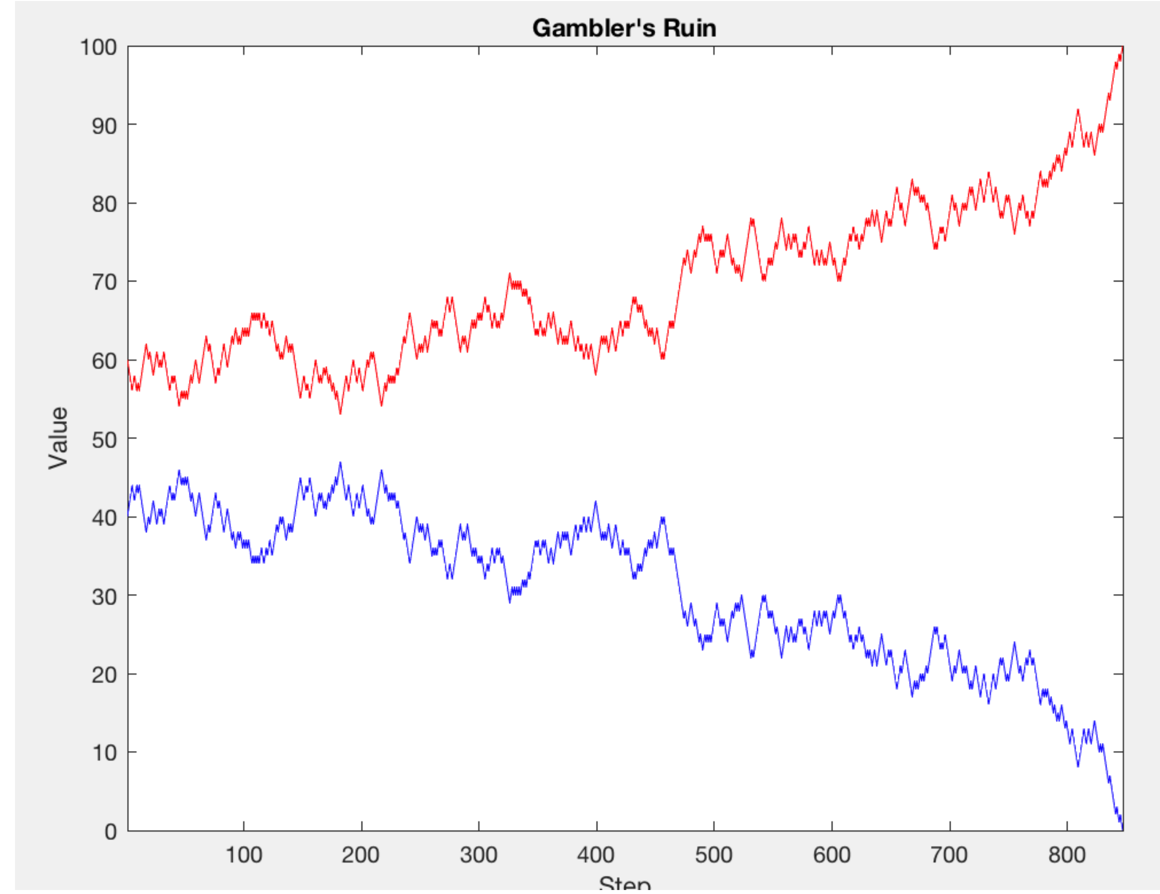
OCTAVE Code

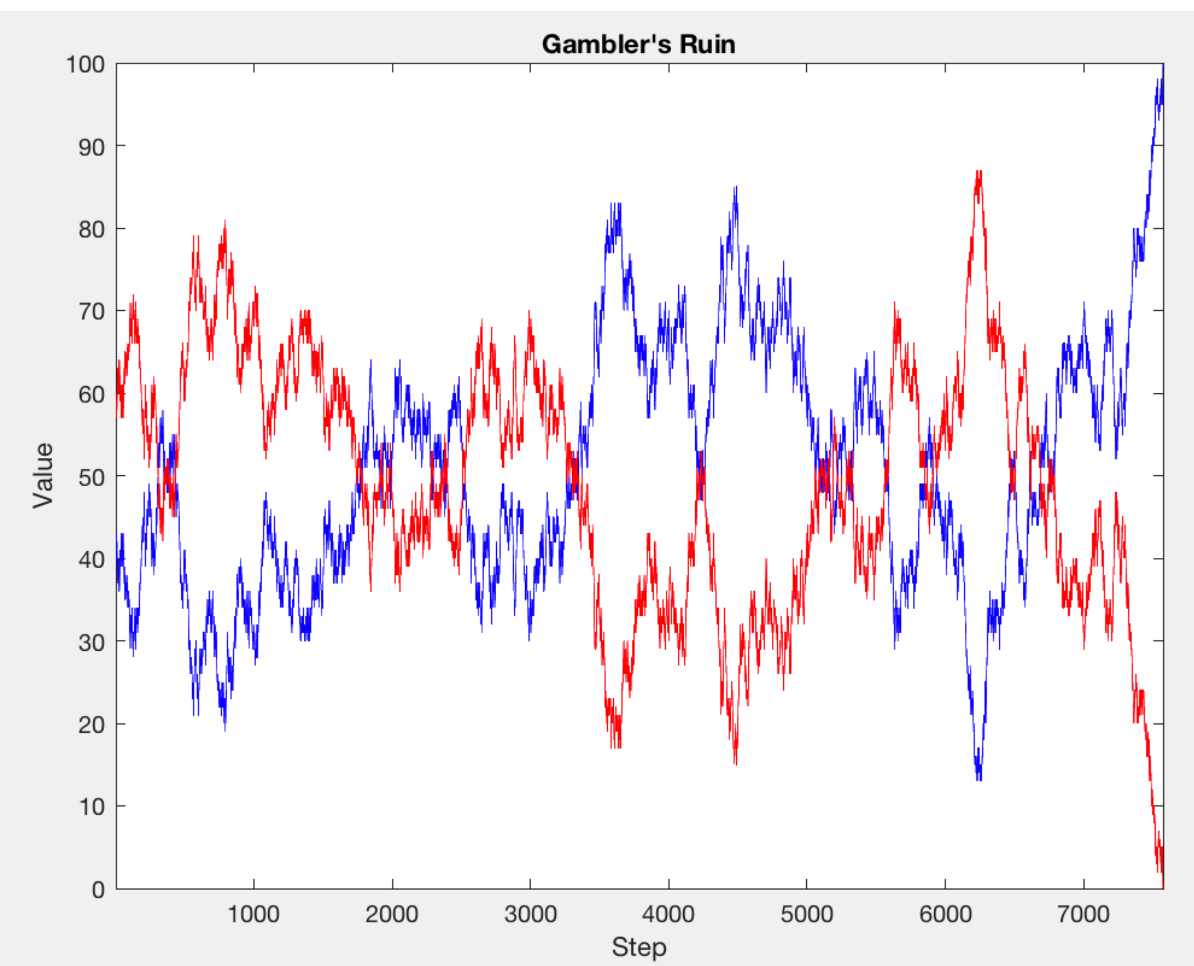
gambruin.m

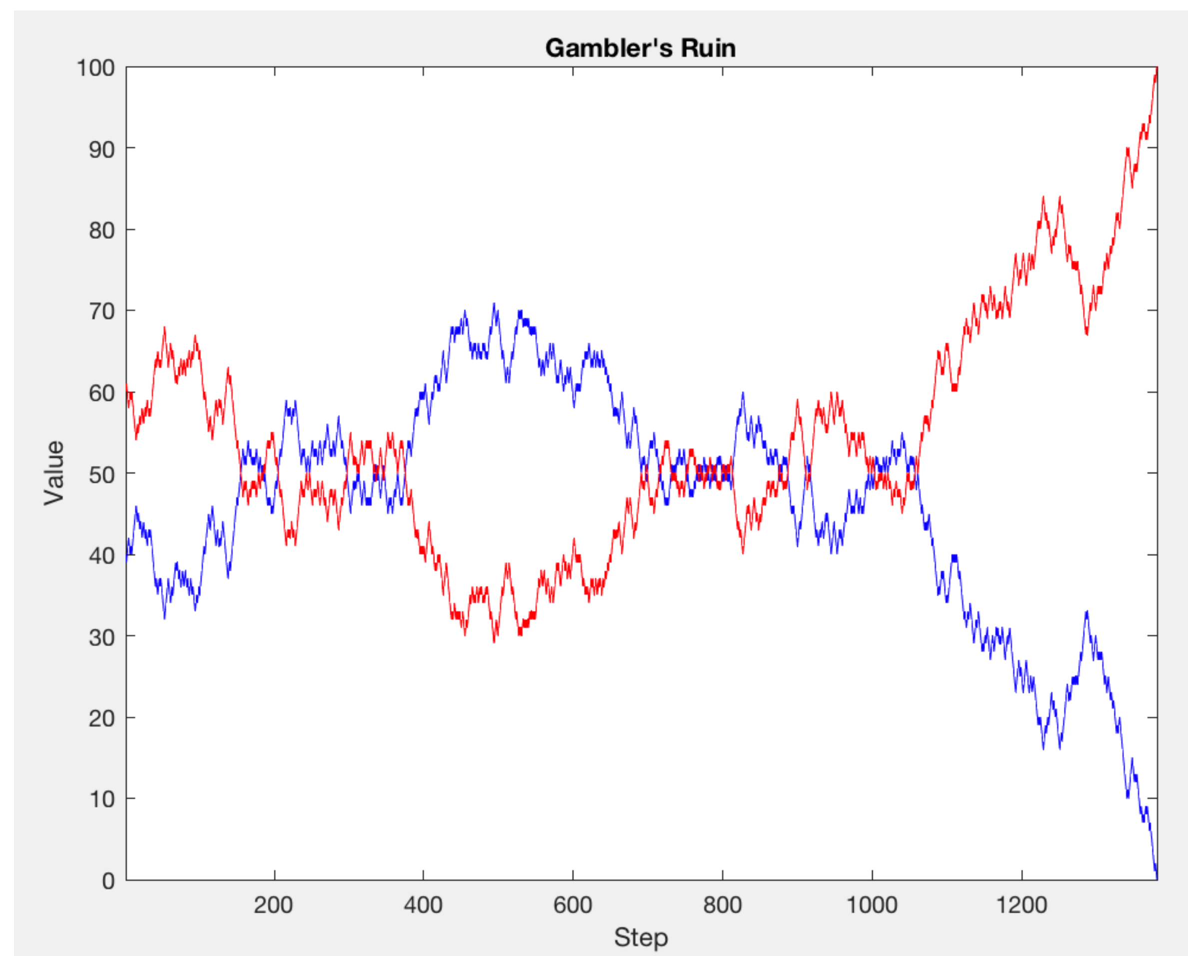
```
x1=[];
y1=[];
x=40;x1=[x1,x];
y=100-x;y1=[y1,y];
count=1;
while (x ~= 0) && (x ~= 100)
r=rand;
x=x+(r <= 0.5)-(r > 0.5);x1=[x1,x];
y=100-x;y1=[y1,y];
count=count+1;
endwhile
figure
plot(1:count,x1,'-b');
axis([1,count,0,100]);
xlabel('Step');
ylabel('Value');
title('Gambler''s Ruin')
hold on;
plot(1:count,y1,'-r');
hold off
count
```

Some sample runs are shown in next slide - **note there is always a winner(loser)**


Gambler's Ruin
Value
Step
0 10 20 30 40 50 60 70 80 90 100
200 400 600 800 1000 1200


Gambler's Ruin
Value
Step
0 10 20 30 40 50 60 70 80 90 100
100 200 300 400 500 600 700 800


Gambler's Ruin
Value
Step
0 10 20 30 40 50 60 70 80 90 100
1000 2000 3000 4000 5000 6000 7000


Gambler's Ruin
Value
Step
0 10 20 30 40 50 60 70 80 90 100
200 400 600 800 1000 1200

One state always wins out!!

Collapse occurs even though there is no direct collapse mechanism!!

An irreversible mark always appears!!

Could this be the way collapse works without a postulate or mechanism?

More Details are in supplementary reading GamblerRuin.pdf

End Lecture #6

Another way to get to the irreversible recording ……

(2) Decoherence (from the physicists)

If we have a quantum system is the state

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

then this(non-zero off-diagonal elements) state **exhibits** quantum interference effects.

"Decoherence" says that as a state interacts with macroworld in its environment (which it has to do) the off diagonal elements go to zero, i.e., the state makes transition to

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

As we already said, this state has zero quantum interference effects and a definite measurement value has been irreversibly recorded somewhere - As Feynman said : the quantum amplitudes have been all smeared out at this point!

If we had quantum dice the state vector would be

$$|Dice\rangle = \frac{1}{\sqrt{6}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle)$$ and $$\rho = |Dice\rangle\langle Dice|$$ **giving**

$$\begin{aligned}\rho = \frac{1}{6}\big[&|1\rangle\langle 1| + |2\rangle\langle 1| + |3\rangle\langle 1| + |4\rangle\langle 1| + |5\rangle\langle 1| + |6\rangle\langle 1| \\ &+ |1\rangle\langle 2| + |2\rangle\langle 2| + |3\rangle\langle 2| + |4\rangle\langle 2| + |5\rangle\langle 2| + |6\rangle\langle 2| \\ &+ |1\rangle\langle 3| + |2\rangle\langle 3| + |3\rangle\langle 3| + |4\rangle\langle 3| + |5\rangle\langle 3| + |6\rangle\langle 3| \\ &+ |1\rangle\langle 4| + |2\rangle\langle 4| + |3\rangle\langle 4| + |4\rangle\langle 4| + |5\rangle\langle 4| + |6\rangle\langle 4| \\ &+ |1\rangle\langle 5| + |2\rangle\langle 5| + |3\rangle\langle 5| + |4\rangle\langle 5| + |5\rangle\langle 5| + |6\rangle\langle 5| \\ &+ |1\rangle\langle 6| + |2\rangle\langle 6| + |3\rangle\langle 6| + |4\rangle\langle 6| + |5\rangle\langle 6| + |6\rangle\langle 6|\big]\end{aligned}$$

Thus, the quantum dice generally has a lot of quantum interference terms.

This corresponds to a density matrix of the form

$$\rho = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{pmatrix}$$

lots of off-diagonal terms
lots of quantum interference

Rolling the dice (a measurement) or "decoherence" if dice left alone, produces

$$\rho = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{pmatrix}$$

where $a_{11} = a_{22} = a_{33} = a_{44} = a_{55} = a_{66} = \frac{1}{6}$

or

$$\hat{\rho} = \frac{1}{6}(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4| + |5\rangle\langle 5| + |6\rangle\langle 6|) \quad \text{mixed state}$$

and as we said earlier this change indicates a measurement has occurred!! .

Maybe that is what Gambler's Ruin is doing and they are the same thing!

Now let us continue on our earlier path.

Now look closely at "which-path" experiments

In so-called "delayed-choice experiment" mentioned earlier,

there is instantaneous (to within some accuracy) fast switching between two states;

Also, each collapse is executed entirely while the photon was inside interferometer.

We can gain considerable insight by studying

how quantum theory describes a which-path measurement.

Note: it is a measurement as defined earlier,

because a detector **registers** "slit 1" or "slit 2" macroscopically for each electron.

Denote state of one electron passing through slit 1 as $|\psi_1\rangle$

and the state of one electron passing through slit 2 as $|\psi_2\rangle$.

John von Neumann, who was the first to

carefully analyze measurement in purely quantum-theoretical terms,

insisted on treating not only the measured quantum but also the macroscopic detector as quantum systems because, after all, detectors are made of atoms and they perform a quantum function by detecting individual quanta.

I agree with that view of the macroscopic detector.

Now we repeat our earlier discussion again, filling in any remaining gaps and eliminating any remaining confusions.

Accordingly, represent "ready to detect" quantum state of detector by $|ready\rangle$,

and state of detector after detecting an electron by $|1\rangle$ if $|\psi_1\rangle$ was detected,

and by $|2\rangle$ if $|\psi_2\rangle$ was detected.

A properly operating detector will surely transition from $|ready\rangle$ to $|1\rangle$

upon measurement of electron that has been prepared

(perhaps by simply shutting slit 2) in $|\psi_1\rangle$ state .

As a limiting idealization,

we assume, with von Neumann,

that the measurement of electron prepared in the state $|\psi_1\rangle$

leaves the electron still in state $|\psi_1\rangle$ after detection.

Such a **minimally-disturbing measurement** would cause

the electron-plus-detector composite system,

initially in the composite state $|\psi_1\rangle\,|ready\rangle$,

to transition into the final state $|\psi_1\rangle\,|1\rangle$.

We summarize the process as (I will be referring to the numbered equations)

$$|\psi_1\rangle\,|ready\rangle \rightarrow |\psi_1\rangle\,|1\rangle \qquad \textbf{(1)}$$

You should make a copy of all the numbered equations for reference as I lecture

Similarly, a minimally-disturbing measurement of an electron initially prepared in $|\psi_2\rangle$ is described mathematically by

$$|\psi_2\rangle\,|ready\rangle \rightarrow |\psi_2\rangle\,|2\rangle \qquad \textbf{(2)}$$

Now suppose that both slits are open so each electron "can pass through either slit", and suppose the preparation and experiment (e.g. slit widths) is symmetric with respect to the two slits.

Then the state of each electron as it approaches the slits prior to detection must be described by a symmetric superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \qquad \textbf{(3)}$$

But quantum physics, including its time dependence, is **linear**.

This implies that $|\psi\rangle\,|ready\rangle$ evolves according to

$$\begin{aligned}
|\psi\rangle\,|ready\rangle &= \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)\,|ready\rangle \\
&= \frac{1}{\sqrt{2}}(|\psi_1\rangle\,|ready\rangle + |\psi_2\rangle\,|ready\rangle) \qquad \textbf{(4)} \\
&\to \frac{1}{\sqrt{2}}(|\psi_1\rangle\,|1\rangle + |\psi_2\rangle\,|2\rangle)
\end{aligned}$$

The final state

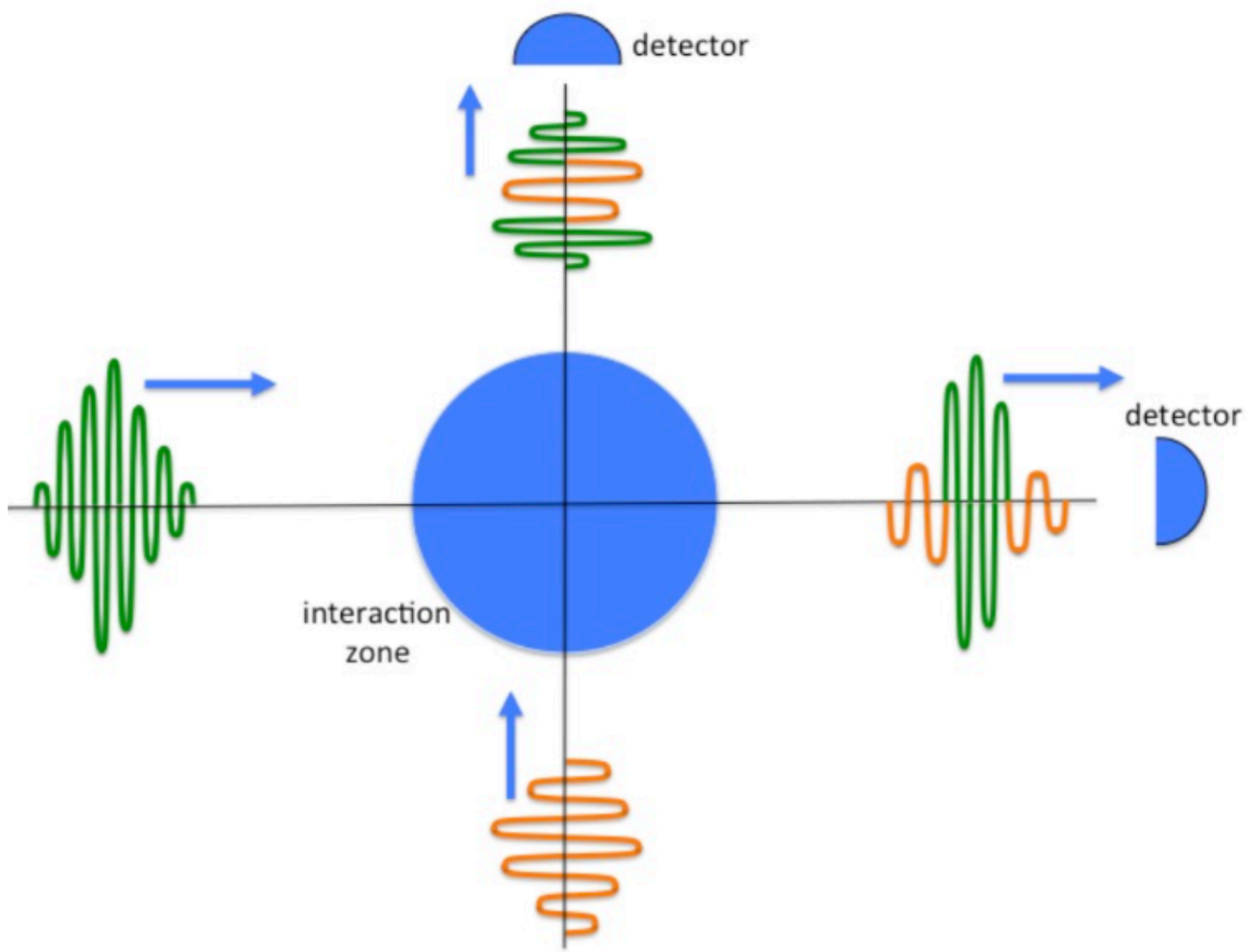
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle\,|1\rangle + |\psi_2\rangle\,|2\rangle\right) \quad \textbf{(5)}$$

following detection is said to be **"entangled"** the state properties are now “correlated”

because it **cannot be factored** into simple product of states of the two sub-systems.

You should make a copy of all the numbered equations for reference as I lecture

As indicated suggestively in the Figure below:



when two independent quanta pass **near** each other, interact, and subsequently separate, the interaction generally **entangles** the two quanta and the entanglement then **persists** after interaction **regardless** of how far apart the two quanta might eventually travel, provided only that the two quanta experience **no further interactions**.

Despite a possibly wide spatial separation, entangled quanta have a **unity** not possessed by non-entangled quanta.

It is this unity that is the source of quantum non-locality!

Entanglement is ubiquitous in nature.

The entangled "measurement state" (5) that is at **heart** of quantum measurement is remarkably **subtle**.

To fully understand **"entanglement"**, we **first** need to understand **"superposition"**.

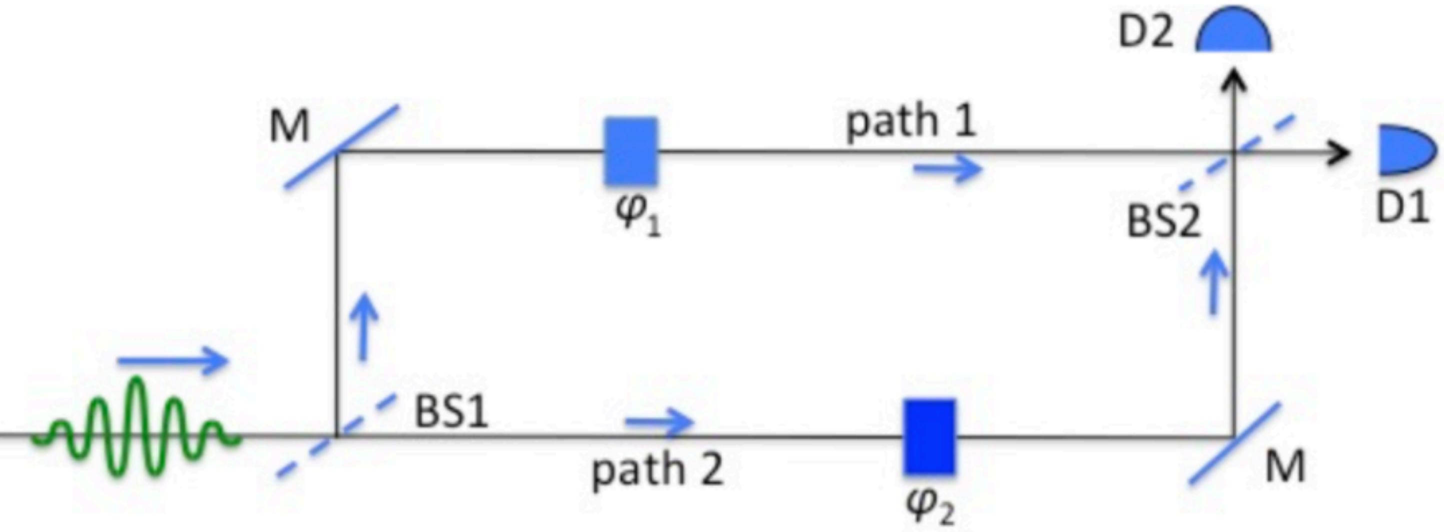
The quantum principle says that

vector sum

any linear combination(= superposition) of possible quantum states of a system,

as in (3) and (5) for example, is also a possible quantum state of that system.

Figure on next slide pictures an experiment that demonstrates such a superposition of states.



This represents a layout of optical paths called a “Mach-Zehnder interferometer” as we have discussed earlier.

Light beam enters at lower left passing through "beam splitter" BS1;

reflected beam(amplitude) makes a right angle with incoming direction

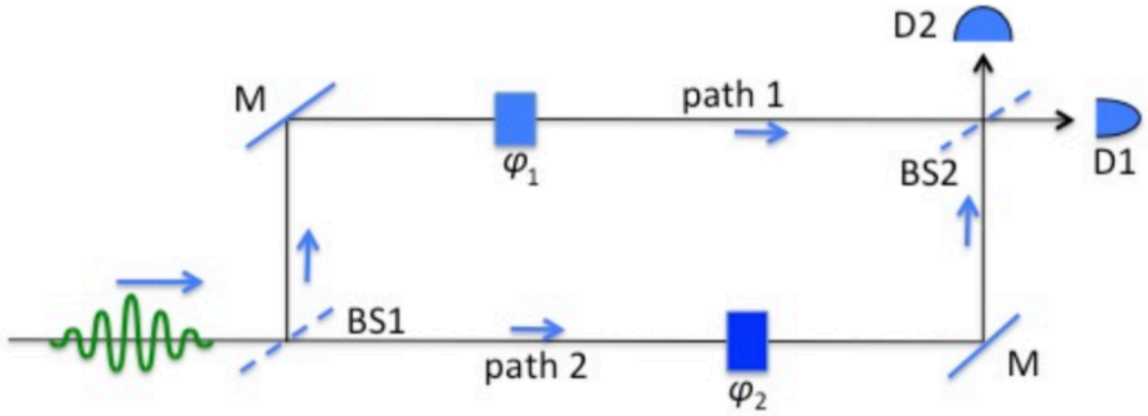
while transmitted beam(amplitude) passes straight through.

So the beam “splits” and each “half” traverses one of two paths (this means amplitudes exist for such to happen);

Mirrors M bring paths back to crossing point as shown.

Devices called "phase shifters,

" denoted by ϕ_1 and ϕ_2, are placed into each path



The phase shifter can add

a short variable length to a path.

A second beam splitter BS2 can be placed at the final crossing point.

Without BS2, each “half”-beam(amplitude) moves straight ahead along one path

to the detector on that path.

Things get more interesting with BS2 in place.

Because 50% of each of the two beams(amplitudes) then goes to each detector,

BS2 mixes the two beams(amplitudes) together so the experiment can show interference.

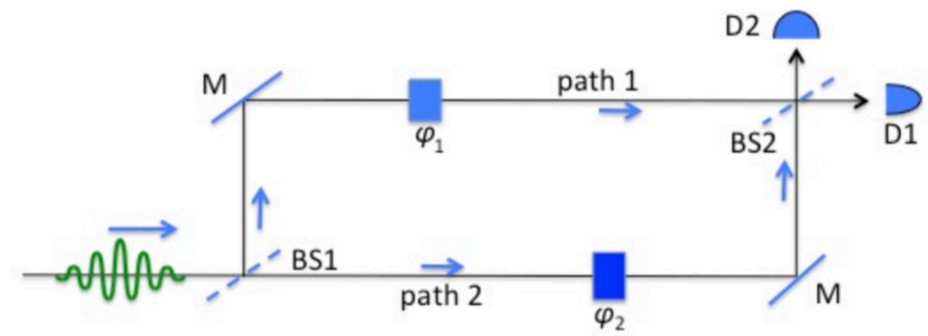
The interferometer is constructed so that, when phase shifters are set to zero, two "optical paths" (number of wavelengths, after accounting for phase changes upon reflection and refraction) from the entry point to D1 are equal while the two optical paths to D2 differ by half a wavelength.

Thus, is found that amplitudes interfere constructively at D1 and destructively at D2,

i.e., all photons go to D1.

If one then uses ϕ_1 or ϕ_2 to add half wavelength to either path, amplitudes then interfere constructively at D2 and destructively at D1,

i.e., all photons go to D2.



As one continuously varies length of one or other path

by varying one or the other phase shifter,

one finds that amount of light(number of photons) arriving at D1 varies continuously from 100% down to 0%,

while amount arriving at D2 varies from 0% to 100%.

Two paths are clearly interfering -> can construct a normal interference pattern from the data!

Experiment is an interferometer-based analog of Young's double-slit interference experiment demonstrating the "wave" nature of light - because we have set the **experiment context** to do so!.

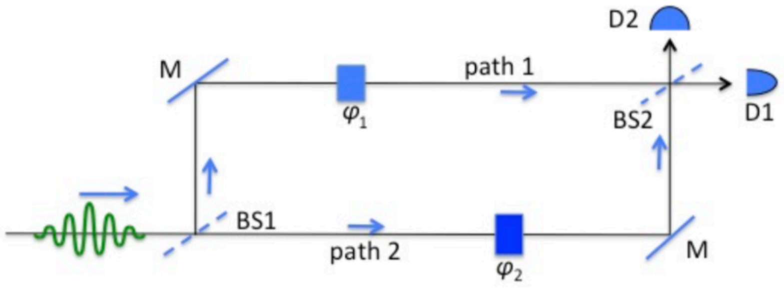
But, as we know, light is really just photons, and photons are indivisible.

How then does **nature** explain this experiment

when we dim the light source to a level where only **one photon at a time** traverses the interferometer? i.e., when the detectors only record one photon at any instant of time!

After all, the photon still traverses BS1, but it cannot split in two because a **quantum is unified and cannot be split.**

very important property



With BS2 removed, one finds either D1 or D2 registers a single entire photon, randomly, with 50-50 probabilities, regardless of how the phase shifters are set.

The randomness is **absolute** - it is more random than any human macroscopic game, such as coin flips, which only mimics randomness.

Nature **has invented** quantum randomness to deal with obstacles such as beam splitters while **preserving the unity of the quantum**.

very important property

Detectors **never** register half a photon.

They **either** get either a whole photon or no photon.

What happens in single-photon experiment with BS2 present?

As discussed earlier,

beginning from equal path lengths,

which give constructive interference at D1

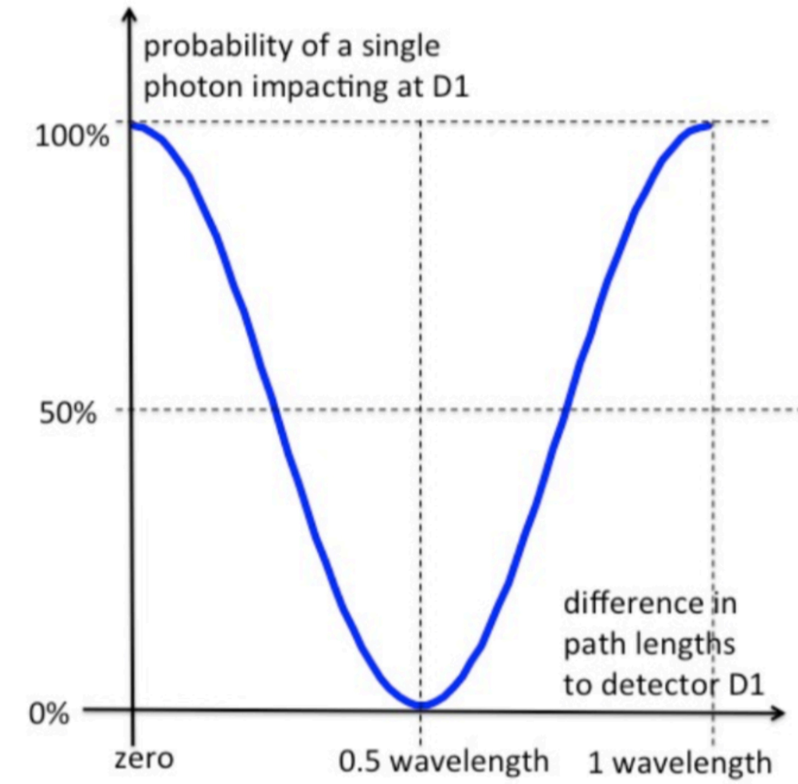
and destructive interference at D2,

as phase shifters vary,

probabilities of detecting a photon at D1 and D2

vary as in Figure,

which gives the percentage of photons impacting D1.



Importantly,

results do not depend on which phase shifter the experimenter chooses to vary.

Since each photon responds to changes in either path length,

each photon **must somehow "follow both paths"**

Whatever that statement in words means!

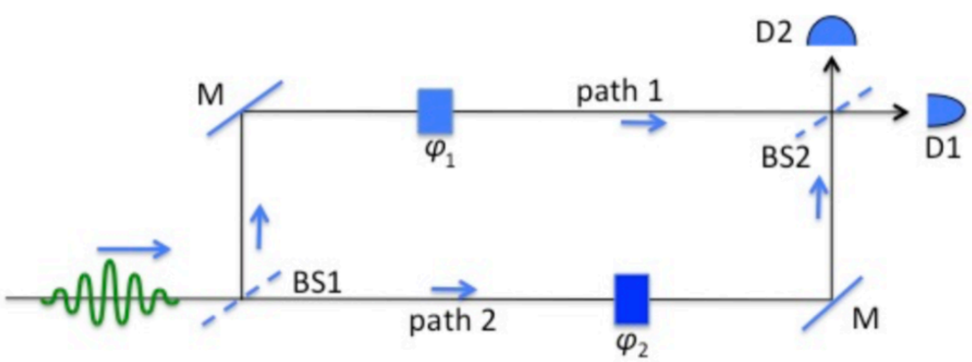
Using quantum math all is clear and works!

This verifies the superposition principle and shows that **quanta** can **seem** to be in "**two places at same time".**

This seems **paradoxical** if we **assume** photons are tiny particles,

but if we assume photons are waves it is **not** paradoxical,

i.e, each photon simply spreads along both paths, interfering with itself at D1 and D2.



One must conclude that each photon travels both paths

even when BS2 is not present to directly verify this,

because once a photon enters interferometer

it **must behave in same manner** regardless of whether BS2 is placed

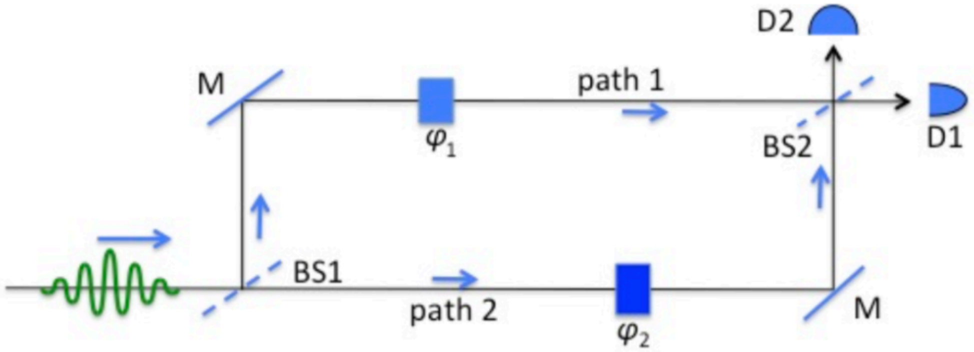
or not placed at far end.

A **delayed-choice** experiment provides further evidence for this conclusion:

Since photons **"do not know"** whether BS2 will be inserted,

they must travel both paths on all trials

including those for which BS2 is not inserted - **this is connected with entanglement**.



With BS2 removed,

the situation is like the double-slit experiment

with a which-slit detector present.

Each photon is entangled

with macroscopic detectors D1 or D2 as in (5).

With BS2 present, the two paths mix and we have a situation

like double-slit experiment with no which-slit detector -

each photon follows two paths to each detector

where it interferes with itself, and we detect an interference state (3).

All of this clearly suggests that measurements affect superposed quantum states

via the entanglement of the superposed quantum with a detector.

Resolving Paradoxes and Understanding Measurement

The apparent paradox of Schrödinger's cat

A cat is penned up in a steel chamber, along with the following setup.

In a Geiger counter there is tiny bit of radioactive substance, so small, that in course of an hour one of its atoms decays, but also, with equal probability, none decay;

if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid (poisonous vapor).

If one left the entire system to itself for an hour,

one would say that the cat still lives if no atom has decayed!

If one uses a state vector approach for understanding the entire system,

one would express this by having a living and a dead cat mixed

or smeared out in equal parts -

a superposition of "dead" and "alive" with other properties of system!

a statement that is as stupid as it sounds!!

It is typical of these cases,

that indeterminacy originally restricted to atomic domain

becomes transformed into a macroscopic indeterminacy,

which can only then be resolved by direct observation,

which prevents us from naively

accepting as valid the “blurred model" for representing reality.

Mathematically, the nucleus and the cat

have become entangled in measurement state (5),

with $|\psi_1\rangle$ and $|\psi_2\rangle$ representing the undecayed and decayed states of the nucleus

and $|1\rangle$ and $|2\rangle$ representing the alive and dead cat.

or lack of understanding in my opinion

According to Schrodinger’s “understanding” of this situation,

the indeterminacy of the microscopic nuclear state "becomes transformed

into the macroscopic indeterminacy" of cat, and he could not comfortably accept this

“blurred cat state” i.e., a cat that is in a superposition of being both alive and dead.

He hoped this would say something is wrong with QM.

As we will show, standard quantum physics says that Schrodinger's 1937 understanding was **incorrect**.

The composite system (cat-plus-nucleus) is **not** predicted to be in superposition of two states of a cat and two states of a nucleus.

Instead, the composite system is predicted to be in a superposition of two **correlations** between the cat and the nucleus; one where live cat is 100% correlated with undecayed nucleus, and second where dead cat is 100% correlated with decayed nucleus.

Entanglement will have transformed a pure state superposition of nuclear states to a pure state superposition of correlations between subsystem states.

We will see this is what one expects from quantum mechanics, and it is **not paradoxical**.

This so-called “problem of definite outcomes”

applies of course to more than Schrodinger’s dramatized example.

Regardless of whether measuring instrument is a which-slit detector,

a Geiger counter, or a cat, the entangled state (5) applies.

This state appears at first glance to represent a quantum superposition

in which the detector is in two macroscopically different states simultaneously.

If so, then there is an inconsistency within quantum physics,

because obviously it **cannot** be this easy to create a **macroscopic superposition**.

The question is: Is it true that (5) really represents a macroscopic superposition?

Turns out, there is more to this entangled state than meets the eye.

If one assumes the detector to be in a superposed state in its own space $a|1\rangle + b|2\rangle$,

one finds that (5) necessitates either $a = 0$ or $b = 0$ (one or the other), implying that the detector is not in an individually superposed state **within its own Hilbert space**.

The **same** applies to the detected quantum:

It is not in a superposed state $a|\psi_1\rangle + b|\psi_2\rangle$ with both $a \neq 0$ and $b \neq 0$.

The entanglement process leaves neither sub-system superposed in **own** space!

So far as I know, this simple fact has long been mostly ignored by physicists studying the measurement problem - except for a few that I have worked with.

The Density Operator to the Rescue

The density operator formalism for quantum physics provides a stronger version of this conclusion(because it is the more appropriate approach).

Density operator for quantum system whose state is $|\psi\rangle$(pure state) is a projection operator

$$\hat{\rho} = |\psi\rangle\langle\psi| \qquad \textbf{(6)}$$

As we saw earlier, if system is in a state whose density operator is $\hat{\rho}$, then standard quantum expectation value $\langle \hat{O} \rangle$ of an arbitrary observable $\hat{O}$ is found from

$$\langle \hat{O} \rangle = \mathrm{Tr}(\hat{\rho}\hat{O}) \tag{7}$$

where "Tr" represents trace operation (sum of diagonal matrix elements).

This approach is especially useful if the quantum system is a composite of two subsystems A and B.

We define the density operator $\hat{\rho}_A$ for subsystem A alone by

$$\hat{\rho}_A = \mathrm{Tr}_B(\hat{\rho}) \tag{8}$$

where "Tr_B" means that trace taken **only** over states of subsystem B (remove all traces of B).

It is then easy to show (see later)

that standard quantum expectation values for subsystem A alone

(values obtained by an observer of A **without** any knowledge of B) are

$$\langle \hat{O}_A \rangle = \mathrm{Tr}(\hat{\rho}_A \hat{O}_A) \tag{9}$$

where $\hat{O}_A$ means any observable operating on system A alone (i.e., operating within A's Hilbert space).

Applying this formulation to measurement state (5), the reduced density operators for the quantum system (call it A) and its detector (call it B), respectively, are

$$\hat{\rho}_A = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|) \quad \textbf{(10)}$$

$$\hat{\rho}_B = \frac{1}{2}(|1\rangle\langle 1| + |2\rangle\langle 2|) \quad \textbf{(11)}$$

Plus signs in (10) and (11) make one think of superpositions such as (3), but these are **not** superpositions.

The density operator for the superposition (3) has cross-terms:

$$\hat{\rho} = (|\psi\rangle\langle\psi|) = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_2| + |\psi_2\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|) \quad \textbf{(12)}$$

Two cross-terms, involving both $|\psi_1\rangle$ and $|\psi_2\rangle$, are missing in (10).

So (10) does not describe a system in a superposition of two quantum states.

However, (10) is precisely the density operator one should use if one **knows** the quantum system **is either** in state $|\psi_1\rangle$ **or** in state $|\psi_2\rangle$ but one didn't know which and so, due your own to lack of information,

—> you simply assign a probability of 1/2 to each of the two possibilities.

Same goes for (11).

(10) and (11) are **"classical"** probabilistic states - **analogous** to "states of knowledge" one would assign to a coin flip when you know outcome to be either heads or tails with equal probability **but don't know yet** which has occurred.

Remember the dice example also!

The situation described by a density operator such as (10) is known as a "mixture" of states $|\psi_1\rangle$ and $|\psi_2\rangle$, as distinct from a "superposition" of states observed in the Mach-Zehnder experiment and represented by (3).

Equation (9) tells us that all correct statistics for subsystem A alone can be found from standard formula (7) applied to subsystem A **alone**.

But we have just seen that (10) is the density operator one should use if one knows A to be in either $|\psi_1\rangle$ and $|\psi_2\rangle$ without knowing which.

The same goes for subsystem B and (11).

In the case of Schrodinger's cat, it follows that the observer of the cat alone sees outcomes appropriate to a cat that is **either** alive **or** dead, **not** both.

For subsystems, the interference terms are missing, and an "ensemble" of repeated trials **must** exhibit a nonsuperposed mixture rather than a superposition.

This is the clear prediction of quantum physics for the entangled state (5).

But one must be careful, because (10) and (11) are not complete descriptions of the quantum states of the nucleus or the cat.

In fact, (10) and (11) are not quantum states at all, but merely **"reduced states"** arising from the actual state (5) of the composite system when one part of the composite system is removed from the equations..

In the case of Schrodinger's cat, (10) and (11) give the correct predictions for observations of either the nucleus alone or the cat alone, but do not represent the state of either subsystem because this given by (5).

In fact, when two quanta are entangled, neither one has quantum state of its own!

But B's state of affairs is certainly **not entirely** described by (11).

Rather, it is described by the composite state (5).

Equation (11) **merely** tells us following:

If the cat and the nucleus are in state (5) then, when one looks at the cat, one is going to see a cat that is either alive or dead.

There is no claim that (11) represents the complete quantum state of the cat.

That is, there is no claim that the cat is really in either state $|1\rangle$ or state $|2\rangle$, because the state it's really in is admittedly (5).

In fact, we do have complete knowledge of the state of both A and B, namely measurement state (5).

Reduced operators admittedly do not represent the state of the composite system.

They tell us only what will observe at the nucleus and at the cat

and they tell us nothing about the correlations between these observations,

so these density operators do not tell us the real state of the system.

And so plot thickens.

The entangled state (5) properly describes both individual subsystems.

However, the plus sign in (5) seems to signify the superposition of two terms.

We know, however, that neither subsystem A nor subsystem B is superposed.

What is meaning of plus sign?

This superposition arose from superposition represented by (3).

We cannot logically ignore this fact -

a strategy known as "shut up and calculate" approach to quantum measurement.

Instead, we must ask:

Exactly what is superposed when two subsystems are in this entangled state?

Superpositions must preserve the all-important **unity of quantum**.

When Max Planck proposed in 1900 that electromagnetic radiation occurs in energy units of magnitude $E = h\nu$, he tacitly implied the central quantum principle:

The unity of an individual quantum.

Energy (electromagnetic energy in case of radiation) comes in spatially extended bundles, each having a definite and identical quantity of energy.

One cannot have half a quantum, or 2.7 quanta.

You must have either 0 or 1 or 2 etc. quanta.

This is a fairly natural notion - apparently nature prefers to sub-divide the universe into a countable or even a finite set of entities as opposed to an uncountable continuum.

The spatial extension of these bundles **then** implies nonlocality.

If we have one quantum and destroy it (by transforming it to something else),

we **must** destroy all of it **everywhere**,

simultaneously, because we **cannot**, at any time, have **just** part of quantum.

Louis de Broglie put it perfectly in 1924, regarding another kind of quantum, namely the electron:

> **The energy of an electron is spread over all space with a strong concentration in a very small region. ...That which makes an electron an atom of energy is not its small volume that it occupies in space – I repeat it occupies all space – but the fact that it is indivisible, that it constitutes a unit.**

When one transforms state of a quantum, one **must** transform the entire extended quantum all at once.

Hence there are observed quantum jumps in experiments.

Furthermore, composite entangled systems such as atoms **also** behave in a unified fashion.

This unity is the source of nonlocality seen in experiments involving entangled pairs of photons.

Nonlocality is exactly what one would expect, given the unity and spatial extension of quantum and unitary (i.e. unity-preserving) nature of entanglement process.

Standard nonrelativistic quantum theory seems to prescribe two kinds of time evolution:

collapse upon measurement,

and Schrodinger equation between measurements.

The key feature of Schrodinger equation is that it prescribes a so-called "unitary" time evolution,

meaning time evolution that preserves pure states,

i.e. transforms unit Hilbert space vectors into other unit vectors.

Some ideas that are required physically by the unity of quantum are expressed as follows:

If quantum is described by pure quantum state at t = 0, it should **remain** pure at later times.

This notion prompts us to **ask** whether the measurement process also preserves pure states.

At least in case of idealized process described in (4), the answer is **"yes"** because **both** "before" and "after" states are pure.

Measurement state (5), since it is pure, **represents** a highly unified state of affairs, **even** though one of its subsystems is a macroscopic detector.

Thus, one suspects that this state, like its progenitor (3), is truly a superposition in which the superposed terms **represent** two situations or states of the same object.

But precisely what is that object, i.e., **what** is superposed?

We have seen that the states of subsystem A are **not** superposed,

nor are the states of subsystem B.

The conventional interpretation (which, as we will see, is subtly incorrect) of a product state such as $|\psi_1\rangle|1\rangle$ is that it represents a state of a composite system AB in which subsystem A is in state $|\psi_1\rangle$ while B is in state $|1\rangle$ (we used such a state in the two-path experiments).

In this case, (5) would represent a superposition in which AB is simultaneously in the state $|\psi_1\rangle|1\rangle$ and also in the state $|\psi_2\rangle|2\rangle$.

The situation for Schrodinger's cat would be: live cat and undecayed nucleus superposed with dead cat and decayed nucleus.

This is at least as physically outrageous as a live cat superposed with a dead cat, and it contradicts physical implications (a cat that is either alive or dead) of reduced states (10) and (11) as described earlier.

Something is still wrong!

Let us repeat some of our discussion and provide more details to work it all out.

Some Repetition and More Intricate Details

Proposal:

The solution to the so-called quantum measurement problem is completely contained within standard quantum mechanics and needs no elaborate new structures and interpretations.

Remember the standard discussion from earlier:

The following discussion is at limits of knowledge you have learned about QM, but I thought you should see what researchers including myself are doing at the present time.

The details of the derivations are not important, ONLY the conclusions.

Total system T = quantum system S (states $|s_i\rangle$) + measuring device A (states $|a_i\rangle$),
where $|a_0\rangle =$ state of measuring device "off"

The unitary time evolution rule then says

$|s_1\rangle\,|a_0\rangle \to |s_1\rangle\,|a_1\rangle$ unitary evolution $\qquad |s_2\rangle\,|a_0\rangle \to |s_1\rangle\,|a_2\rangle$ unitary evolution

The linearity of QM then says that

$$|\psi\rangle_{SA} = (c_1 |s_1\rangle + c_2 |s_2\rangle) |a_0\rangle = c_1 |s_1\rangle |a_0\rangle + c_2 |s_2\rangle |a_0\rangle \rightarrow c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

In this derivation

$$|\psi\rangle_S = c_1 |s_1\rangle + c_2 |s_2\rangle$$

is state of quantum system(superposition), while

$$|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

is state of combined system(entanglement/superposition).

The quantum measurement problem is the realization that the state

$$|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

is **not** observed as outcome of measurement!

What is seen is not a so-called superposition, but either

$$|s_1\rangle\,|a_1\rangle \;\text{ or }\; |s_2\rangle\,|a_2\rangle$$

That is the so-called **“problem of outcomes”**.

The state

$$|\psi\rangle_{SA} = c_1\,|s_1\rangle\,|a_1\rangle + c_2\,|s_2\rangle\,|a_2\rangle$$

is usually referred to as a superposition, which is **misleading**.

Entanglement **outweighs** superposition as the **defining** feature of this state.

Without entanglement-correlations - we would **not** have a measurement problem!

Careful investigation of this state in a 2007 experiment(Roch, et al) (a Wheeler-type delayed choice experiment) demonstrates its strikingly non-local character.

We will discuss the experiment in detail shortly.

A photon jumps from state $|\psi\rangle_S = c_1 |s_1\rangle + c_2 |s_2\rangle$ to state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ precisely when A(measuring device) switches on and while photon is still inside interferometer, and jumps from $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ to $|\psi\rangle_S = c_1 |s_1\rangle + c_2 |s_2\rangle$ when A switches off during the experiment.

Quantum jumps, removal of interferences, and the observed non-locality are due to entanglement.

Of course, an entangled state = superposition, but it is a very **special** superposition.

To call $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ simply a "superposition" **misses** the crucial physics of entanglement and makes all the difference in the understanding of the state.

Entanglement is the characteristic trait of quantum mechanics; one that enforces the entire departure from classical lines of thought.

It is well known that, for 2-part systems, **all** non-product states exhibit non-locality (Bell inequality).

Thus when S and A are entangled $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$, they **share** a so-called non-local **channel**.

The measurement state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ is very subtle.

Although $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ is often called a superposition of S and/or superposition of A, **neither** case is true.

Thus, when S and A in measurement state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$, neither S nor A in a superposition. $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ = superposition, but neither a superposition of S nor of A and also not superposition of states of composite system SA.

In $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$, S is in both states $|s_1\rangle$ and $|s_2\rangle$ simultaneously, as is known from observed interference between 2 states.

However, in case of $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ **experiment** now shows that SA is not in both two-part states $|s_1\rangle |a_1\rangle$ and $|s_2\rangle |a_2\rangle$ simultaneously, **but only simultaneously in two correlations between the states.**

That is the crucial experimental result!

The entanglement of **two** systems is quite **different** from superposition of **one** system.

Experiments(Rarity) demonstrated the **precise** sense in which $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ **represents** a superposition.

Experiments answer question "given that S and A are in state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$, **what** (if any) entities interfere and **what** is nature of interference?"

Let us see exactly how………

The Local State Solution(due to Jauch) of the Problem of Definite Outcomes

Consider a **single** quantum S(electron or photon), passing through double-slit experiment, with a "downstream" viewing screen.

Suppose an ideal "which-slit detector" A is present so that, upon detection, S and A become entangled in measurement state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ with **orthogonal** detection states $|a_i\rangle$.

Imagine S and A are possibly separated by meters or kilometers.

Complete observation of experiment **requires** two “local observers”, The 1st observer is S and the 2nd observer is A.

Such a non-local setup **has been carried** out experimentally.

What did the **local** observers observe this experiment?

A well-known prediction of quantum physics says

that the 1st observer observes implications of the **“local state of S,”**

represented by a **“reduced”** density operator,

where the degrees of freedom for the 2nd system

are averaged over(removed) by the “trace” (Tr) operation (see below)

$$\rho_S = \mathrm{Tr}_A(\rho_{SA}) = |c_1|^2 |s_1\rangle \langle s_1| + |c_2|^2 |s_2\rangle \langle s_2|$$

and 2nd observer would observe the implications of **“local state of A”**, represented by the reduced density operator

$$\rho_A = \mathrm{Tr}_S(\rho_{SA}) = |c_1|^2 |a_1\rangle \langle a_1| + |c_2|^2 |a_2\rangle \langle a_2|$$

where a density operator gives probabilities via the relation

$$P(b) = \mathrm{Tr}(\rho P_b) = \mathrm{Tr}(\rho \, |b\rangle \langle b|)$$

The “local state of S” is found by **completely removing** from the density operator any effects of A and vice versa.

This is the important theoretical idea!

Derivation of reduced density operator (for completeness)

$$\rho_{SA} = |\psi_{SA}\rangle \langle \psi_{SA}| = (c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle)(c_1^* \langle s_1| \langle a_1| + c_2^* \langle s_2| \langle a_2|)$$

$$\begin{aligned}
\rho_S = \mathrm{Tr}_A(\rho_{SA}) &= \sum_{k=1}^{2} \langle a_k| \rho_{SA} |a_k\rangle \\
&= \sum_{k=1}^{2} \langle a_k| (|\psi\rangle_{SA} \; {}_{SA}\langle\psi|) |a_k\rangle \\
&= \sum_{k=1}^{2} \langle a_k| ((c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle)(c_1^* \langle s_1| \langle a_1| + c_2^* \langle s_2| \langle a_2|)) |a_k\rangle \\
&= \sum_{k=1}^{2} (c_1 |s_1\rangle \delta_{k1} + c_2 |s_2\rangle \delta_{k2})(c_1^* \langle s_1| \delta_{k1} + c_2^* \langle s_2| \delta_{k2}) \\
&= |c_1|^2 |s_1\rangle \langle s_1| + |c_2|^2 |s_2\rangle \langle s_2|
\end{aligned}$$

where we have used orthonormality of measurement states via $\langle a_i \,|\, a_j \rangle = \delta_{ij}$

We note that the Tr (trace) operation = sum over all designated states

removes all knowledge of the designated(traced) system from the equation.

This is clearly useful i**f we do not know much** about the reduced system.

A similar result holds for ρ_A .

Now, continuing our discussion.

Clearly, the reduced states are **mixtures**, not superpositions(no cross-terms).

QM predicts both LOCAL observers **find(see)** mixtures **not** superpositions.

An ensemble of experimental trials verifies

this via mixed-state patterns in agreement with the assertion made earlier

that neither S nor A is in a superposition.

For a different example,

$$\rho_A = \mathrm{Tr}_S(\rho_{SA}) = |c_1|^2 \, |a_1\rangle \langle a_1| + |c_2|^2 \, |a_2\rangle \langle a_2|$$

predicts Schrödinger's cat is in a mixture of either dead or alive,

not a superposition of both dead and alive.

The **local states**

$$\rho_S = \mathrm{Tr}_A(\rho_{SA}) = |c_1|^2 \, |s_1\rangle \langle s_1| + |c_2|^2 \, |s_2\rangle \langle s_2|$$

and

$$\rho_A = \mathrm{Tr}_S(\rho_{SA}) = |c_1|^2 \, |a_1\rangle \langle a_1| + |c_2|^2 \, |a_2\rangle \langle a_2|$$

must be taken seriously as implying outcomes predicted to be observed at the two sites.

The local states **cannot be dismissed** simply

by the argument that the only "real" state is the "global state"

$$|\psi\rangle_{SA} = c_1 \, |s_1\rangle \, |a_1\rangle + c_2 \, |s_2\rangle \, |a_2\rangle$$

There is no contradiction between the predicted local **mixtures**

$$\rho_S = \mathrm{Tr}_A(\rho_{SA}) = |c_1|^2 \, |s_1\rangle \, \langle s_1| + |c_2|^2 \, |s_2\rangle \, \langle s_2|$$

$$\rho_A = \mathrm{Tr}_S(\rho_{SA}) = |c_1|^2 \, |a_1\rangle \, \langle a_1| + |c_2|^2 \, |a_2\rangle \, \langle a_2|$$

and the unitarily-evolving global **pure state** .

$$|\psi\rangle_{SA} = c_1 \, |s_1\rangle \, |a_1\rangle + c_2 \, |s_2\rangle \, |a_2\rangle$$

Important:

We are saying: To consider the combined system SA as a single system only evolving unitarily **misses** the essential physics of nonlocality.

Haroche and Raimond weighed in on the question of

when a two-part system should be considered as a composite of two subsystems,

versus when it should be considered a single system B = SA.

“The composite system should be considered as single whenever the binding between the parts is much stronger than the interactions involved in dynamics, so that the internal structure of the composite system is left unchanged as it travels through the experiment.”

By this criterion, SA not single system.

Not only does the relation between S and A change during experiment, the relation is entangled and thus nonlocal.

The implication is that $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ **must be** considered an entanglement of two separate systems S and A, not superposition of single composite system SA.

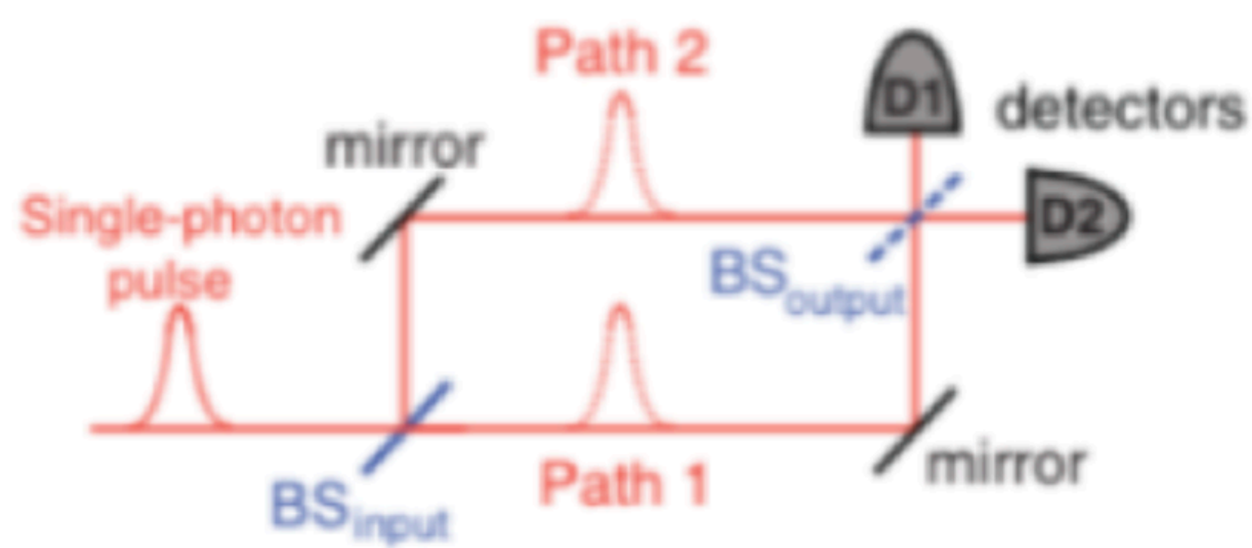
That's the basics of the theory at this point in the discussion(at this point in my tortuous development!) It follows completely from standard QM.

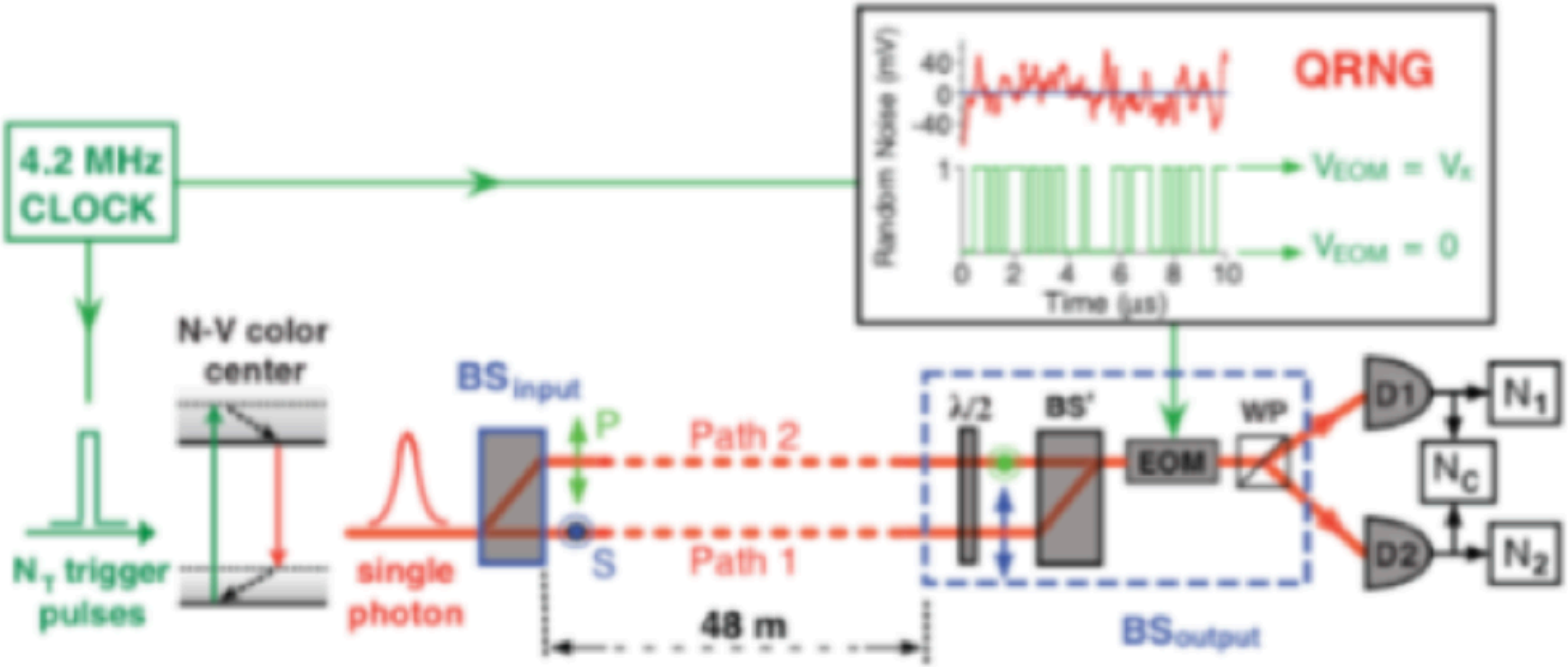
Not sure yet! Let us do it again!

Now for even more details and some possible new interpretations arise.

Start new discussion with a new experiment as we always should!!!!!

Roch's "delayed choice" experiments used a Mach-Zehnder interferometer (figure right) rather than the logically equivalent double-slit setup, to observe photons. It is shown in detail in the figure on next slide.





While a photon is on the **48-meter-long** interferometer paths,

a quantum-based random number generator "decided" whether the second beam splitter (positioned at the end of paths - note the spatial separation) would be incorporated or omitted,

i.e., it decided whether detectors would not or would (respectively) determine "which path."

In trials incorporating second beam splitter, an interference pattern is observed,

indicating the photon passed through device as superposition along both paths.

On trials omitting second beam splitter, no interference is observed, indicating the photon passed through device along one or other path.

The two parallel paths were 5 millimeters apart.

Precisely (so far as the experiment could determine)

when the second beam-splitter switched from "on" to "off,"

the photon collapsed in **mid-flight**

from being on both paths to being on one or the other path.

That means that the subsequent quanta reaching the detectors change abruptly at that moment.

The quantum jump was **correlated** with and **coincident** with

the incorporation or the omission of the second beam splitter

(i.e., with the **decision** to not entangle **or** to entangle the detectors with the photon).

All experimental results **are** just as predicted correctly by the reduced states

$$\rho_A = \mathrm{Tr}_S(\rho_{SA}) = |c_1|^2 |a_1\rangle \langle a_1| + |c_2|^2 |a_2\rangle \langle a_2|$$

$$\rho_S = \mathrm{Tr}_A(\rho_{SA}) = |c_1|^2 |s_1\rangle \langle s_1| + |c_2|^2 |s_2\rangle \langle s_2|$$

Switching the second beam-splitter off entangles the photon and the detector in measurement state

$$|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

causing the photon to collapse from a superposition

$$|\psi\rangle_S = c_1 |s_1\rangle + c_2 |s_2\rangle$$

into the mixture

$$\rho_S = \mathrm{Tr}_A(\rho_{SA}) = |c_1|^2 |s_1\rangle \langle s_1| + |c_2|^2 |s_2\rangle \langle s_2|$$

that is observed at the detector.

At this point we are in the diagonal density matrix stage.

Thus, we are doing a classical measurement(like dice) and it should be interpreted as such!

This should resolve problem of definite outcomes!

This was my belief at this point in my tortuous path to understanding!

Quantum theory predicts and experiment verifies that, with the detector in operation, observers of S and of A find them to be in definite mixtures, **not** indefinite superpositions.

Any strategy of imagining the quantum and the detector to be widely separated obviously changes nothing - it does not matter whether the quantum and the detector are close together or far apart.

The key to understanding quantum measurements comes from understanding nonlocal relationship that develops between S and A when they evolve unitarily into entangled measurement state

$$|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

In experiments, this global state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ violates Bell's inequality, implying instantaneous non-local transfer of **correlations** across arbitrarily large distances.

Not instantaneous information transfers, but CORRELATIONS which we can only see later!

Without entanglement one would observe different results

- the interferences would not disappear as they do in the experiments.

But entanglement "decoheres-collapses" coherent states

so that S and A impact their detectors randomly.

Locally, entanglement "decoheres" each photon so that they exhibit definite outcomes.

But quantum dynamics is unitary, implying that the global state

$$|\psi\rangle_{SA} = c_1\,|s_1\rangle\,|a_1\rangle + c_2\,|s_2\rangle\,|a_2\rangle$$

remains coherent despite the incoherence of subsystems.

Since the individual photons are now incoherently mixed, what has happened to coherence?

Answer:

It is the experimentally observed global coincidence measurements

which compare the impact points of entangled pairs.

In the double-slit experiment with two screens, each photon “knows” the impact point (i.e., phase shift) of other photon and instantly adjusts own impact point in order to form an interference pattern as a function of difference between two photons’ phase shifts!

This is strikingly non-local, and the experimental results violate Bell’s inequality.

Thus the coherence of the entangled state resides in correlations between subsystems, rather than in the subsystems themselves.

Entanglement **transforms** the coherence of states of S into coherence of correlations **between** states of S and A, **allowing** S and A to exhibit definite outcomes while **preserving** the global coherence as demanded by unitary evolution.

We can now answer the question:

Precisely what is superposed and what interferes in the measurement state?

The answer is surprisingly simple:

Only the correlations between S and A are superposed.

Thus the measurement state $|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$ should be read as:

The state $|s_1\rangle$ is positively correlated with the state $|a_1\rangle$,
and the state $|s_2\rangle$ is positively correlated with the state $|a_2\rangle$
Only correlations are superposed, not states.

When the superposition $|\psi\rangle_S = c_1 |s_1\rangle + c_2 |s_2\rangle$ of S entangles with states of A, the superposition **shifts**, from a superposition of states of S to superposition of correlations between S and A, so S can be in an incoherent mixture **while** maintaining unitary global dynamics.

This is how nature resolves problem of definite outcomes.

So the coherence exhibited by the measurement state

$$|\psi\rangle_{SA} = c_1 |s_1\rangle |a_1\rangle + c_2 |s_2\rangle |a_2\rangle$$

must be **invisible** to local observers,

and **yet show up** in the global measurement state in order to **preserve** unitary dynamics.

One could regard this as the **underlying reason** why entanglement (i.e., measurement)

must shift coherence from the states of S and A to the correlations between S and A.

"Collapse" or the "appearance of definite values" (a reduced density operator)

can be viewed as a consequence of the measurement state's **nonlocality**

plus special relativity's **ban** on instant signaling - that is WHY!

We note that the global measurement state is a very **different** animal from local states.

While the local mixed states are immediately observed at both local sites,

the global state can be "observed" only at some time after measurement by traveling

to both local sites, gathering data from both, and then assembling information

and noting the correlations between two sets of data.

Our conclusions so far:

When the detector measures the superposed quantum,
quantum physics predicts that the states actually observed are
local (i.e. mixed or reduced) states of subsystems,
it is not the superposed global state that follows from Schrödinger's equation.

The local states directly observed in the measurement
must contain **no** hint of nonlocal correlations
between two subsystems since this would violate relativity's prohibition on instant signaling.

Thus the local states describe what **actually** happens at both subsystems.

The global state predicts these local states(they can be derived from it),
and **also** predicts indirectly-observable (by gathering global data at **later** time)
nonlocal correlations between these states.

Continuing (we still have 1-1/3 lectures) along my path to final understanding…..

An even more dramatic experiment — Experimental nonlocality and entanglement

Now we demonstrate that, according to standard quantum theory (**and** experiment), measurement state (5) represents none of paradoxical situations we have mentioned.

Again let us repeat material from earlier - understanding this topic is so important that repetition is warranted.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle|1\rangle + |\psi_2\rangle|2\rangle)$$

The **unity** of the quantum suggests that the measurement state (5) represents a unified, hence superposed and pure, quantum state of composite system.

We asked the question: precisely what is superposed?

Studied simple (i.e. non-composite) superposition (3) via interference exhibited in M-Z experiment.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$$

Varying the length of either path 1 or path 2

created varying interference effects in detectors,

demonstrating each photon really must travel both paths to its detector.

Quantum theory agrees entirely with these conclusions,
as can be shown by using photon wavelengths
to show that path differences correctly predict interferences observed at each detector.

This implies that to understand the measurement state,
one needs to find and analyze entanglement experiments that demonstrate interference.

This has been done for several decades in connection with quantum non-locality.

Many nonlocal interference experiments have been done
beginning with Clauser and Freedman,
culminating in experiments demonstrating nonlocality across great distances
and that simultaneously closed all possible loopholes in all previous experiments.

By now, it is well known that entangled state (5) predicts nonlocal effects
between two subsystems, and that phase variations of either subsystem
cause instantaneous(non-signaling), i.e., non-local, readjustments(correlations)
of the possibly-distant other subsystem.

When macroscopic systems are involved (i.e., cats) we have a problem.

it is not easy to **vary the phase of cat**,

and as we saw in the Mach-Zehnder experiment,

one **cannot** understand superposition without varying phases of the superposed parts.

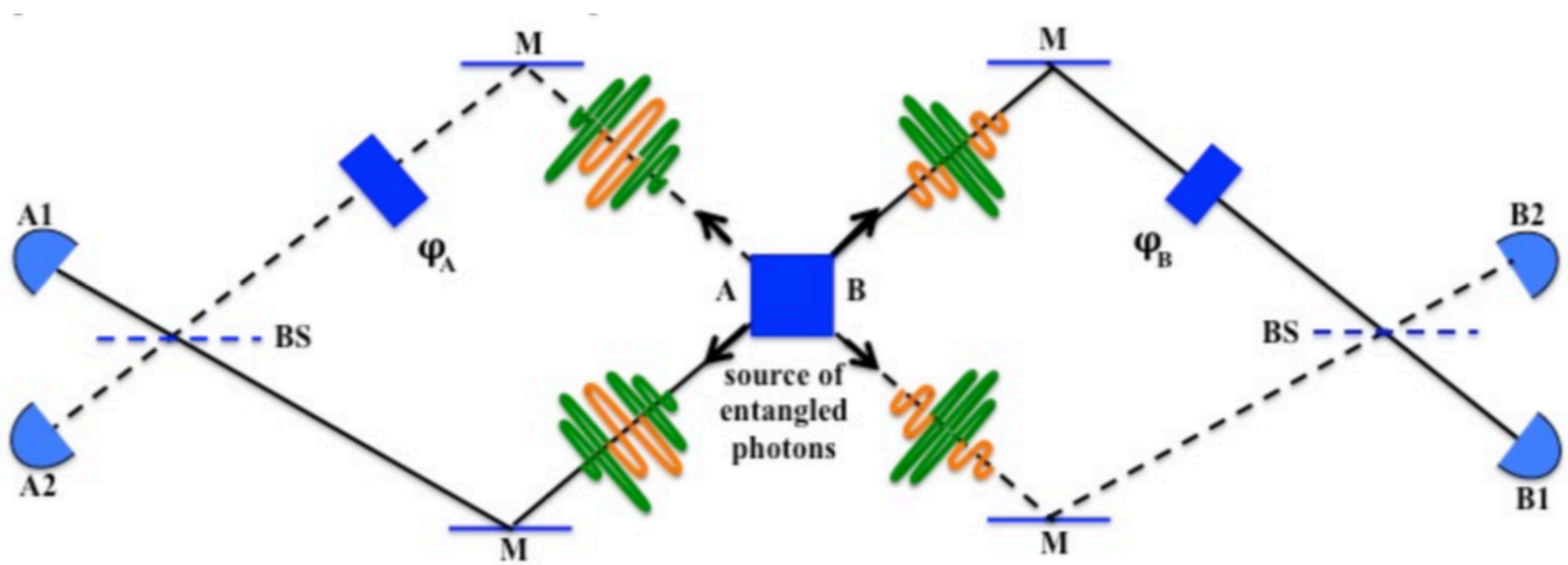
Thus, all nonlocality experiments carried out with pairs of simpler quanta such as photons.

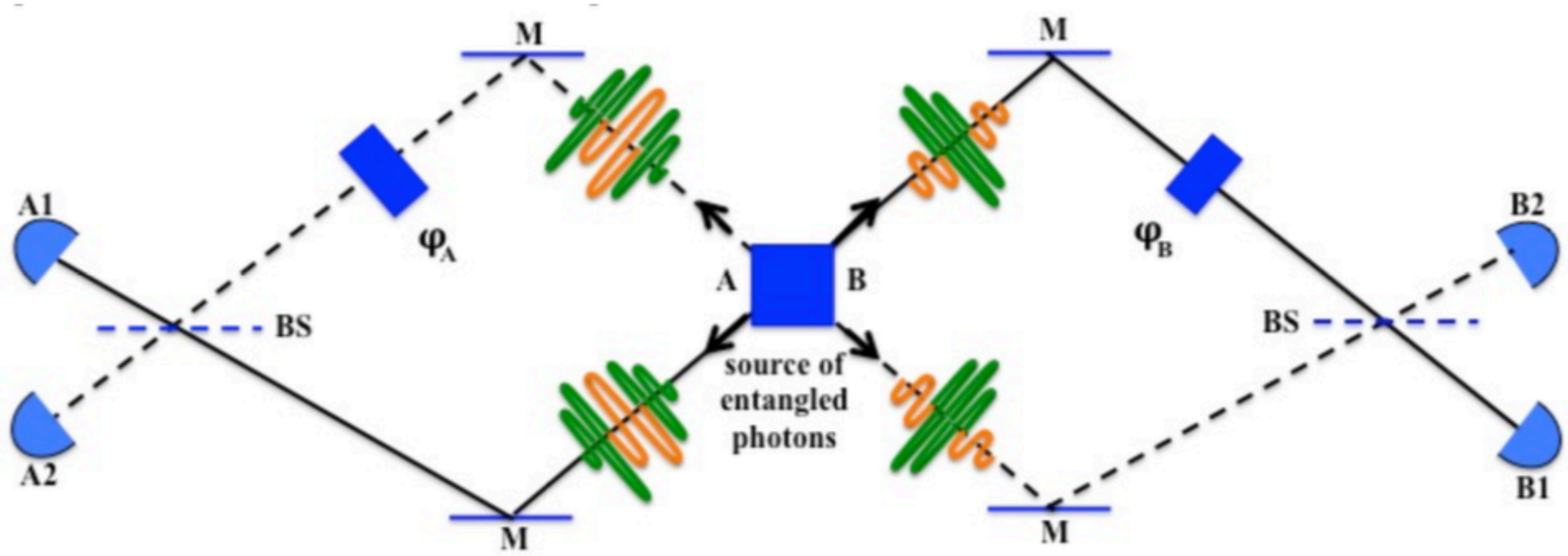
The best nonlocal entanglement experiments

most appropriate for investigating measurement

were conducted nearly simultaneously by Rarity,Tapster and Ou, Zou, Wang (RTO).

Figure shows layout for these "RTO" experiments.





The “Source" creates entangled photon pairs by "parametric down-conversion”, a process which we discussed earlier.

The RTO experiment is **two back-to-back MZ-interferometer experiments** but with first beam splitter for each photon located **inside** the source of entangled photons.

Without entanglement, each single photon (either A or B) would interfere with itself at its own detectors according to own phase shifters ϕ_A or ϕ_B.

In the experiment, two entangled photons are emitted into a superposition of solid paths connecting detectors A1 and B1, and dashed paths connecting detectors A2 and B2.

Remember that the two photons are already entangled when emitted.

Entanglement changes everything.

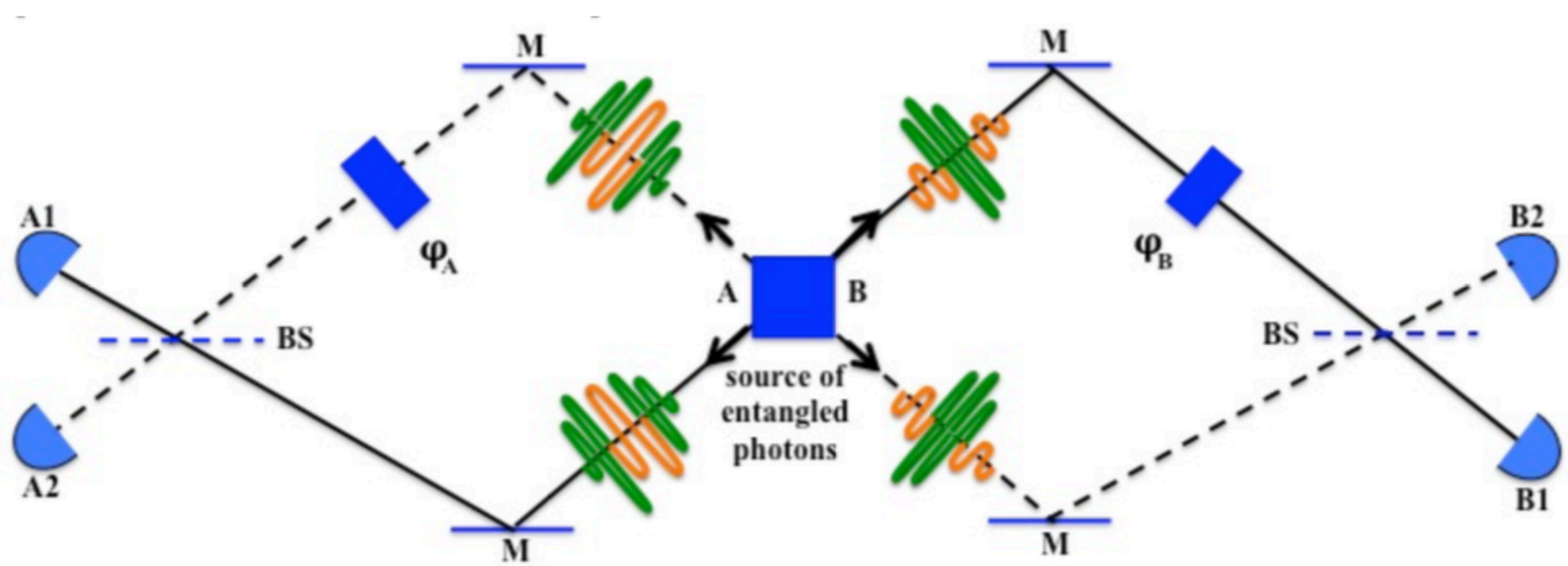
No longer does either photon interfere with itself at own detectors.

Instead, photons are entangled in measurement state (5) with $|\psi_1\rangle$ and $|\psi_2\rangle$

representing (say) solid-line and dashed-line states of A and $|1\rangle$ and $|2\rangle$

representing solid-line and dashed-line states of B,

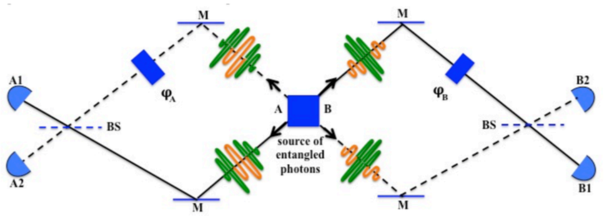
although in RTO experiments **neither** subsystem is macroscopic.



Each photon **now** acts like a which-path detector for the other photon.

Recall the double-slit experiment:

When the which-slit detector is switched on, the pattern on screen switches abruptly from a striped interference pattern indicating the pure state nature of each electron across both slits, to phase-independent sum of two non-interfering single-slit patterns.



Entanglement between the electron and the which-slit detector breaks the pure state into two single-slit parts, so that measured electron comes through either slit 1 or slit 2.

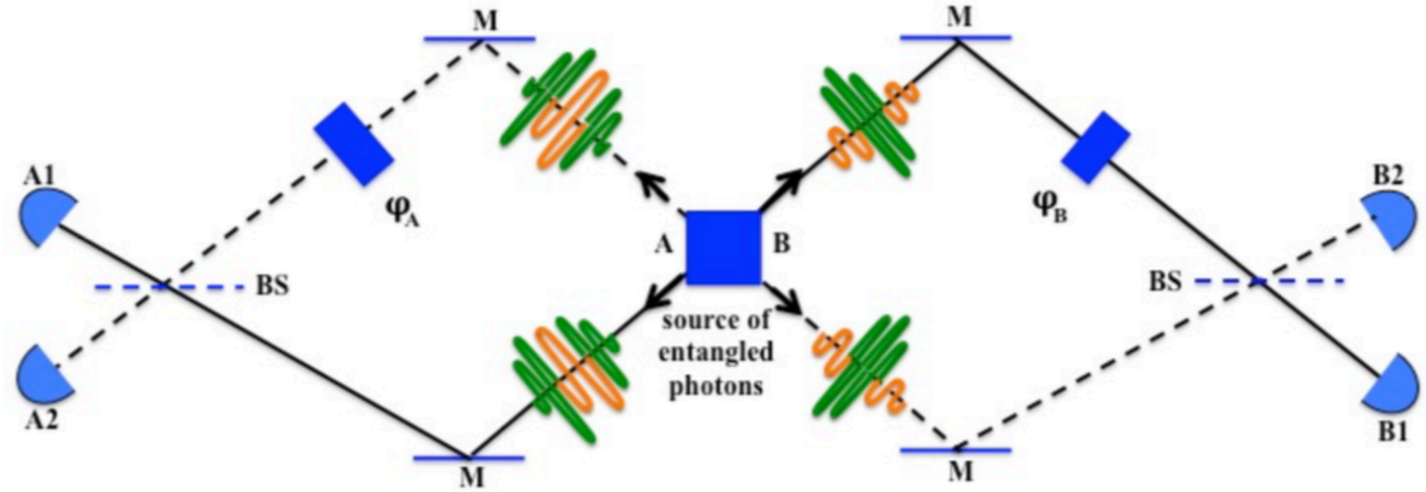
This suggests that in the RTO experiment, the entanglement should break the pure-state superposition (3) into two non-interfering parts.

This is exactly what is observed.

Both photons impact their detectors as random 50-50 mixtures, just like a flipped coin.

Entanglement **breaks** the single-photon pure state (3) observed in the Mach-Zehnder experiment, causing each photon to behave "incoherently" with no dependence on its phase setting.

But (5) is pure state.



Where has the phase dependence gone?

The **answer** lies in the phase-dependent but nonlocal relationship observed **between** the solid and the dashed branches.

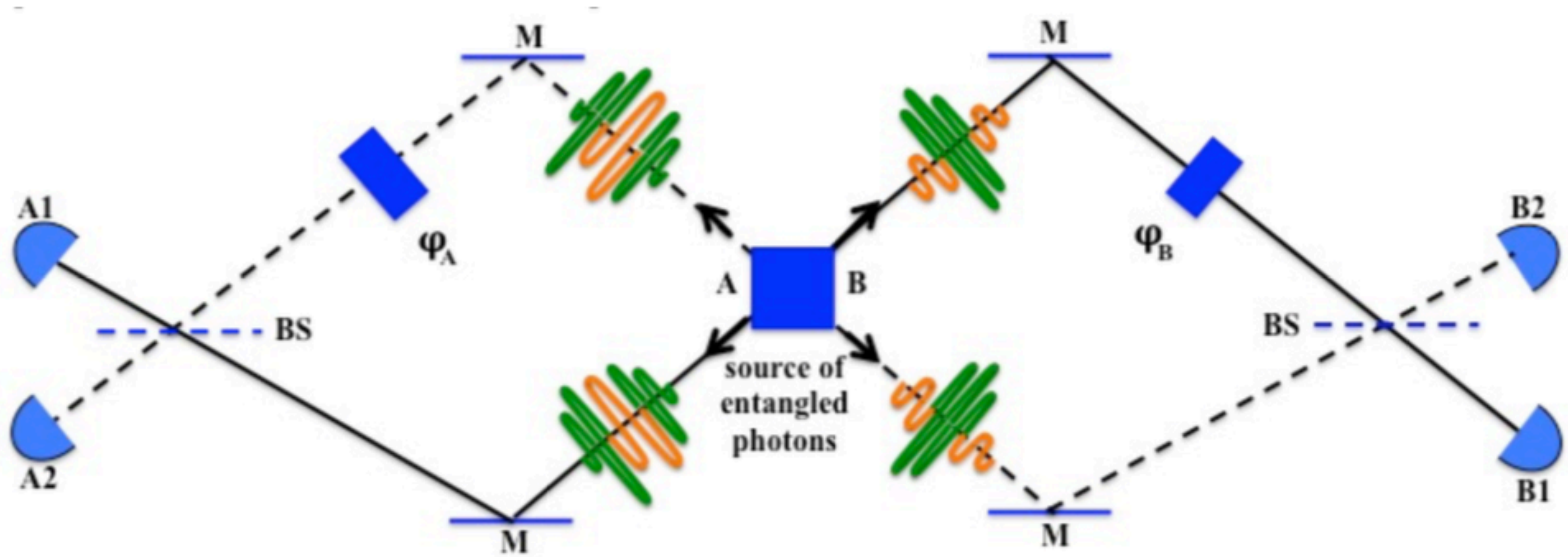
This phase dependence is observed experimentally in coincidence (or **correlation**) measurements comparing the detections of entangled pairs.

The "flipped coins" mentioned above turn out to be correlated with each other.

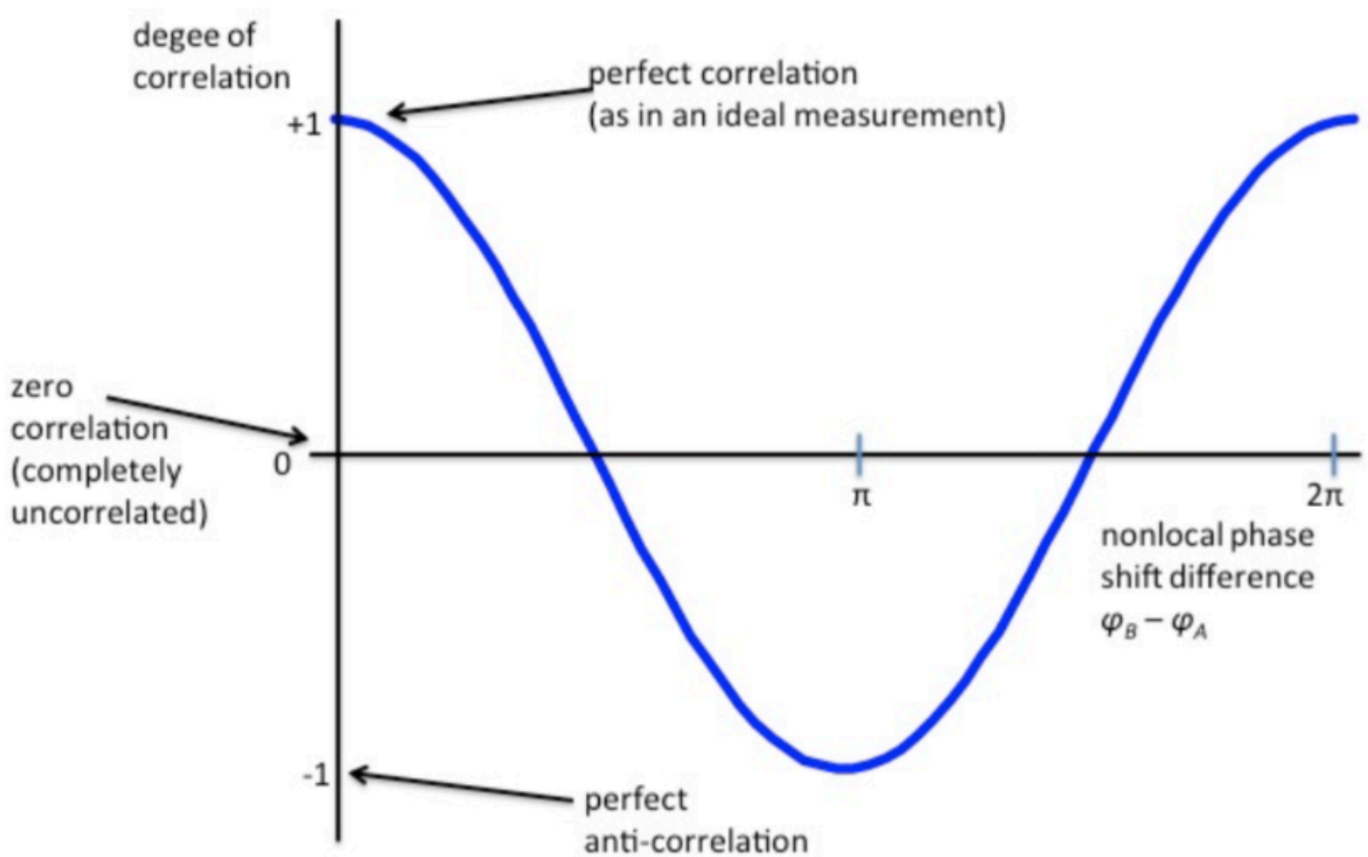
This phase dependence across two separated subsystems is **essential to preserve** unity of (now entangled) quantum.

This is **not** an easy experiment to perform:

The source creates a stream of photon pairs, and one must compare the impact of single photon A at detectors A1, A2 with impact of corresponding entangled photon B at detectors B1, B2.

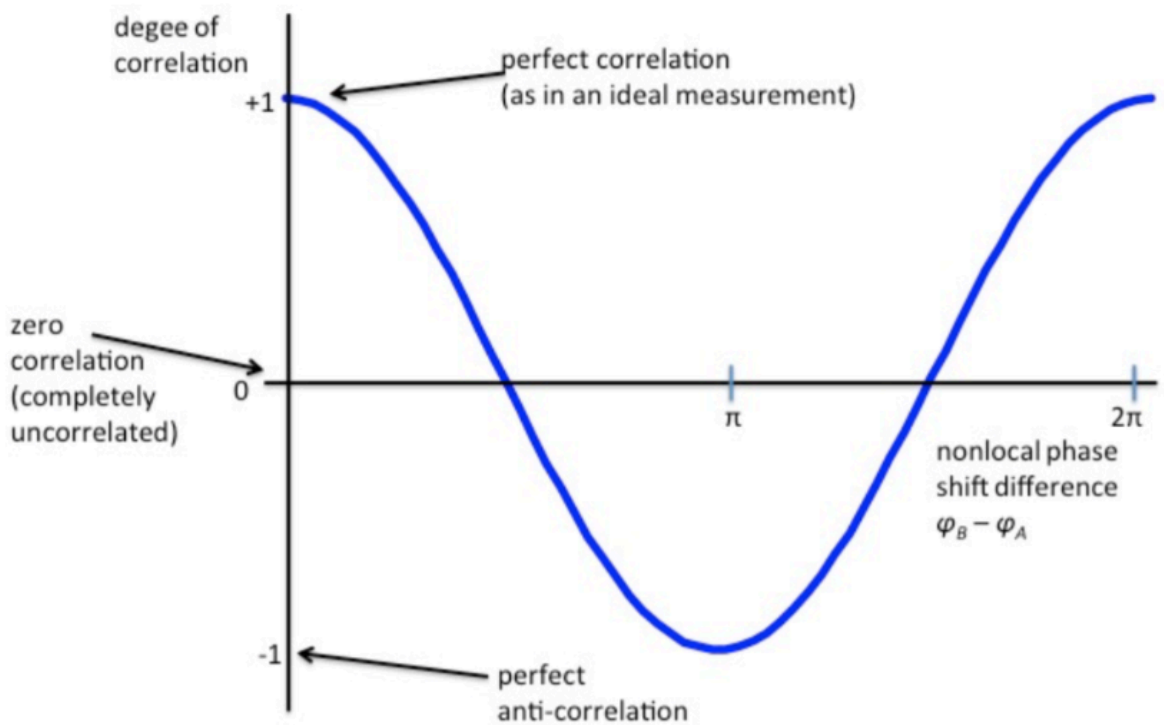


RTO figured out how to do this, with result shown in Figure.



The figure graphs the **degree of correlation** between A and B.

This is a **measure of the agreement** between the outcomes at A's detectors and B's detectors.



A correlation of +1 means perfect, or 100%, agreement:

Either both sets of detectors register outcome 1 (i.e., A1 and B1 click)

or both register outcome 2.

The opposite extreme is a correlation of -1, meaning 100% disagreement: If one detector registers 1, the other registers 2.

Either correlation, +1 or -1, implies that **either** photon's outcome is **predictable** from the other photon's outcome.

A correlation of zero means one photon's outcome does not at all determine other's outcome: Each photon has random 50-50 chance of either outcome regardless of the other photon.

Correlations between 0 and +1 mean the outcomes are more likely to agree than to disagree,
with larger correlations denoting higher probability of agreement;
for example, correlation of +0.5 means 75% probability of agreement.

Similarly, correlations between 0 and -1 mean outcomes are more likely to disagree than to agree; a correlation of -0.5 means a 75% probability of disagreement.

Every experimental result in the RTO experiment agrees exactly with the predictions of standard quantum physics.

QM does not have a problem - observers holding on to old ideas and words do!

When accounting is made of optical paths for both photons, QM obtains the following result:

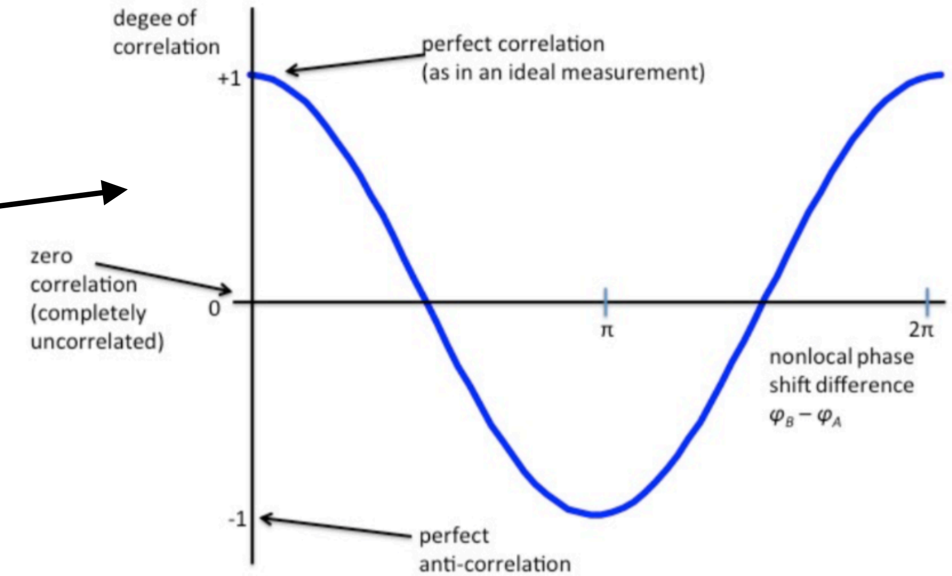
$$P(correlated) = P(A1\ and\ B1) + P(A2\ and\ B2) = \frac{1}{2}[1 + \cos(\varphi_B - \varphi_A)]$$

$$P(anticorrelated) = P(A1\ and\ B2) + P(A2\ and\ B1) = \frac{1}{2}[1 - \cos(\varphi_B - \varphi_A)]$$

where P(correlated) is single-trial probability that A's and B's detectors will agree,

and P(anticorrelated) is a single-trial probability that A's and B's detectors will disagree.

The degree of correlation, defined as
P(correlated) – P(anticorrelated),
is then simply $\cos(\phi_B - \phi_A)$, as graphed in last figure.



In 1964, John Bell, as we discussed earlier, published a ground-breaking article stating the sufficient condition for a statistical theory such as quantum physics to meet a condition known as "locality."

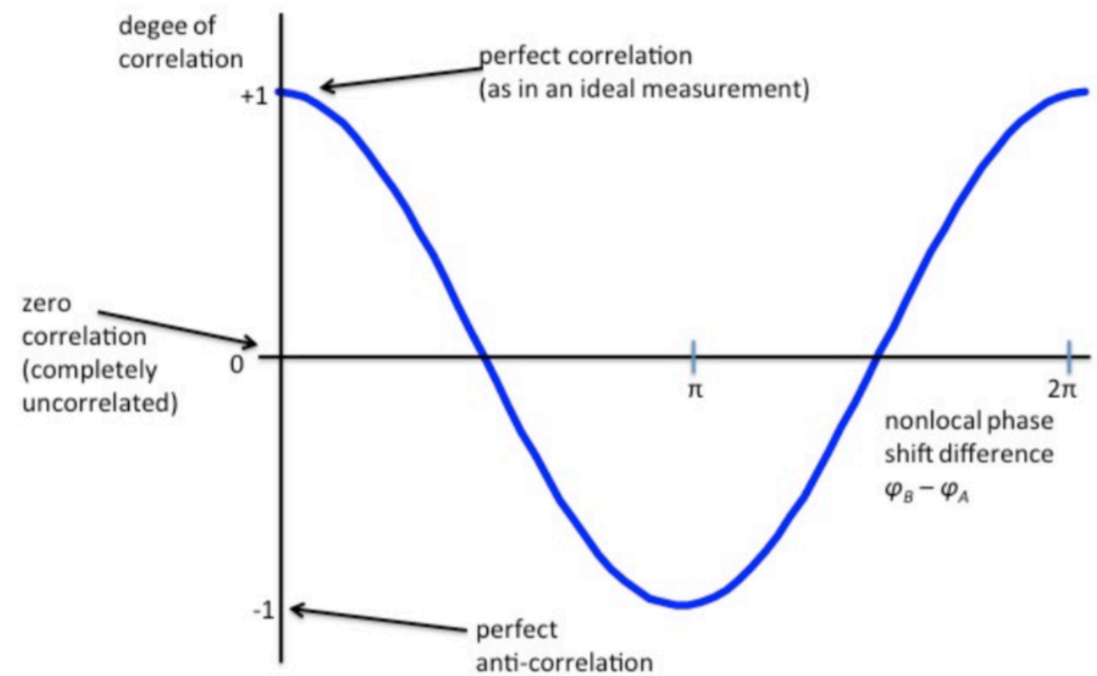
He defined locality to mean **"that the result of measurement on one system be unaffected by operations on distant system with which it has interacted in past"**.

Bell expressed a sufficient condition in the form of inequality that any local theory must obey.

He demonstrated that certain statistical predictions of quantum physics violate Bell's inequality, i.e., quantum physics makes nonlocal predictions.

The results in last figure implies the case in point: the figure violates Bell's inequality at all phase differences $\phi_B - \phi_A$ other than 0, π, and 2π.

Let me **underline** meaning of this:

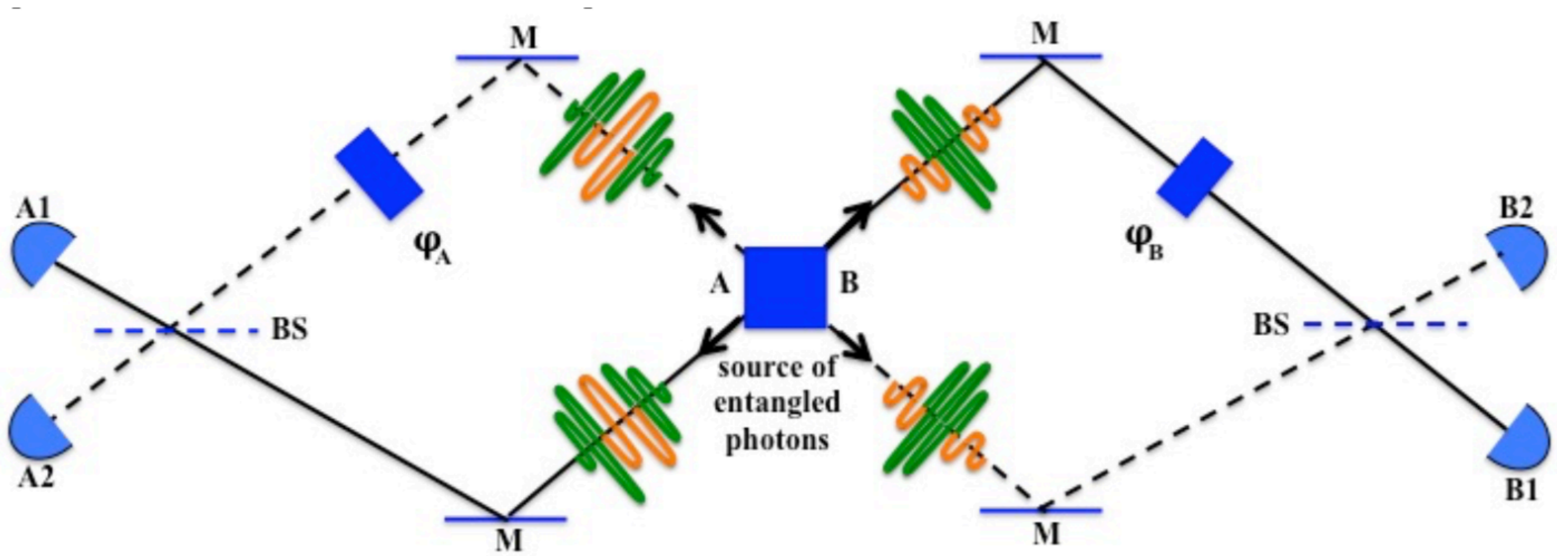


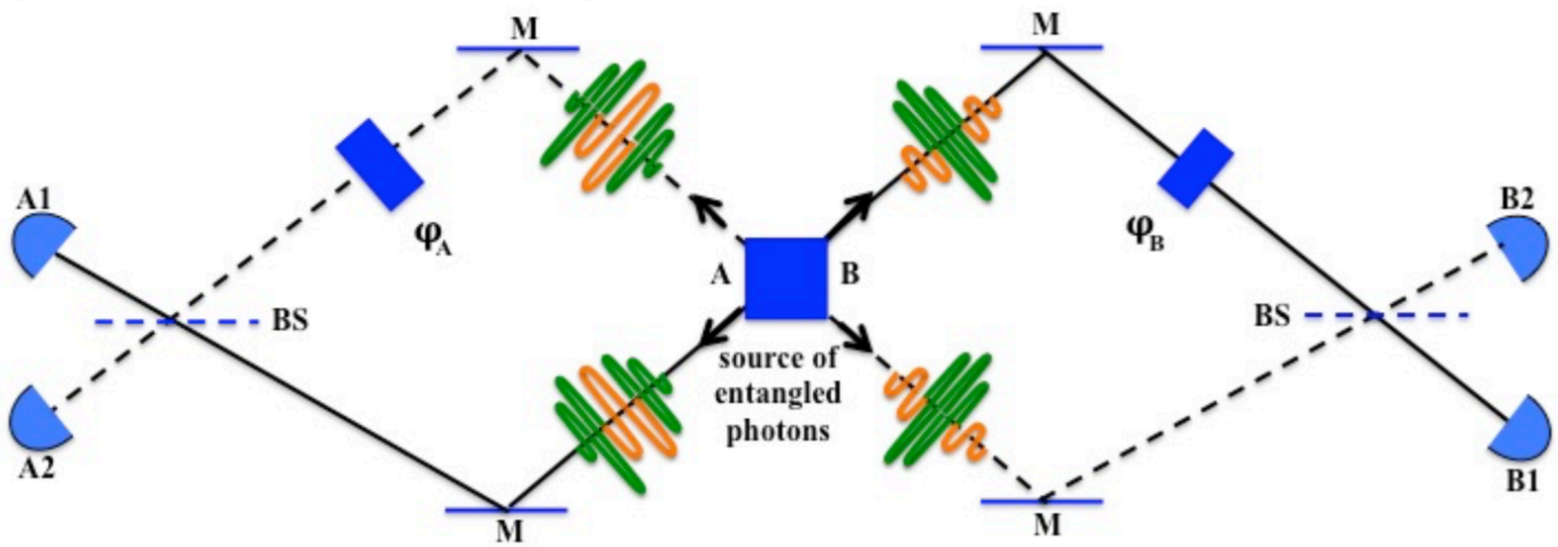
Violation of Bell's inequality means that the statistics of the measurements on photon A - photon A's **"statistical behavior"** - is necessarily affected by the setting of photon B's phase shifter.

In fact, even without Bell's condition, the nonlocality of the experiment is intuitively obvious.

Here's why:

Suppose we set the phase shifters to zero and that all four optical paths (two solid, two dashed) in are then equal; thus $\phi_B - \phi_A$ is zero.





Without the two beam splitters BS,

the two photons emitted into the solid pair and the dashed pairs of paths

would impact either detectors A1 and B1 or A2 and B2

because of the symmetry of experiment and conservation of momentum.

This is neither surprising nor nonlocal,

and would happen even if the photons were not entangled.

But the beam splitter is a **randomizing** device

that mixes the solid and dashed paths;

—> any photon passing through it has 50-50 chance of reflection or transmission.

With non-entangled photons and both beams splitters in place, there would then be no correlation between photon A's outcome and B's outcome because the two photons are independent of each other.

With entanglement, the correlation is perfect.

How does one photon know which path the other photon took at the other photon's beam splitter? **again a poor question!!**

Each photon is now "detecting" the quantum state of the other photon, from a distance that could be large.

Perfect correlation certainly "feels" nonlocal even though (as mentioned above) this perfect correlation at $\phi_B - \phi_A = 0$ does not violate Bell's inequality.

Note that such a violation is a sufficient but not a necessary condition for nonlocality.

Non-locality is **written** all over the RTO experiment.

Each photon "knows" which direction the other photon takes at its beam splitter and adjusts its selection accordingly.

The key nonlocal feature of graph, which is simply a cosine function, has (ϕ_B – ϕ_A) as its independent variable.

Thus any desired shift in correlations can be made by an observer at either of
possibly-widely-separated phase shifters.

Bell suspected that this kind of situation meant that observer A (call her Alice)
could use her phase shifter to alter outcomes that would have occurred at both her own and
observer B’s (call him Bob) detector and, following up on this hypothesis, he derived his
inequality involving probabilities at both Alice’s and Bob’s detectors
which, **if violated**, implied that both photons must have readjusted their states.

Such a readjustment is just what we expect,
given the **unity** of the quantum
and thus the unity of atoms and other entangled systems such as our two photons.

The two photons form a single **“bi-quantum", an "atom of light”**, in the pure state (5).

When Alice varies her phase shifter, both photons "know" both path lengths and **readjust** their behavior accordingly to produce the proper correlations.

Analogously, a single photon "knows" both path lengths in single-photon interferometer experiment.

Finally, we come to the central question of the discussion:

What is actually superposed in the entangled superposition (5)?

A Mach-Zehnder experiment tests the simple superposition (3), while the RTO experiment tests the entangled superposition (5).

We know what is superposed in Mach-Zehnder, namely quantum states $|\psi_1\rangle$ (path 1) and $|\psi_2\rangle$ (path 2).

This is deduced from the effect that either phase shifter has on both states.

Now consider the RTO experiment.

What is the effect of shifting either phase shifter?

One thing that does not change is the state ("local state" would be a better term, as discussed earlier) of either photon A or photon B:

As we know, both photons remain in 50-50 mixtures regardless of either phase setting.

What does change with variations in either phase shifter is the correlations between A and B.

With $\phi_B - \phi_A = 0$ they have perfect correlation:

Either A1 and B1 (which we denote (11)) or A2 and B2 (denoted (22)).

As we vary either ϕ_B or ϕ_A we obtain non-zero probabilities of anti-correlated individual trials, denoted (12) (outcomes A1 and B2) and (21) (A2 and B1).

When the non-local phase angle difference ($\phi_B - \phi_A$) reaches $\pi/2$, we have zero correlation, and when it reaches π we have perfect anti-correlation.

Table summarizes the crucial points in more detail-meaning will become clearer as we proceed.

Simple superposition:		**Entangled superposition of two sub-systems:**		
φ	State of photon	$\varphi_B - \varphi_A$	State of each photon	Correlation between the two photons
0	100% "1", 0% "2"	0	50-50 "1" or "2"	100% corr, 0% anti
$\pi/4$	71% "1", 29% "2"	$\pi/4$	50-50 "1" or "2"	71% corr, 29% anti
$\pi/2$	50% "1", 50% "2"	$\pi/2$	50-50 "1" or "2"	50% corr, 50% anti
$3\pi/4$	29% "1", 71% "2"	$3\pi/4$	50-50 "1" or "2"	29% corr, 71% anti
π	0% "1", 100% "2"	π	50-50 "1" or "2"	0% corr, 100% anti

Table 1. In a simple superposition, the photon's state varies with phase angle. In an entangled superposition, the relationship between states of the two photons varies, while individual states of both photons are phase-independent (or "mixed").

The column titled "simple superposition" shows how the superposition state of single photon (M-Z) varies from "100% state 1" to "100% state 2" as phase angle between two states varies.

Column titled "entangled superposition of two subsystems" shows that the state of each photon remains unchanged throughout entire range of both phase settings, while nonlocal correlation between the states of two photons varies from "100% correlated" to "zero correlation" and then to "100% anticorrelated" as either of the two local phase angles varies.

So, once again, what is superposed in the RTO experiment?

The hallmark of a superposition is the dependence on phase difference between the objects superposed.

But Table exhibits **no** such phase dependence of the states of the two photons.

Simple superposition:		**Entangled superposition of two sub-systems:**		
φ	State of photon	$\varphi_B - \varphi_A$	State of each photon	Correlation between the two photons
0	100% "1", 0% "2"	0	50-50 "1" or "2"	100% corr, 0% anti
$\pi/4$	71% "1", 29% "2"	$\pi/4$	50-50 "1" or "2"	71% corr, 29% anti
$\pi/2$	50% "1", 50% "2"	$\pi/2$	50-50 "1" or "2"	50% corr, 50% anti
$3\pi/4$	29% "1", 71% "2"	$3\pi/4$	50-50 "1" or "2"	29% corr, 71% anti
π	0% "1", 100% "2"	π	50-50 "1" or "2"	0% corr, 100% anti

Table 1. In a simple superposition, the photon's state varies with phase angle. In an entangled superposition, the relationship between states of the two photons varies, while individual states of both photons are phase-independent (or "mixed").

Each photon remains in an unchanging 50-50 mixture of own "path 1" and "path 2" states

- a situation that is radically at odds with the true superposition of path 1 and path 2 exhibited by the M-Z experiment.

Thus, in the entangled RTO state, **neither** photon is superposed.

We see here the source of the "classical" or non-superposed nature of reduced density operators (Eqs. (10) and (11)), not to mention the non-superposed and hence non-paradoxical nature of Schrodinger's cat.

Examination of the phase-dependence of the measurement state (5), as demonstrated by nonlocality experiments such as the RTO experiment,

reveals the true nature of Schrodinger's cat.

The last column of Table shows us what actually is superposed when two subsystems are entangled in measurement state (5).

Simple superposition:		**Entangled superposition of two sub-systems:**		
φ	State of photon	$\varphi_B - \varphi_A$	State of each photon	Correlation between the two photons
0	100% "1", 0% "2"	0	50-50 "1" or "2"	100% corr, 0% anti
$\pi/4$	71% "1", 29% "2"	$\pi/4$	50-50 "1" or "2"	71% corr, 29% anti
$\pi/2$	50% "1", 50% "2"	$\pi/2$	50-50 "1" or "2"	50% corr, 50% anti
$3\pi/4$	29% "1", 71% "2"	$3\pi/4$	50-50 "1" or "2"	29% corr, 71% anti
π	0% "1", 100% "2"	π	50-50 "1" or "2"	0% corr, 100% anti

Table 1. In a simple superposition, the photon's state varies with phase angle. In an entangled superposition, the relationship between states of the two photons varies, while individual states of both photons are phase-independent (or "mixed").

Since the correlations between two photons vary sinusoidally as the non-local phase angle between the two photons varies, clearly these are correlations between the states of two photons, and not the states themselves, that are interfering.

Entanglement has **shifted** superposition, from states of one photon A (Eq. (3), M-Z) to correlations between photon A and photon B (Eq. (5), RTO).

More learning, More repetition, More Details and Some Tentative Conclusions

We are getting closer……But I have not as yet found the final correct road!

But I perservered in my efforts!

In order to resolve the problem of definite outcomes of measurements, aka Schrodinger's cat, our discussion analyzed the entangled state (5) of a microscopic quantum and its macroscopic measuring apparatus.

This state is a superposition of two composite entities $|\psi_1\rangle|1\rangle$ and $|\psi_2\rangle|2\rangle$, with a phase angle between these entities that can range over 2π radians.

To resolve problem of definite outcomes we must ask:

Precisely what does the composite superposition (5) actually superpose, physically?

In order to understand a simple non-composite superposition (3), we looked at the effect of varying the phase angle between superposed entities $|\psi_1\rangle$ and $|\psi_2\rangle$ in an experimental setting such as M-Z interferometer.

The theoretically predicted and experimentally observed results then made it obvious that the quantum whose state is (3) flows simultaneously along two separate paths described by $|\psi_1\rangle$ and $|\psi_2\rangle$.

To understand superposition (5), one should proceed similarly by studying situations in which the phase angle between superposed entities $|\psi_1\rangle|1\rangle$ and $|\psi_2\rangle|2\rangle$ varies.

Theorists and experimentalists studying the phenomenon of nonlocality have been doing this for decades, but quantum foundations specialists have not particularly noticed how the work is connected with the measurement problem.

In fact, the nonlocal aspects of the state (5) have been studied since Bell's 1964 theoretical paper and Clauser and Freedman's 1972 experiment.

The 1990 experiments of Rarity and Tapster, and of Ou, Zou, Wang, and Mandel, furnish an ideal vehicle for such an analysis and are the central feature of this discussion

One lesson of this analysis is that, in order to understand the measurement problem, one must understand the significance of nonlocality.

This is because the key measurement state (5) that caused Schrodinger and other experts so much concern has nonlocal characteristics.

It must be understood as a superposition of correlations, rather than a superposition of states, but this cannot become apparent until one considers the effect of variations in phase angle between its superposed terms.

Experimental or theoretical studies of such phase variations will have nonlocal ramifications, because such variations are inherently nonlocal!

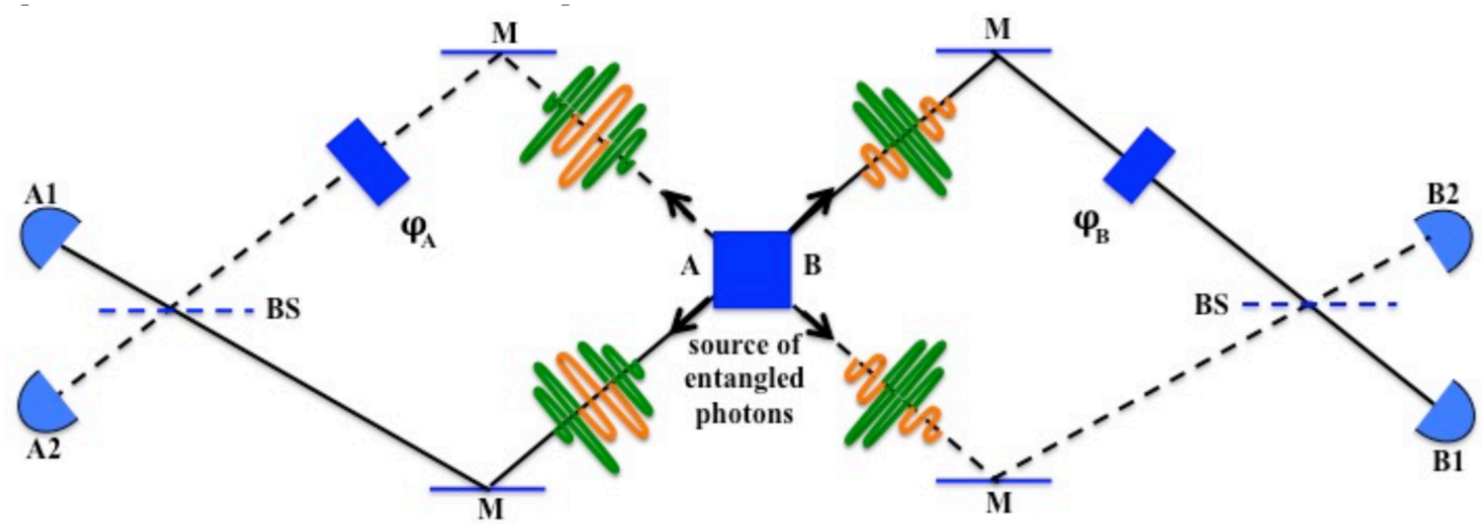
This situation would have prevented Schrodinger in 1935, or indeed anyone prior to Bell's 1964 paper and the experimental confirmations of reality of nonlocality beginning in 1972, from understanding entangled superposition (5).

It's worth emphasizing that, when two subsystems are entangled in measurement state (5), neither subsystem is superposed.

Only the correlations between subsystems are superposed.

In RTO experiments, the two correlations in question are represented by solid and dashed paths connecting pairs of outcomes.

A pair of photons entangled in state (5) follows both of these paths simultaneously.



The subsystems themselves, however, are not in superpositions, but instead, in indeterminate mixtures of definite states.

Thus observers of either subsystem will observe only definite outcomes, as predicted by local mixtures (10) and (11).

RTO experiments are the entangled analog of the M-Z interferometer experiment: a pair of back-to-back interferometer experiments, with entangled pair of quanta of which one quantum passes through each interferometer.

As we said earlier, the experiment and its theoretical analysis shows that, when superposed photon A becomes entangled with second photon B to form state (5), the nonlocal aspect of A's superposition is transferred to correlations between A and B.

Thus, an entangled state such as (5) is neither a superposition of states of A nor of states of B, but is instead a superposition of correlations between states of A and states of B.

To see most clearly, compare simple superposition (3) with entangled superposition (5).

In the simple superposition, the state observed by a "which- state" detector varies smoothly from 100% $|\psi_1\rangle$, through 50% $|\psi_1\rangle$ and 50% $|\psi_2\rangle$, and finally to 100% $|\psi_2\rangle$ as the phase angle ϕ between $|\psi_1\rangle$ and $|\psi_2\rangle$ varies from 0 to π.

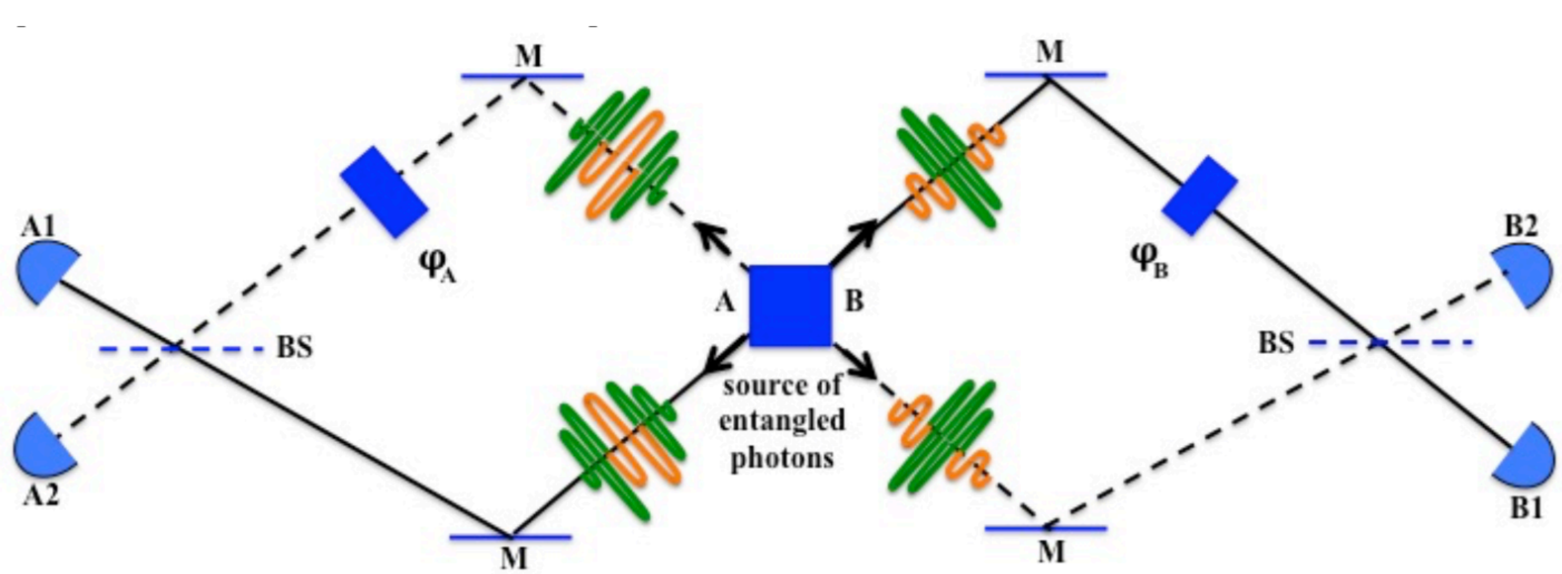
In entangled superposition, neither the state of A nor the state of B varies as ϕ_A or ϕ_B varies;

both A and B remain in 50-50 mixtures throughout.

What does vary is the correlation between A and B.

A non-local "correlation detector" (i.e., an RTO-type of experiment!)
would find the relation between the two subsystems varies from 100% positively correlated (either pair state 11 or 22, pictured by solid and dashed paths in figure), to 50% positively correlated and 50% anti-correlated, and finally to 100% anti-correlated (12 or 21), as the nonlocal phase difference $\phi_B - \phi_A$ varies from 0 to π.

This is a superposition of correlations,
not a superposition of composite states or of non-composite (single-system) states.



At least in idealized case of a minimally-disturbing von Neumann measurement, the initial stage of the measurement process (through formation of the measurement state (5)) can be **described** as follows:

A quantum in a simple superposition such as (3) entangles with a macroscopic which-path detector.

At the instant of entanglement, the local states of both the quantum and the detector undergo a radical change, **a quantum jump**.

Locally, the detector and the quantum jump into mixtures (10) and (11).

Simultaneously, the global state (5) continues evolving smoothly according to the Schrodinger equation.

Entanglement causes the superposed single quantum to be **instantly transformed** into superposed correlations between the quantum and the detector.

This stage of measurement process is **entirely describable** in terms of pure global states following Schrodinger equation.

"Collapse" from the local superposition to the local mixtures occurs because of the formation of the entangled state (5) **and** the resulting formation of subsystems whose local states (Eqs. (10) and (11)) have definite outcomes.

Note that the phenomenon of nonlocality is essential to preserving the pure-state nature (the unity) of the composite system.

To put this more intuitively, a reorganization throughout the entire extent of the composite entangled system is required in order to preserve unity of the (now entangled) quantum.

According to Table 1, when

two systems entangle to form the state (5),

both "collapse" into phase-independent

local mixtures.

Simple superposition:		**Entangled superposition of two sub-systems:**		
φ	State of photon	$\varphi_B - \varphi_A$	State of each photon	Correlation between the two photons
0	100% "1", 0% "2"	0	50-50 "1" or "2"	100% corr, 0% anti
$\pi/4$	71% "1", 29% "2"	$\pi/4$	50-50 "1" or "2"	71% corr, 29% anti
$\pi/2$	50% "1", 50% "2"	$\pi/2$	50-50 "1" or "2"	50% corr, 50% anti
$3\pi/4$	29% "1", 71% "2"	$3\pi/4$	50-50 "1" or "2"	29% corr, 71% anti
π	0% "1", 100% "2"	π	50-50 "1" or "2"	0% corr, 100% anti

Table 1. In a simple superposition, the photon's state varies with phase angle. In an entangled superposition, the relationship between states of the two photons varies, while individual states of both photons are phase-independent (or "mixed").

Relativity **requires** this phase independence:

If any phase-dependent aspect of entangled state were locally observable, instant information-containing messages could be sent, violating special relativity.

Local states of entangled subsystems must be invariant to phase changes.

Thus, only the relationship - the correlations - between A and B, but not A or B themselves, can vary with phase angle.

Since local observers cannot detect these correlations, the entangled state cannot be used to send superluminal signals.

This is, ultimately, the reason Schrodinger's cat must be either alive or dead rather than a superposition of both.

A phase-dependent superposition involving both local states would permit nonlocal signaling, violating relativity.

This conclusion implies that the standard physical description of composite non-entangled (i.e. factorable) product state such as $|\psi 1\rangle|1\rangle$ **has, for a long time, been mistaken**.

Usually many regard $|\psi 1\rangle|1\rangle$ as state of the composite system AB, where subsystem A is in state $|\psi_1\rangle$ and subsystem B is in state |1>.

But this leads us into the **paradox** of Schrodinger's cat, where (5)

$$(|\psi_1\rangle|1\rangle + |\psi_2\rangle|2\rangle)/\sqrt{2}$$

represents a state in which two macroscopically different composite states exist **simultaneously** as a superposition.

According to the present discussion, quantum theory and quantum experiments **imply** this entangled state to be a **superposition** of correlations between states rather than a superposition of composite states.

Thus $|\psi_1\rangle|1\rangle$ is **not** a state of composite system, but **instead** a correlation between two subsystems.

That is, $|\psi_1\rangle|1\rangle$ means **"subsystem A is in state $|\psi_1\rangle$ if and only if subsystem B is in state $|1\rangle$,"** an important departure from usual description.

Even if one of two subsystems happens to be macroscopic detector, entangled state (5) is simply a non-paradoxical superposition of correlations.

It says merely that state $|\psi_1\rangle$ of A is correlated with state $|1\rangle$ of B, and state $|\psi_2\rangle$ of A is correlated with the state $|2\rangle$ of B, with **non-local phase angle** $\phi_B - \phi_A$ determining degree of each correlation.

Regardless of the value of the phase angle, neither subsystem is in a superposition.

The entangled measurement state (5) is best described as a "macroscopic correlation":

a pair of superposed (i.e. phase-dependent) quantum correlations in which one subsystem happens to be macroscopic.

It is technically very difficult to create a macroscopic superposition, but macroscopic which-path detectors routinely achieve state (5).

It’s not paradoxical, even though many physicists have puzzled over it.

In entanglement, nature employs an ingenious tactic.

She must not violate relativistic causality, yet she must be nonlocal in order to maintain the pure-state nature of the original single-quantum superposition over composite objects such as bi-photons.

Thus she accomplishes nonlocality entirely via the superposition of correlations, because correlations cannot be locally detected and thus their superposition cannot violate relativity.

This tactic lies behind the nonlocal spread of phase-dependence over large spatial distances.

By means of the superposition of correlations - entanglement - nature creates a phase-dependent pure-state quantum structure across extended quantum systems such as bi-photons.

I’ve frequently used the term "local" as contrasted with "global."

For composite systems, and especially the entangled measurement state, it’s a crucial distinction.

Entangled states such as (5) have distinct local and global (nonlocal) aspects.

The local description corresponds to two observers, each observing only one subsystem.

In the case of (5), this "local description" is fully captured by the reduced density operators (10) and (11) - **each local observer detects a mixture, not a superposition, of one subsystem.**

The "global description" means the evolving pure state of the entire composite system, in our case Eq. (5).

It is a superposition of nonlocal correlations that can only be detected by observing both subsystems and, via an ensemble of trials that individually record corresponding outcomes at both subsystems, determining the state of the correlations between them.

Although the global state implies the local description, the local description cannot hint at the global correlations because any such hint would violate Einstein causality.

Thus, when an electron shows up in your lab, neither an examination of the electron nor an examination of an ensemble of identically-created electrons can give you the least hint of whether or how this electron is entangled with other quanta elsewhere in the universe.

This clarification of entanglement resolves the problem of definite outcomes, aka the Schrodinger's cat.

An ideal measurement of a superposed microscopic system A by a macroscopic detector B establishes the measurement state (5) at 100% positive correlation.

This state is equivalent to the logical conjunction "A is in local state $|\psi_1\rangle$ if and only if B is in local state $|1\rangle$, AND A is in local state $|\psi_2\rangle$ if and only if B is in local state $|2\rangle$," where AND indicates the superposition.

This conjunction is precisely what we want following a measurement.

Schrodinger's cat is not in the least paradoxical.

Still, this analysis **does not** entirely resolve the quantum measurement problem.

It resolves the problem of definite outcomes associated with the measurement state (5), but this state continues to obey Schrodinger's equation and is hence reversible.

In fact, the entangled state between a quantum and its which-path detector can actually be reversed in the Stern-Gerlach experiment.

In my view, a quantum measurement must result in a macroscopic indication such as a recorded mark, and a mark is **irreversible**.

The above analysis shows the entangled state (5) describes a mixture of definite, not superposed, outcomes of measurements, but these outcomes remain indeterminate and the global state remains reversible.

The irreversibility problem is the question of how this nonlocal superposition of correlations then further "collapses" irreversibly to just one of its possible outcomes, a "collapse" that occurs in the RTO experiment only when one photon impacts a detector.

Present analysis does not seem to resolve this problem.

I think Gambler's Ruin might do it!

In the case of the RTO experiment, however,

it seems fairly clear that the non-local superposition described by Eq. (5)

must irreversibly decohere when either of its subsystems A or B interacts with a detector.

The RTO experiment furnishes a particularly good setting for this question,

because the two photons remain in the reversible entangled state (5)

throughout their flights from the source to detectors,

and thus the two key questions of the measurement problem

(the problem of definite outcomes and the problem of irreversibility)

can be analyzed individually.

End Lecture #7

Let us now **attempt** to resolve the seeming existence of an "**irreversiblity**" problem.

The Environment as Monitor

I hope you are by now convinced that quantum physics describes the microscopic world entirely, consistently and with unparalleled experimental accuracy.

But still, there’s one slight problem:

Quantum physics seems to fail utterly in describing the world around us!

We never see tables or teapots, not to mention ice cream, as wavy, space-filling fields that are possibly here and possibly there and possibly both here and there.

Tables and teapots don't quantum jump, nor does ice cream.

Classical physics may be inaccurate at the microscopic level, but it does seem to explain how such ordinary objects move in response to forces.

If quantum physics describes the microscopic world correctly, and if the macroscopic world is made of microscopic objects, then the quantum principles should lead ultimately to tables and teapots.

Let us now explain, at least in part, **one way that might work**.

As we have seen, quantum physics began with Max Planck's hypothesis that eventually led to the quantum.

The crucial quantum principle is the universe is made of these highly unified, extended **bundles** of field energy.

To the extent that any given phenomenon depends on the spatially extended field nature of quanta, the phenomenon should be considered as "quantum".

To the extent that quanta can be represented by “pointlike” objects, the phenomenon can usually be considered "classical".

But the existence of a classical-quantum boundary is a useful metaphor, not a law of nature.

Most physicists, me included, regard the world as fully quantum(as you will see later).

The clearest expression of the extended field nature of quanta is the principle of superposition.

Because electrons, photons, atoms, and so on are simply disturbances in fields, these objects superpose just as disturbances in the surface of a tub of water superpose.

Several states can all be present at the same time.

We will describe how the process called **decoherence** converts these superpositions into classical-like mixtures later.

The quantum measurement problem must be part and parcel of any discussion of how quantum physics explains our normal world.

After all, quantum measurements - quantum phenomena that cause macroscopic changes - are the bridge from the micro- to the macroworld.

Schrodinger's cat was a dramatic example.

The microscopic decay of a radioactive atom triggers a device that can kill a macroscopic cat.

Schrodinger launched the measurement problem when he noted that the quantum rules seem to imply something we never see: a macroscopic superposition - namely, a cat that is both alive and dead.

Earlier, we suggested what I believe is a resolution of this problem of definite outcomes.

But there's more than that to the measurement problem.

Macroscopic processes are **irreversible**.

The moving finger moves on.

"Things run down".

The second law of thermodynamics demands it.

So quantum measurements must be irreversible, even though there appears to be no trace of irreversibility in the microscopic world.

As we'll see, decoherence **seems** to solve this mystery.

The Problem of Irreversibility

Every measurement involves a macroscopic change of some sort, and such changes must obey the second law of thermodynamics, so entropy increases.

Furthermore, we can observe this entropy increase as a permanent (i.e., irreversible) mark made by the measurement - pointer points, counter clicks, etc…..…

But when a quantum is measured, the quantum and its detector obey the time evolution equation as they **entangle** to form the measurement state, so it seems that entropy doesn't increase.

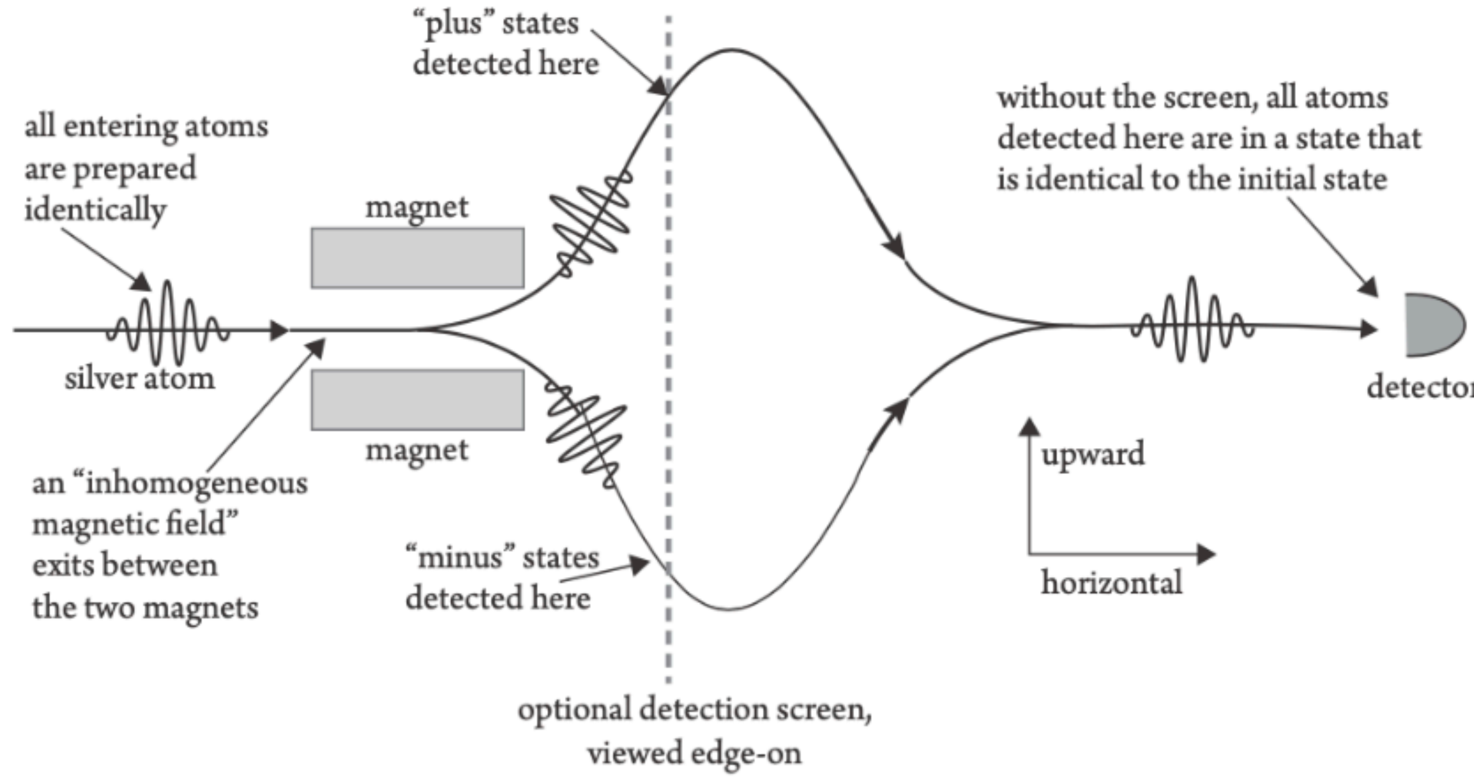
How can we resolve this conundrum?

A variation on the Stern-Gerlach experiment demonstrates the connection between quantum measurements and the second law.

I'll focus only on the features of this experiment relevant to the irreversibility of measurements.

Here's the experiment.

A horizontal stream of silver atoms passes between a pair of magnets (see figure).



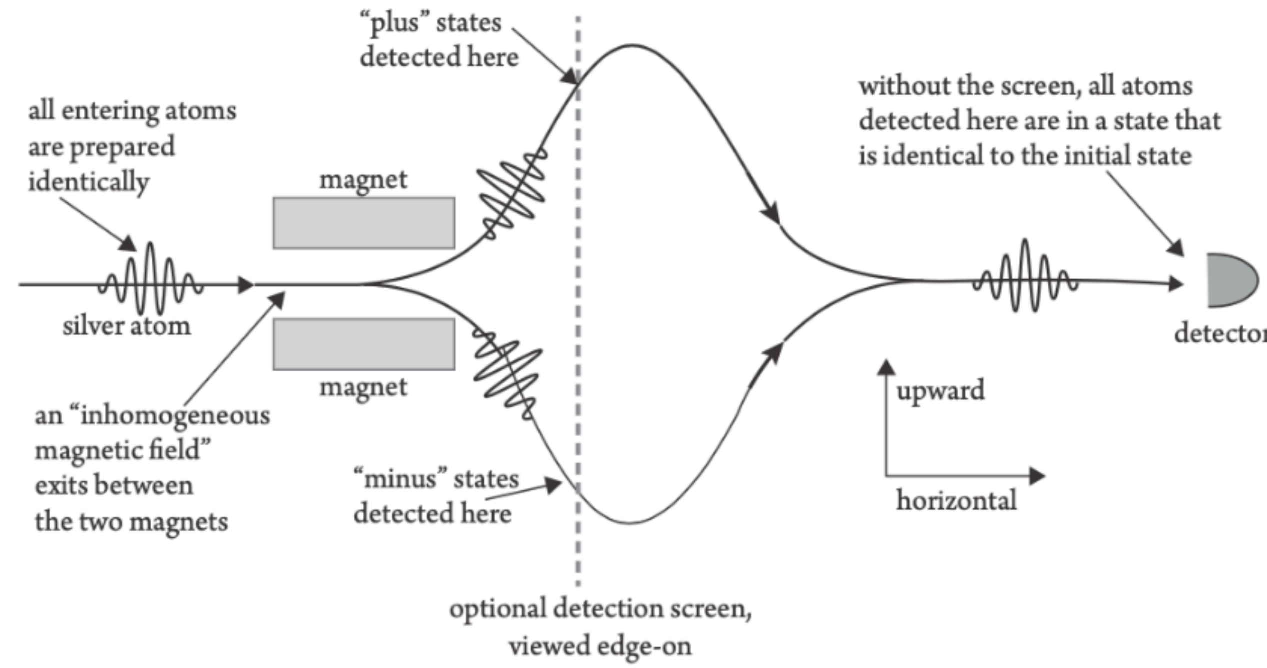
The entering atoms, at the left, have all been prepared previously in identical states that I'll call the "zero state".

There's no need for us to concern ourselves with the precise nature of this initial state.

The magnets are shaped to create a so-called "inhomogeneous magnetic field" in the space between the magnets, as shown in the figure.

This field separates the stream in such a way that **if** a detection screen (dashed line) is placed "downstream" from the magnets as shown, atoms make **visible impacts** on the screen at **two** different spots, one above and the other below the original direction.

Individual atoms impact randomly at one or the other spot with a 50% probability - a striking example of quantum indeterminacy - because the atoms were all prepared identically.



Examination of the two beams shows that atoms striking the upper spot are no longer in the zero state but are in a different state called the "plus state", whereas the atoms striking the lower spot are in a new state called the "minus state".

So the magnet-plus-screen combination acts as a detector to determine which atoms are in the plus state and which are in the minus state. This is entirely analogous to a double-slit experiment with a which-slit detector putting quanta into the "slit 1 state" or the "slit 2 state" before the quanta strike a viewing screen.

There's more, however.

If one removes the detection screen and instead installs some appropriately chosen magnets (not shown in figure) at certain points along both paths, one can bend each stream back onto its original horizontal path, as shown.

Removing screen —> **No** measurements are made now.

On studying the atoms in the converged stream, a perhaps surprising result emerges; none of these atoms are in the plus state and none are in the minus state.

Instead, every one is in the zero state from which it started!

We created a **"do-nothing" box**!

So this experiment, without the detection screen(without the measurement), **is** reversible.

But with the screen, the experiment is obviously **irreversible** because the atoms make an **irreversible** mark when they strike the screen.

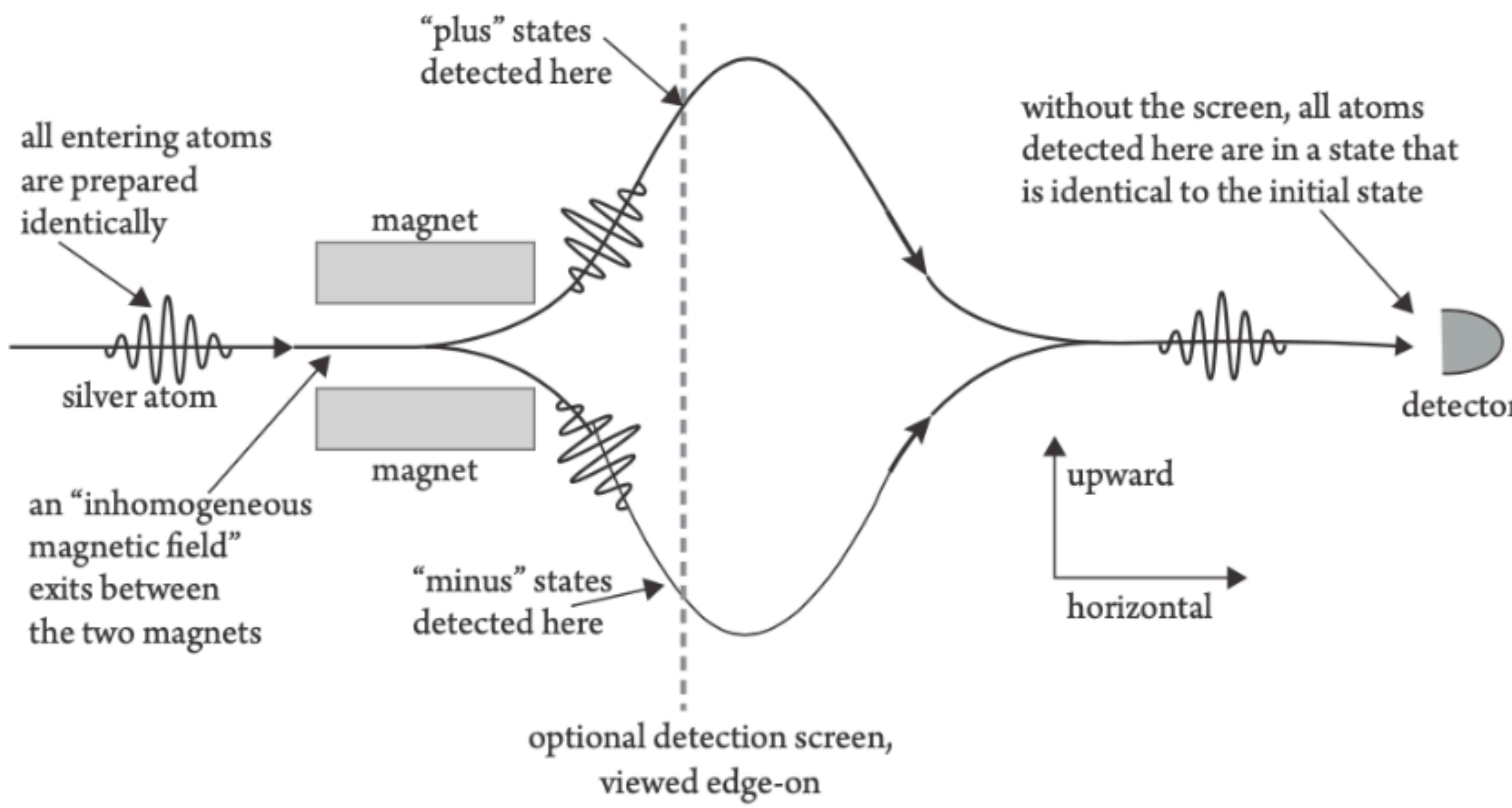
It follows that any irreversibility and corresponding entropy increase in this experiment are **entirely due** to the impact on the screen, because we have seen that, without the screen, the experiment is reversible.

Thus the macroscopic detection makes all the difference, and it is here that we must search for all the irreversible effects of measurement.

Specifically, mere entanglement, such as occurs when a which-slit detector operates in the double-slit experiment, is **not responsible** for irreversibility.

The detection at the screen works as follows:

The detection screen interacts with each atom to form the entangled measurement state: atom is in the plus state, flash appears at the upper spot entangled (superposed); with atom is in the minus state, flash appears at the lower spot



This is the controversial Schrodinger's cat state that we analyzed earlier (the radioactive nucleus is now the silver atom, and the cat is now the screen), where we argued that this state is only a superposition of correlations.

This experiment demonstrates that the macroscopic recording of a measurement is no small detail.

No measurement is complete until it makes a mark on the macroscopic world, and making such a mark must, because of the second law, involve an entropy increase, implying irreversibility.

As John Wheeler repeatedly stressed,

> No elementary quantum phenomenon is a phenomenon until it is brought to a close by an irreversible act of **amplification(so we can see it!!)**

If the measurement is recorded by the audible click of a detector, for example, the irreversible mark is a sound wave spreading out in all directions into the air around the detector.

This sound wave warms the air a little and eventually disperses (vanishes, **for all practical purposes**) into a large volume of air.

There is no way nature is going to spontaneously gather up every last bit of this wave's dispersed energy (while necessarily cooling the air), reverse the entire dispersal process, and use this energy to restore the detector and the air to their states before the click.

In fact, the reversed process is prohibited by the second law because it would reduce the total entropy of the universe!

We conclude that microscopic quantum processes, including entanglement, remain reversible as long as they remain microscopic, and that the irreversibility of measurements must be rooted in the macroscopic recording(amplification) process.

What have we learned:

There is no collapse unless a macroscopic system has made an irreversible mark.

With the screen in place, the system collapses to one value.

Without the screen, the system is unchanged.

We now include these ideas into our thinking!

We now look at a macroscopic recording process that goes on all over the universe all the time.

How Environmental Decoherence Collapses Superpositions

A pebble lies on a sunny beach, immersed in an environment that includes atmospheric molecules, photons from the sun, cosmic rays from stars, and even photons from the Big Bang.

During every second, many such quanta interact with the pebble, reflecting or otherwise scattering off the pebble in every possible direction.

The scattered photons must carry away data about the pebble's orientation, structure, and color, because otherwise the pebble could not be seen.

Such natural "measurement" processes occur all the time, regardless of the presence or absence of humans to consciously observe the gathered data.

To study the quantum features of such a natural measurement, instead of a pebble let’s consider an atmospheric atom that is in a highly nonclassical state of being superposed at two or more macroscopically separated locations.

We know that such a superposition can be created in the laboratory, for instance by passing the atom through a double-slit apparatus, but it could also occur naturally, for example if the atom passes through an opening that is sufficiently narrow to cause the atom to diffract widely.

What happens when such an atom interacts with quanta in the surrounding environment?

The superposed atom interacts with, say, an environmental photon in a manner analogous to the way a superposed electron coming through double slits interacts with a which-path detector.

If the interaction is significantly different at the atom’s two superposed locations, a photon can "measure" the atom by entangling with it in a measurement-like state.

Such an entangling interaction is just like a which-path measurement we discussed earlier!

The scattered photon carries away which-path(location) data about the atom, just as a which-path detector carries away data about an electron coming through the slits.

As discussed earlier(via reduced density operator), this measurement collapses or decoheres (removes, via a series of small environmental interactions, the interference pattern from) the superposed atom, converting its state from a superposition of being at both locations to a local mixture of being **either** at the first location **or** at the second location.

In the natural environment, a single scattered photon is not likely to entirely measure (entirely decohere) the atom, because environmental quanta are not specifically organized, the way a laboratory detector is organized, to measure superpositions.

A which-path detector in a laboratory is carefully constructed to respond in detectably different ways at different locations of the measured quantum, quite unlike environmental quanta, which interact randomly in all kinds of ways.

Although a good which-path detector requires only one interaction to reliably distinguish between - reliably decohere - the superposed states of the detected quantum, a large number of environmental quanta must scatter from a typical superposed atom to completely decohere it and turn it into a local mixture.

Careful analysis shows that decoherence of typical naturally occurring superpositions requires many environmental interactions.

So decoherence by the environment **involves** a series of partial measurements and partial collapses, **each** of them instantaneous, nonlocal, and similar to the single-step measurement that occurs at the slits in a double-slit experiment with a which-path detector.

It's through this environmental decoherence process that everyday objects such as pebbles and this slide lose their extended quantum field nature and behave classically, with no obvious trace of superposition, interference, or nonlocality.

Small objects, of atomic dimensions, are less susceptible to environmental decoherence simply because fewer environmental quanta scatter from them as a result of their smaller size, and because each scattering event can cause only a tiny amount of decoherence because the entanglement involves nearly indistinguishable locations.

Thus, a superposed photon from the Big Bang might travel the universe for 13.8 billion years without decohering, while a grain of sand on Earth, should it happen to show any signs of superposition, is decohered environmentally and essentially instantly because of the myriad environmental interactions it experiences.

Widely extended superpositions of small objects, such as a fine dust grain superposed in two locations separated by a millimeter, are also decohered nearly instantly because the branches of such a superposition are so distinct that a single photon reflecting from one branch but not the other can turn the entire superposition into a mixture.

In a similar way, a single detector at only one of two parallel slits is sufficient to turn a quantum that's initially in a superposition of coming through both slits into a mixture that comes through either one or the other slit.

For mesoscopic objects such as dust grains, environmental decoherence turns superpositions quickly into mixtures.

How quick?

This has been calculated theoretically and measured in a few experiments.

A fine dust grain is some 10^{-5} m across.

If it's in a superposition of being in two places with its two superposed branches separated by this same distance so that the two branches are right next to each other, as though the grain had come through two closely adjacent slits, it would be decohered entirely by normal air on Earth in only 10^{-31} second.

Even the best laboratory vacuum (which still contains plenty of air molecules) would decohere it within 10^{-14} second - a hundredth of a trillionth of a second.

If this grain were in deep outer space, cosmic background radiation from the Big Bang would decohere it in 1 second.

For a smaller object, such as a large molecule with a diameter of 10^{-8} m, these decoherence times are longer (still assuming it's in a superposition with two branches that are separated by the molecule's diameter): 10^{-19} second if the molecule is in normal air, 0.01 second if it's in the best laboratory vacuum, but much longer than the age of the universe if it's in deep space.

The message is that sufficiently small superpositions can survive awhile, but meso- or macroscopic superpositions are fragile and are decohered quickly by tiny environmental interactions.

This is why the "quantum world" is usually identified with the microworld.

When typical quantum features become meso- or macroscopic, they generally vanish quickly.

So the quantum universe appears classical at the macroscopic level because the enveloping quantum environment "monitors" every object constantly, and macroscopic objects are especially susceptible to this decoherence process.

Nature is full of which-branch detectors! Note that humans aren’t required in any of this - no physicists, no laboratories needed.

Nature has been collapsing superpositions, and quanta have been losing their quantum field nature, at least since the Big Bang.

The role of the environment as an ever-present which-path monitor that turns mesoscopic and macroscopic superpositions into mixtures was first clearly recognized by Zurek during the early 1980s.

The work of Zurek, his colleagues, and others has, in large part, explained how the quantum world leads(via reduced density operators) to the classical world of our experience.

A wide variety of experiments have demonstrated environmental decoherence and convinced physicists that decoherence really is the mechanism that converts the quantum world into tables and teapots.

One beautiful example is an experiment by the University of Vienna group under Zeilinger and Arndt.

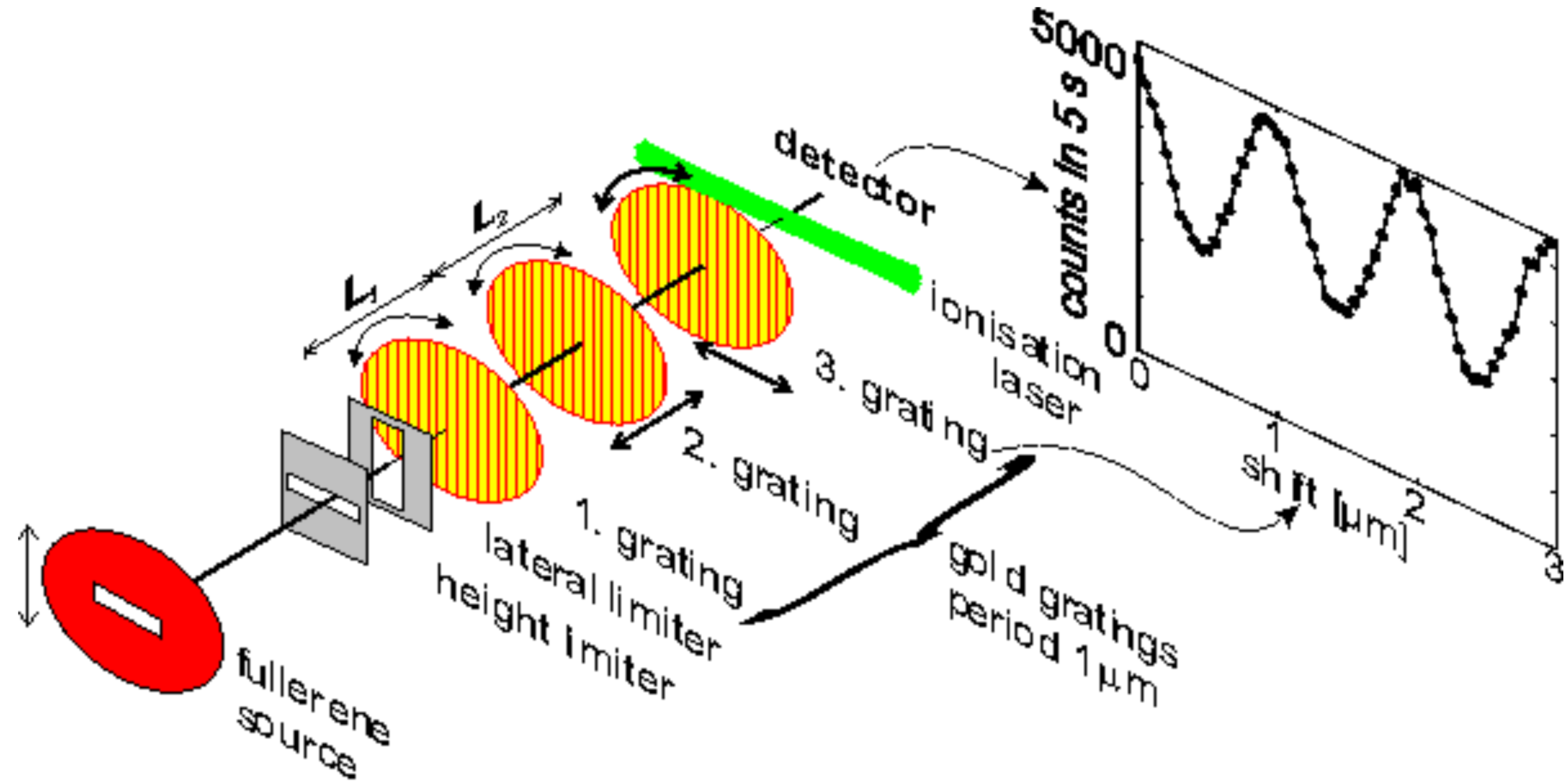
They use the Talbot-Lau interferometer technique (see next slide) that demonstrated, in 2002, interference in large molecules such as C70 and certain biological molecules.

The interference showed each of these large molecules to be in a superposition of following more than one path on its journey through the interferometer.

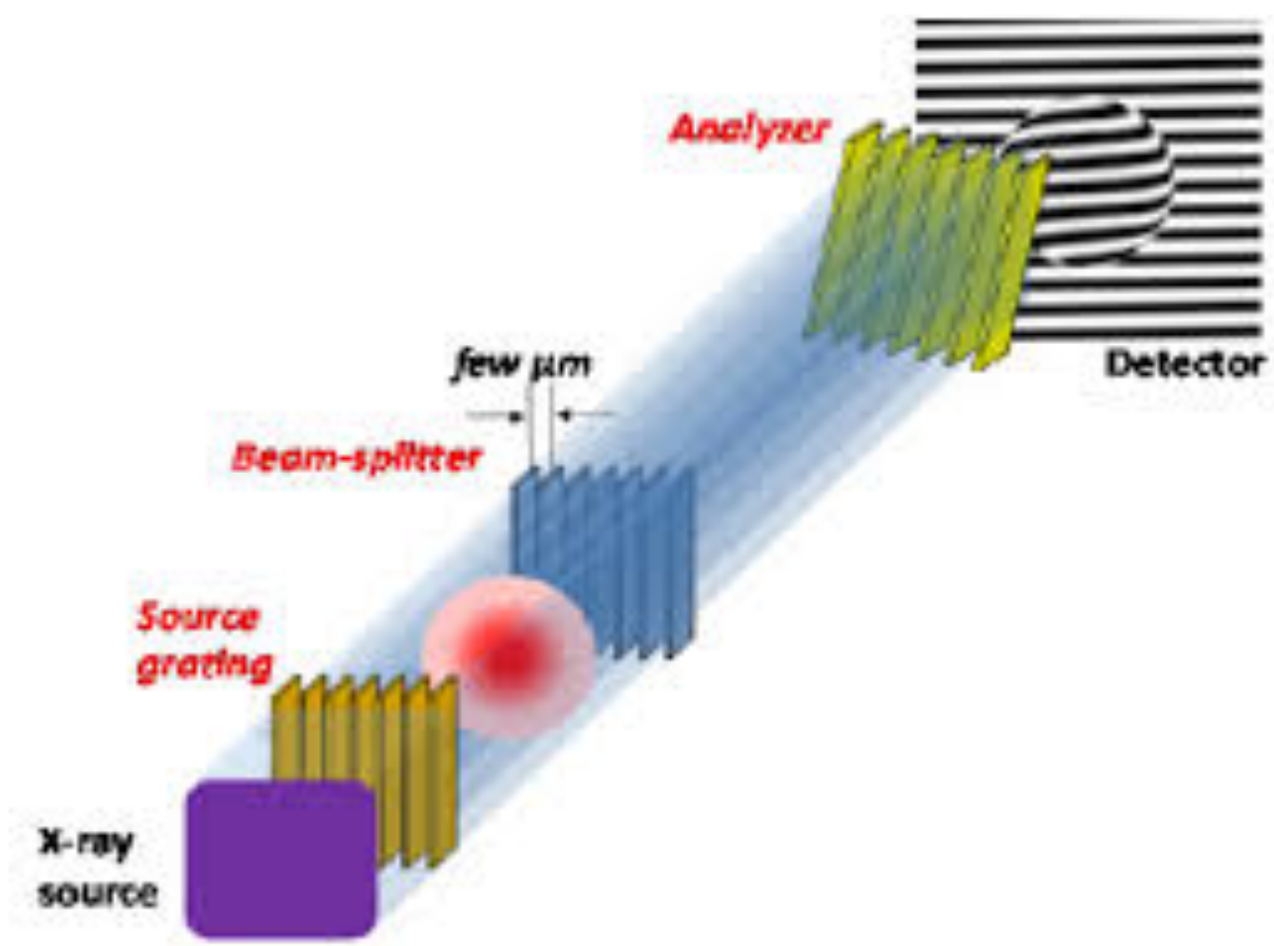
Using the same technique in 2004, this group was able to demonstrate convincingly the "environmental" decoherence of superpositions of C70 molecules.

I put environmental in quotation marks because in this experiment the environment came from within the molecules themselves rather than from an external environment.

Individual C70 molecules passed through the interferometer, as in the 2002 experiment.



5000
counts in 5 s
0
0
1
2
3
shift [µm]
detector
ionisation laser
L_1
L_2
3. grating
2. grating
1. grating
lateral limiter
height limiter
gold gratings period 1 µm
fullerene source



Analyzer
Detector
few µm
Beam-splitter
Source grating
X-ray source

But there was a new twist; the experimenters heated the molecules just before sending them through the interferometer.

They expected that, with sufficient heating, the molecules would themselves emit thermal radiation in the form of visible and infrared "thermal photons", just as an electric hot plate emits thermal radiation (you can feel its warmth at a distance, and it might glow red) when heated.

The radiation comes from the random thermal motion of the many atoms and other quanta within the molecule.

According to decoherence theory, each of these radiated photons should act as a partial which-path detector, collapsing the molecule's superposition state partially by carrying a certain amount of which-path data from the molecule into its surroundings.

If enough such data are transferred, this which-path measurement should decohere the molecule, causing the interference pattern to dim or vanish - the signature of a superposition evolving into a mixture.

Their results demonstrated decoherence in action.

With the molecules only slightly heated, the emitted photons had low energies and thus low frequencies and long wavelengths - too long to distinguish between the possible paths, which for such a massive molecule are separated by extremely small distances.

But when the molecules were heated to a few thousand kelvins, the emitted photons' wavelengths became short enough to distinguish between the possible paths, so the which-path data transmitted to the surroundings was sufficient to partially decohere the superposed molecules, causing the interference pattern to partially vanish.

At sufficiently high temperatures, theory and experiment showed that a mere two or three emitted photons sufficed to decohere each molecule.

There was quantitative agreement between the theory predictions and observations.

The interference pattern began decohering at just the predicted temperature, and the degree of visibility of the remaining interference was just as predicted.

The experiment revealed the step-by-step action of decoherence as data leaked, photon by photon, into the environment.

It also demonstrated the extreme sensitivity of superpositions to decoherence.

Just a few high-temperature photons turn a massive superposed molecule into an incoherent mixture.

Superposed quanta are very delicate.

Decoherence and the Measurement Problem

Decoherence provides a solution of the irreversibility problem for **natural** environmental measurements.

Decoherence was first introduced during the early 1980s to explain how our apparently classical surroundings arise from measurement-like interactions with the environment.

The local state argument, presented earlier, shows that such measurements turn superpositions into entangled nonlocal measurement states with definite outcomes, resolving the problem of definite outcomes for environmental measurements.

But just as for laboratory measurements, we must ask if this also resolves the irreversibility problem associated with **natural** measurements.

The answer is yes.

Here's why.

As we've seen, when a small superposed dust grain is decohered by the surrounding environment, the object undergoes a series of small entanglement-caused partial collapses.

These environmental measurements are "recorded" by the many environmental photons and air molecules that interact with the grain and then disperse widely.

Data about the superposition state of the grain before it decohered are now scattered randomly far and wide.

Just as one cannot unscramble an egg, one cannot reversibly gather these pieces back together and reconstruct the global state of the air plus the superposed grain.

This argument is especially compelling in the case of environmental decoherence, because of the environment's enormous size.

This is an obvious example of the second law in action; **For all practical purposes**, the process is irreversible and entropy has increased.

Because there is famous opposition to introducing **for-all-practical-purposes** arguments into physics, it needs to be noted that the second law itself is inherently a **for-all-practical-purposes** principle.

We know that a box full of gas could evolve spontaneously into separate regions of hot gas and cold gas simply by chance, with no external assistance.

The chances of this are ridiculously small, but they are not zero, and if we consider boxes containing smaller and smaller amounts of gas, these odds increase.

Such **for-all-practical-purposes** arguments that trace back to the second law are entirely in keeping with the principles of physics.

So the local state resolution plus decoherence combine to resolve the measurement problem entirely for the case of natural environmental measurements.

The environment measures a superposed object (molecule, dust grain, and so on) when myriad environmental quanta convert the superposition into a measurement state that now takes the form of a nonlocal entanglement between the object and a very dispersed state of the many environmental quanta.

The local state argument implies this measurement state represents a mixture of definite properties, whereas the highly dispersed nature of the environmental quanta guarantees the process is irreversible.

There has, for years, been a question about what is the exact relationship between decoherence and the solution of the measurement problem.

Some accounts appear to imply that decoherence alone resolves the measurement problem, but this is not true.

The connection of decoherence to the measurement problem is that it resolves the irreversibility problem in the case of natural environmental measurements, and it shows how environmental interactions transform a superposed quantum into the measurement state, but it does not solve the problem of definite outcomes associated with this measurement state.

The problem of definite outcomes is, however, resolved by the local state analysis, as we explained earlier.

So the local state solution combines with environmental decoherence to solve the measurement problem for the case of environmental measurements.

What about the case of laboratory measurements?

Here, the preceding discussion resolved the definite outcomes problem, but not the irreversibility problem.

Irreversibility poses slightly different problems in the two cases, because lab measurements are sufficiently controlled that the natural environment has little effect(by design).

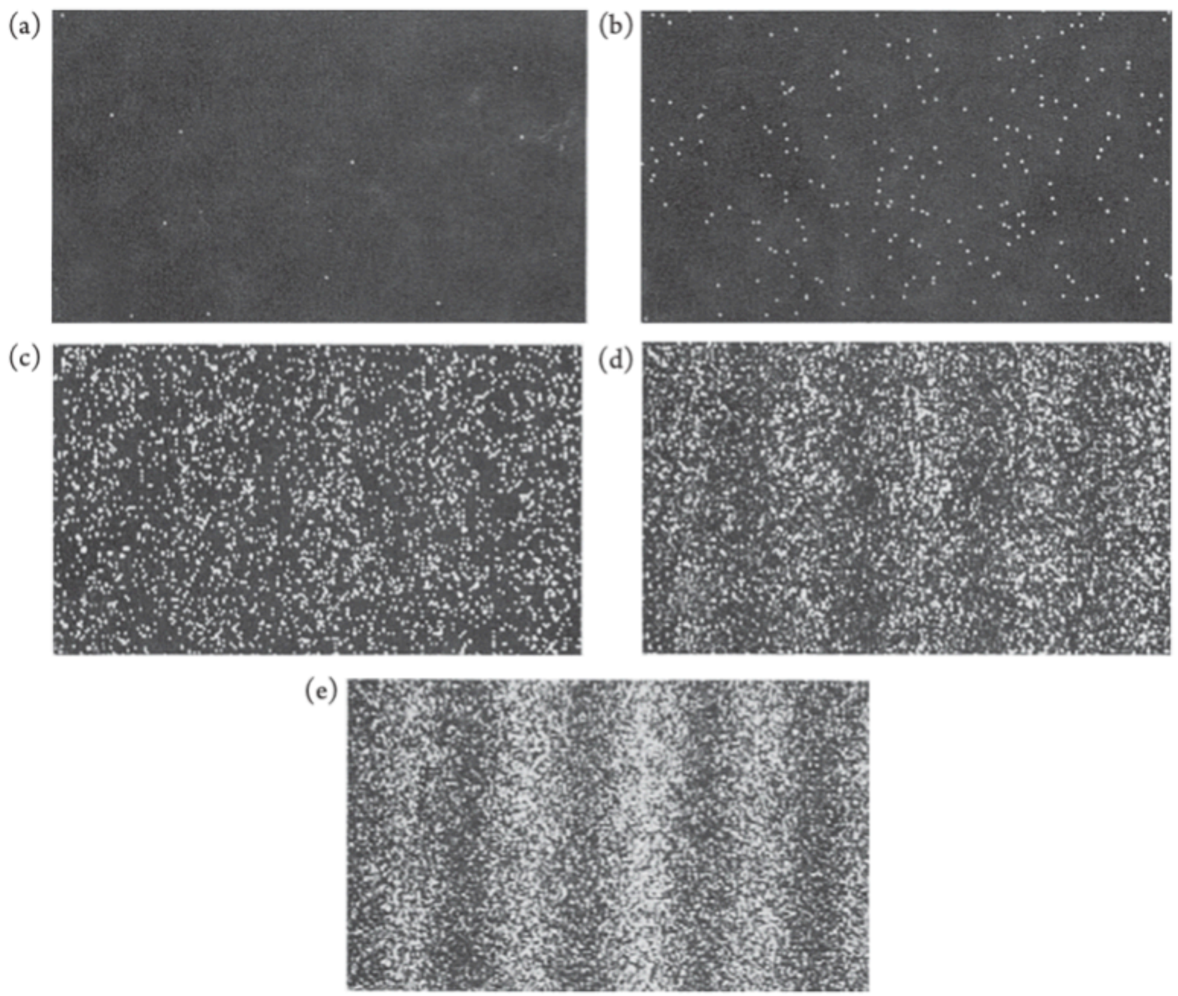
In fact, much of an experimentalist's efforts go precisely into ensuring that random environmental interactions have no significant effect on an experiment's outcome.

Our discussion of the Stern-Gerlach experiment suggested that the answer lies in the irreversible nature of the macroscopic detection process.

This suggestion resembles the resolution of the irreversibility problem for environmental measurements:

Environmental measurements are "recorded" by innumerable environmental quanta whereas lab measurements are recorded by detection screens, electronic clicks, or, perhaps, cats.

Let's focus on a single impact made on a detection screen by one electron in a double-slit experiment - one of the small spots in the figure below:



(a–e) Buildup of an interference pattern in the double-slit experiment using electrons.

As in a regular interference experiment, an interference pattern builds up from individual impacts over longer-time exposures.

How was it recorded?

This mark was initiated by a single electron, but to observe it macroscopically the original impact had to be detected and then **amplified** sufficiently for humans to see it.

This is the purpose of every laboratory detector of microscopic events, and it's why an **"irreversible act of amplification"** (as Wheeler puts it) is essential to quantum measurements.

In the 1989 experiment that produced the patterns of dots on a screen, each electron was emitted in a single coherent state, the same state for each electron.

EM fields accelerated these electrons to high speeds, about 180,000 kilometers per second, or 60% of light speed - much faster than the electron in a normal hydrogen atom, which orbits at an average speed of "only" 2000 kilometers per second.

High energies were needed to make a detectable impact.

Each electron went through a pair of parallel slits, then spread out into an interference state that interacted indeterminately with the screen.

The interaction entangled the electron with the screen, decohering the electron and collapsing it instantaneously from its earlier superposition state over the entire screen to a locally observed mixture(a flash).

In other words, the electron collapsed randomly into one or the other of many small atom-size regions in the screen.

For the fluorescent screen used in the experiment, this interaction created some 500 photons that marked and amplified the location of the impact - already an irreversible process because the photons are emitted randomly in all directions.

In a process called the photoelectric effect, each photon then struck a metal plate and dislodged an electron from it, converting each of the 500 photons into a low-energy electron.

These 500 low-energy electrons were then focused into a point image that could be displayed on a TV monitor in much the same way that old (before flat screens) TV tubes operated.

Every step of this process - creation of 500 photons, conversion to photoelectrons, and amplification to create the final display - creates entropy and is irreversible thermodynamically.

It’s the second law of thermodynamics in action.

This illustrates the general case:

Laboratory detection and amplification of a quantum event necessarily involves irreversibility and entropy production.

The laboratory detector, a macroscopic device made of many microscopic quanta, plays the decohering role that the environment plays in natural measurements.

To summarize:

The quantum measurement problem, in both its laboratory and environmental senses, seems to be resolved entirely by the local state solution of the problem of definite outcomes and by the irreversibility of the decoherence process.

For more than a century, much has been made of the odd and supposedly paradoxical nature of the quantum.

This presumed quantum spookiness has led to an excess of attempted fixes and interpretations.

Many experts have even declared the theory to be not a description of reality at all, but only a mathematical recipe that helps humans predict the results of experiments.

As Niels Bohr put it, **"There is no quantum world. There is only an abstract quantum description."**

According to this hypothesis, quantum theory doesn't describe anything real at all, so there's no cause for concern about collapse of the quantum state and other odd quantum behaviors.

We now collect everything we have proposed together, eliminate all that is unneeded and finally present a straightforward picture of the solution to the quantum measurement process

We will see that the final understanding arises because we looked at so many different approaches, went down so many dead-ends and along the way uncovered many new features of the quantum world.

So, now we will present only what we need with a more detailed analysis and a deeper and, what I believe, is a correct final interpretation.

1 - INTRODUCTION to the FINAL ROAD

Physicists agree that Schrodinger's equation describes the unitary evolution of non-relativistic quantum states between measurements, but there is no agreement on how states change during measurements.

In fact, an apparent problem arises when one applies standard quantum theory (minus the collapse postulate) to measurements.

John von Neumann, in 1932, analyzed the problem and we follow his argument here.

> "Measurement" is the **experimental determination and macroscopic recording** of the value of a physical observable associated with a quantum system.
>
> von Neumann showed that, **unless** the system happens to be in an eigenstate of the measured observable, measurement leads to a **"measurement state" (MS)** whose mathematical representation is an **entangled state** that **seems to predict** the detector to be in a **macroscopic superposition** of exhibiting all of the possible outcomes, a paradox known as the **"problem of definite outcomes."**

We will now show, based on standard quantum theory without a collapse postulate, that this is a **pseudo-problem** and that, far from predicting superposed outcomes, von Neumann's MS actually **predicts an instantaneous collapse** to a single eigenstate.

Specifically, we will demonstrate, with no assumptions other than standard quantum physics (minus the collapse postulate)

(i) The MS has been **misinterpreted** (for 90 years) and **does not** in fact predict paradoxical multiple macroscopic *outcomes;* it instead correctly predicts non-paradoxical multiple ***statistical correlations between*** system and detector outcomes.

(ii) Exactly **one** outcome actually occurs.

(iii) The entanglement says that, when one outcome occurs, the **other outcomes** simultaneously and nonlocally remain "dark" (i.e. **do not occur**).

This **resolves** an objection to quantum physics first raised by Einstein in 1927.

That is, we will show von Neumann's enigmatic MS **to be in fact** the collapsed state expected upon measurement.

The collapse is derived (i.e. demonstrated) with **no assumptions** beyond the other standard principles of quantum physics(i.e., without a collapse postulate!).

We will show the problem of outcomes arises from a **mistaken understanding** of entangled superpositions, **not only** in measurements **but also** in purely microscopic processes.

The MS is obviously a **superposition** of subsystem product states.

But the MS is poorly understood because no previous work has analyzed its complete **phase dependence**. Remember our earlier discussion of state vectors for boxes of electrons!

Do the states of individual subsystems vary with phase, as they do in simple superpositions?

If not, then what does vary with phase, i.e., precisely which entities occupy indefinite states when two or more subsystems are entangled?

Such questions show that we do not fully understand the MS's phase dependence, i.e., we don't fully understand the MS.

We investigate these questions by studying earlier Bell-test quantum-optics experiments that measure momentum-entangled two-photon states.

These experiments study a **purely microscopic** entangled superposition (mathematically **identical** with the MS) **across all phases**.

The results show that entangled superpositions **differ sharply** from simple superpositions, i.e., the implication of the **"plus"** sign differs surprisingly.

The MS **does not** represent multiple detector readings, but instead represents multiple ***statistical correlations between*** system states and detector readings.

These correlations are **not paradoxical**: a macroscopic detector that simultaneously exhibited two states **would be paradoxical**, but a detector that simultaneously participates in two correlations is **not paradoxical**.

Furthermore, the MS **directly implies** that **exactly one** of these correlations is **realized** as the measurement **outcome**.

This **resolves** the so-called "Schrodinger's cat paradox," which in turn **resolves** the quantum measurement problem.

Our discussion follows from quantum theory and from experimental evidence(already discussed) that an instantaneous nonlocal collapse takes place, **resulting in one outcome occurring while the other outcomes simultaneously do not occur**.

Thus, while the present discussion **does not *postulate* collapse**, it ***derives an instantaneous collapse*** as a **consequence** of **entanglement**.

Many papers, and our earlier discussions, express the effect of measurement as a **trace** on the composite-system density operator representing the MS.

This trace operation, which **predicts definite outcomes** at the subsystems, seems to yield just what we want, namely that measurement **transforms** the subsystem states into **mixtures** over the possible eigenstates.

However, **several** objections are commonly raised against this proposal, so we now go into more detail to understand what is the **correct** interpretation that overcomes objections.

This discussion, in contrast to earlier discussions, argues that the MS **directly** represents the collapsed state of the composite system, that the collapse is a **consequence** of entanglement, and that the entanglement is **required** in order to ensure a **simultaneous** (hence instantaneous) collapse over the separated branches of the experiment.

We will **present** von Neumann's derivation of the MS, **pose** the measurement problem, **review** eight presumed measurement problem insolubility proofs, and **explain** why the conclusions cannot be correct.

We then present a **crucial** clue — In 1927, **Einstein** noted that quantum theory **implies** that measurements entail **instantaneous** collapse and suggested this would **violate** Special Relativity.

However, today (unlike 1927) we **know** that **instantaneous** nonlocal changes of **correlations** violating Bell's inequalities really **occur**, that they **do not violate** special relativity, and that they occur when disparate systems are **entangled**.

They have been observed experimentally!

Thus, from today's perspective, Einstein's **argument** implies that **nonlocal correlations** are **required** during measurements and also saying that **entanglement** is also **required**.

Is von Neumann's MS in fact precisely what we **want**?

The answer can only come from **fully understanding** the MS, particularly its **full phase dependence**.

To this end, we will review two 1990 quantum optics experiments exploring an entangled two-photon microscopic state that is **mathematically identical** to the MS, that we discussed earlier.

The **correct** understanding of the microscopic version of the MS will then be worked out.

We **find** that both entangled sub-systems are in **definite** (i.e. non-superposed) states, but that the ***degree of correlation between these states is in an indefinite state***.

We will then apply this new **insight** to the MS.

We will find the MS is **not** a macroscopic superposition of different detector *states*.

It instead **represents** *different correlations between* detector states and system states.

This is **not paradoxical**.

Furthermore, quantum theory directly implies that precisely **one outcome is realized**.

This **resolves** the Schrodinger's cat paradox.

We will then conclude the analysis by studying a particularly **simple measurement example** that also typifies the **essentials** of quantum measurements.

Finally we will summarize the results and consider **why it has taken so long to overcome this simple misunderstanding of von Neumann's MS**.

2 - ENTANGLEMENT AND MEASUREMENT

The superposition postulate entails that, if $|A1\rangle$ and $|A2\rangle$ are Hilbert space vectors ("kets") representing possible states of a quantum system *A*, then all normalized linear superpositions of $|A1\rangle$ and $|A2\rangle$ also represent possible states of *A*. For example,

$$|\Psi_A\rangle = \frac{1}{\sqrt{2}}(|A1\rangle + |A2\rangle) \tag{1}$$

You should again make a copy of all the new numbered equations for reference as I lecture

($|A1\rangle$ and $|A2\rangle$ are orthonormal) represents a possible state of *A*.

The superposition postulate is prerequisite to the Hilbert-space representation of quantum states, and the basis for conceptualizing quantum states as physically real waves in a quantum field that fills the universe.

$|\Psi_A\rangle$ represents a situation in which *A* is represented neither by $|A1\rangle$ nor by $|A2\rangle$ but incorporates aspects of both, including "overlap" effects such as interference.

As **Dirac** put it, *A* goes "partly into each of the two components" and "then interferes only with itself."

If quantum system *A* interacts with another quantum system *B*, it frequently happens that the situation of *A* and *B* are then represented by an entangled superposition state vector such as

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|A1\rangle\,|B1\rangle + |A2\rangle\,|B2\rangle) \tag{2}$$

where $|Ai\rangle$ and $|Bi\rangle$ (*i = 1, 2*) are orthonormal kets representing the "subsystems" *A* and *B*, respectively.

Although the physical interpretation of simple superpositions such as $|\Psi_A\rangle$ is clear, the physical interpretation of entangled superpositions such as $|\Psi_{AB}\rangle$ is not comparably clear.

$|\Psi_{AB}\rangle$ is a superposition of two products $|A_i\rangle\,|B_i\rangle$ (*i =1, 2*). $|A_1\rangle\,|B_1\rangle$ is commonly interpreted to represent a state of the composite system *AB* in which *A* has the properties associated with $|A1\rangle$ and *B* has the properties associated with $|B1\rangle$, and similarly for $|A2\rangle\,|B2\rangle$.

But if this is the case, then the physical interpretation of the state $|\Psi_{AB}\rangle$ would seem to be that *AB* simultaneously exhibits properties associated with $|A1\rangle$ and $|B1\rangle$ AND properties associated with $|A2\rangle$ and $|B2\rangle$, where “AND” represents the superposition.

In the case of Schrodinger's iconic cat, this would imply that the nucleus is both decayed and undecayed and, more disturbingly, the cat is both alive and dead.

Here we will demonstrate that both quantum experiment and quantum theory show that this is actually *not* the case.

Instead, the state $|\Psi_{AB}\rangle$ entails merely that $|A1\rangle$ and $|B1\rangle$ are *coherently* (in a phase-dependent manner) *correlated* with each other AND $|A2\rangle$ and $|B2\rangle$ are coherently correlated with each other (see later discussion).

This is not paradoxical in any way.

Quantum measurements are important examples of entanglement.

As first discussed by John von Neumann, a "measurement" is the determination of the value of an observable associated with a quantum system A.

If A happens to be represented by an eigenvector of the measured observable, then a good measurement will detect the associated eigenvalue.

But what if A is represented by a superposition of eigenvectors of the measured observable?

It will suffice for this discussion's purpose to assume that *A*'s Hilbert space has only two dimensions, and that *A* is represented by the superposition Equation (1).

The kets $|Ai\rangle$ *(i =1, 2)* define the eigenvectors of the measured observable.

We assume the existence of a **detector** *B* designed to **distinguish** between the $|Ai\rangle$.

For example, $|A1\rangle$ and $|A2\rangle$ could represent the paths of an electron passing through the slits of a double-slit apparatus, and *B* could be an electron detector for the "which-slit" observable whose eigenvectors are the $|Ai\rangle$.

To make a which-slit measurement, *B* must distinguish between the states represented by $|A1\rangle$ and $|A2\rangle$, so *B* must contain a specific quantum detection component with quantum states represented by kets $|Bi\rangle$ such that, if *A* is in the state represented by $|Ai\rangle$, then detection yields the state represented by $|Bi\rangle$*(i = 1 or 2)*.

Assuming a minimally-disturbing measurement that leaves eigenstates unaltered, and letting $|B_{ready}\rangle$ represent the state of *B*'s quantum component prior to measurement, the process

$$|Ai\rangle\,|B_{ready}\rangle \Rightarrow |Ai\rangle\,|Bi\rangle \quad (i = 1, 2) \tag{3}$$

describes a measurement of the which-slit observable when *A*'s state is represented by an eigenstate.

When *A* is in the state represented by $|\Psi_A\rangle$ and *B* measures the which-slit observable, simple linearity of the time evolution implies

$$\frac{1}{\sqrt{2}}(|A1\rangle + |A2\rangle)\,|B_{ready}\rangle \Rightarrow \frac{1}{\sqrt{2}}(|A1\rangle\,|B1\rangle + |A2\rangle\,|B2\rangle) = |\Psi_{AB}\rangle \tag{4}$$

Thus von Neumann's straightforward argument(1932) shows the measurement creates the entangled superposition $|\Psi_{AB}\rangle$ of Equation (2), where "*B*" now refers to the quantum detection component of the detector.

But does von Neumann's measurement postulate imply that, when the which-slit observable is measured, *A* collapses into one of its eigenstates while *B* collapses into the corresponding detector state.

It is by no means clear that $|\Psi_{AB}\rangle$ (Equation (4)) represents such a measurement outcome.

As many physicists put it, "The problem of what to make of this is called the 'measurement problem'."

This discussion will show that $|\Psi_{AB}\rangle$ does in fact represent the collapsed state and the single definite outcome expected from von Neumann's measurement postulate.

We have used the same notation, $|\Psi_{AB}\rangle$, for the arbitrary entangled state represented by Equation (2) (where *A* and *B* are arbitrary quantum systems) and for the specific case of the entangled state that develops when a detector measures a quantum system, represented by Equation (4) (where *B* is now a detector).

We will refer to this state in the context of Equation (4) as the "measurement state" (MS).

We will also, however, need to refer to the arbitrary entangled state Equation (2), especially later where we analyze an experiment involving two microscopically entangled photons.

Thus the question of how to interpret entangled states looms large in the foundations of quantum physics.

As noted above, the interpretation of general entangled states such as the one represented by Equation (2) is already murky as compared with the interpretation of simple superposition states such as the one represented by Equation (1).

The problem of interpreting the MS is especially important, because **macroscopically distinct** states now lie on each side of the "plus" sign on the right-hand side of Equation (4).

As already discussed, the superposition $|\Psi_A\rangle$ can be interpreted to represent a situation in which *A* incorporates properties represented by both $|A1\rangle$ and $|A2\rangle$.

And a product state such as $|A1\rangle|B1\rangle$ represents a state of the composite system *AB* in which *A* is represented by $|A1\rangle$ and *B* is represented by $|B1\rangle$.

Thus $|\Psi_{AB}\rangle$ appears to describe a detector that simultaneously "points" to two macroscopically different outcomes $|B1\rangle$ and $|B2\rangle$!

The detector seems to display no definite outcome, a conundrum known as the **"problem of outcomes"**

Such a superposition state would be paradoxical.

Schrodinger, who imagined a cat attached to the detector in such a way that $|B1\rangle$ included a live cat and $|B2\rangle$ included a dead cat, described $|\Psi_{AB}\rangle$ as representing a “living and dead cat ...smeared out in equal parts.

As one quantum foundations expert writes,

The crucial difficulty is now that it is not at all obvious how one is to regard the dynamical evolution described by [Equation (4)] as representing measurement in the usual sense.

This is so because [Equation (4)] is ...not sufficient to directly conclude that the measurement has actually been completed.

In fact, while measurement should lead to a specific eigenstate of the measured observable, Equation (4) appears to entail that the system has been **sucked into a vortex of entanglement** and **no longer** has its own quantum state.

On top of that, the entangled state **fails to indicate** any particular measurement outcome.

As noted, it seems paradoxical that quantum measurements lead to a state represented by the MS.

Measurement should lead to a situation in which *A* is represented by one of its eigenvectors $|Ai\rangle$ and *B* is represented by the corresponding $|Bi\rangle$.

Since quantum uncertainty typically implies **unpredictable** measurement outcomes, it is reasonable to conclude that measurement should lead to a state represented by an **ignorance-interpretable mixture** of the products $|A1\rangle|B1\rangle$ and $|A2\rangle|B2\rangle$.

Assuming the initial state is represented by $|\Psi_A\rangle$, such a post-measurement mixture would be represented by the density operator

$$\rho_{mixed} = \frac{1}{2}(|A1\rangle|B1\rangle\langle B1|\langle A1| + |A2\rangle|B2\rangle\langle B2|\langle A2|) \tag{5}$$

This mixture can be interpreted(as we did earlier) as "**the system is represented by a single component** $|Ai\rangle|Bi\rangle$ **, but we cannot know whether *i = 1* or *2* until we look at the outcome.**"

Beginning with von Neumann's analysis, at least eight published "**measurement problem insolubility proofs"** have assumed that, in order to obtain definite outcomes, the measurement process should transform the composite system *AB* into a mixture analogous to Equation (5) **(so have we so far!!)**

The initial state of *A* is assumed to be pure and to be represented by a superposition such as Equation (1).

The analysis then investigates whether a suitable composite-system post-measurement mixture can be reached via a unitary process.

To achieve this, the detector must be represented by a mixture initially, because unitary processes **cannot turn a pure state into a mixture**.

Since B is macroscopic, such an initial mixture **seems appropriate**.

Thus von Neuman and seven succeeding analysts asked:

> Is there an initial mixed-state density operator ρ_{ready} of B and a unitary process U acting on AB such that U transforms the initial composite density operator $|\Psi_A\rangle\langle\psi_A| \otimes \rho_{ready}$ into the desired composite mixture?

The eight insolubility proofs, with varying assumptions, say the answer is "no," presumably demonstrating the measurement problem to be insoluble(according to them!).

We will show later, however, that the premise of these insolubility proofs, namely that Equation (5) represents the appropriate post-measurement state, was doomed from the start, precisely because it is *not* entangled and thus cannot have the properties required if quantum theory is to describe the measurement process as we have seen in our discussions.

That last statement represents the crucial point!!

To put this another way, there are reasons why the post-measurement state *must* be an entangled state, which implies that it *cannot* be a mixture such as Equation (5), which just represents what we measure!

We will then show that the MS does have the desired properties.

3 - A CRUCIAL CLUE FROM EINSTEIN

At the 1927 Solvay Conference, five years prior to von Neumann's analysis of quantum measurement, Einstein asked the audience to consider an experiment in which electrons pass through a tiny hole in an opaque screen and then impact a large hemispherical detection screen centered at the hole (Fig. 1).

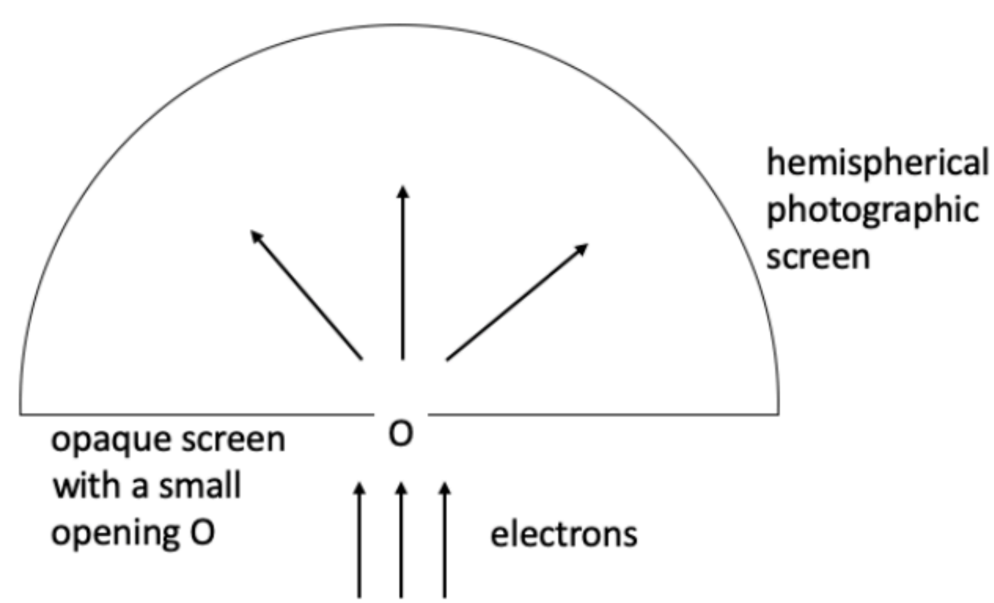


Figure 1. Einstein's thought experiment. Each electron diffracts widely, reaching the entire screen simultaneously, yet only one point shows an impact. How do the other points *instantaneously* remain dark? Does this violate Special Relativity?

According to the Schrodinger equation, each electron's state diffracts widely, spreading and reaching the entire screen simultaneously.

Yet each electron registers at only a **single point**.

How, Einstein asked, do the other points ***instantaneously* remain "dark," i.e., not show an impact**?

As Einstein put it in his notes, this "**entirely peculiar mechanism of action-at-a-distance, which prevents the wave continuously distributed in space from producing an effect in *two* places on the screen,**" presents a fundamental problem.

It appears to imply instant signaling, **violating** special relativity.

Einstein's argument shows that, under a **realistic and objective** (**independent of humans**) interpretation of quantum physics, the Schrodinger equation is **at odds** with experimental facts **unless** the electron's state collapses instantaneously and nonlocally upon measurement.

Thus, realistic quantum physics **implies** instantaneously-established nonlocal correlations are **essential** to the *measurement process*.

Indeed, modern experiments have **verified the nonlocal character** of the measurement transition.

Since nonlocality is essential to measurement, the presumed post-measurement mixed state(5) was doomed from the start precisely because it **does *not* exhibit** the required nonlocality.

But entangled superpositions **do exhibit** the required nonlocality.

So from our **modern point of view(after almost a century of research)**, Einstein's argument shows that **entanglement, far from being an unwelcome paradox, is *required* in measurements.**

This is a **crucial clue** and good news for quantum foundations, because von Neumann's predicted MS is **just such an entangled superposition**!

One particular argument, at least, should be mentioned. Its resolution of the measurement problem "diagonalizes the density operator" as we discussed earlier on one of our adventures.

They form the exact density operator $\rho = |\Psi_{AB}\rangle\langle\Psi_{AB}|$, which can be written

$$|\Psi_{AB}\rangle \langle\Psi_{AB}| = \rho_{\text{diagonal}} + \rho_{\text{off-diagonal}} \tag{6}$$

where $\rho_{diagonal} = \rho_{mixed}$ (Equation (5)) and

$$\rho_{off-diagonal} = \frac{(|A1\rangle|B1\rangle\langle B2|\langle A2| + |A2\rangle|B2\rangle\langle B1|\langle A1|)}{2}. \tag{7}$$

Recall that the exact expectation value of any observable F is

$$\langle F\rangle = \mathrm{Tr}(\rho F) = \sum_j \sum_k \rho_{jk} F_{kj} \tag{8}$$

where ρ_{ijk} and F_{ijk} are matrix elements of ρ and F .

This idea argues that off-diagonal terms in Equation (8) can be ignored because they involve matrix elements such as $\langle B1|\langle A1| F |A2\rangle |B2\rangle$ that are non-zero only for a "**fantastic**" observable F because $|B1\rangle$ and $|B2\rangle$ represent the states of **widely separated detectors**.

It says that matrix elements for such **fantastic** observables can, **for all practical purposes**, be neglected so that we can replace ρ by ρ_{mixed}.

But we have seen that this premise is doomed because ρ_{mixed} lacks the required nonlocal properties, so this type of proposal fails (it leaves our original discussions in limbo for the moment also!!).

4 - EXPERIMENTAL STUDIES OF STATES HAVING ENTANGLED SPATIAL PATHS

Earlier we presented the measurement problem and some previous thoughts on the problem.

We now will present a suggested resolution of all difficulties.

We now review interferometry experiments and theory that investigate the *microscopic* entangled superposition $|\Psi_{AB}\rangle$ Equation (2) over its full 0 - to - π range of phases, for a system of two momentum-entangled (i.e. path-entangled) photons.

The results provide a key insight into solving the measurement puzzle.

As preparation, we first study the simple superposition Equation (1).

Consider the interferometer experiment of Figure 2.

On each experimental trial, a photon enters a 50-50 beam splitter *BS1* which transforms the photon's state into the superposition Equation (1) where $|A1\rangle$ and $|A2\rangle$ respectively represent paths 1 and 2.

A series of single-photon trials probes this state using mirrors *M* that bring the two branches together, phase shifters ϕ_1 and ϕ_2 that lengthen the two paths by phases ϕ_1 and ϕ_2, and a second beam splitter *BS2* that mixes the branches together.

Measurement occurs at photon detectors *B1* and *B2*.

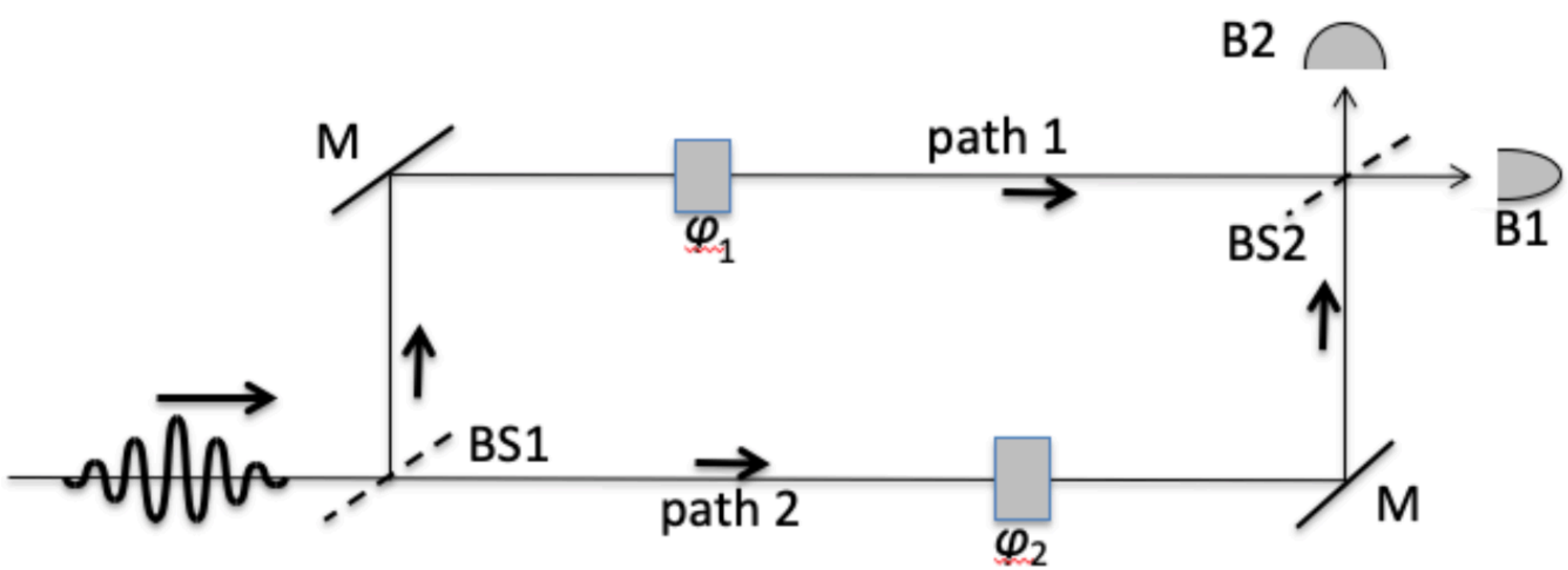


Figure 2. Mach-Zehnder interferometer experiment. A photon traverses a beam splitter, travels on two phase-shifted paths to another beam splitter, and is detected.

Figure 3 shows the results(discussed earlier).

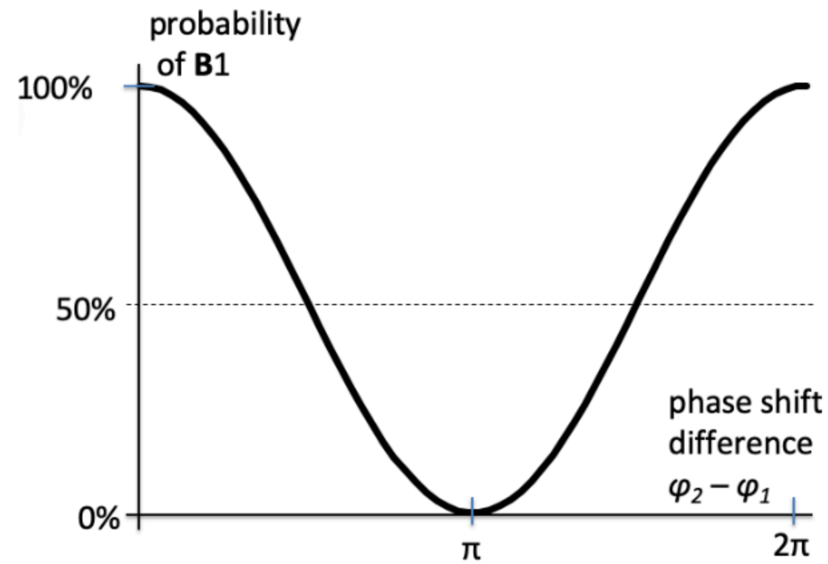


Figure 3. Single photon interference, pointing to Dirac's conclusion that "each photon ...interferes only with itself," i.e. each photon follows both paths.

Varying ϕ_1 through 180 degrees causes the photon's state to shift from 100% probability of detection at *B1*, through diminishing probabilities at *B1* and increasing probabilities at *B2*, finally reaching 100% probability of detection at *B2*.

The photon exhibits similar interference upon varying ϕ_2 .

Note that *A*'s state depends **only** on the phase *difference* $\phi_2 - \phi_1$.

Since single-trial results vary regardless of which phase shifter varies, it is hard to avoid the conclusion that each photon follows both paths.

In fact, let us assume the contrary, namely that each photon follows only one path.

Suppose the phase shifters are set to ensure 100% probability of detection at *B1*.

Under our assumption, this setting guarantees that every photon following path 1, and every photon following path 2, is detected at *B1*.

Suppose path 2 is now blocked without changing the phase settings, so that (still under our one-path assumption) every detected photon must now follow path 1 and be detected at *B1*.

But the experiment shows that, to the contrary, 50% of the detected photons now go to *B2*.

Conclusion: each photon follows both paths.

For a full discussion, see earlier parts of these lectures.

As Paul Dirac put it,

"The new theory, which connects the wave function with probabilities for one photon, gets over , the difficulty by making each photon go partly into each of the two components.

Each photon then interferes only with itself."

This illustrates why the "plus" sign in a superposition such as Equation (1) is interpreted by the word “AND".

We turn now to the entangled state Equation (2).

Many Bell inequality tests used polarization-entangled photon pairs to study the full phase dependence of this state.

More useful for this discussion are two interferometer experiments conducted in 1990.

Both of these experiments used *momentum*-entangled photon pairs to conduct Bell inequality tests of the entangled superposition Equation (2) (see earlier discussions).

Figure 4 shows the layout.

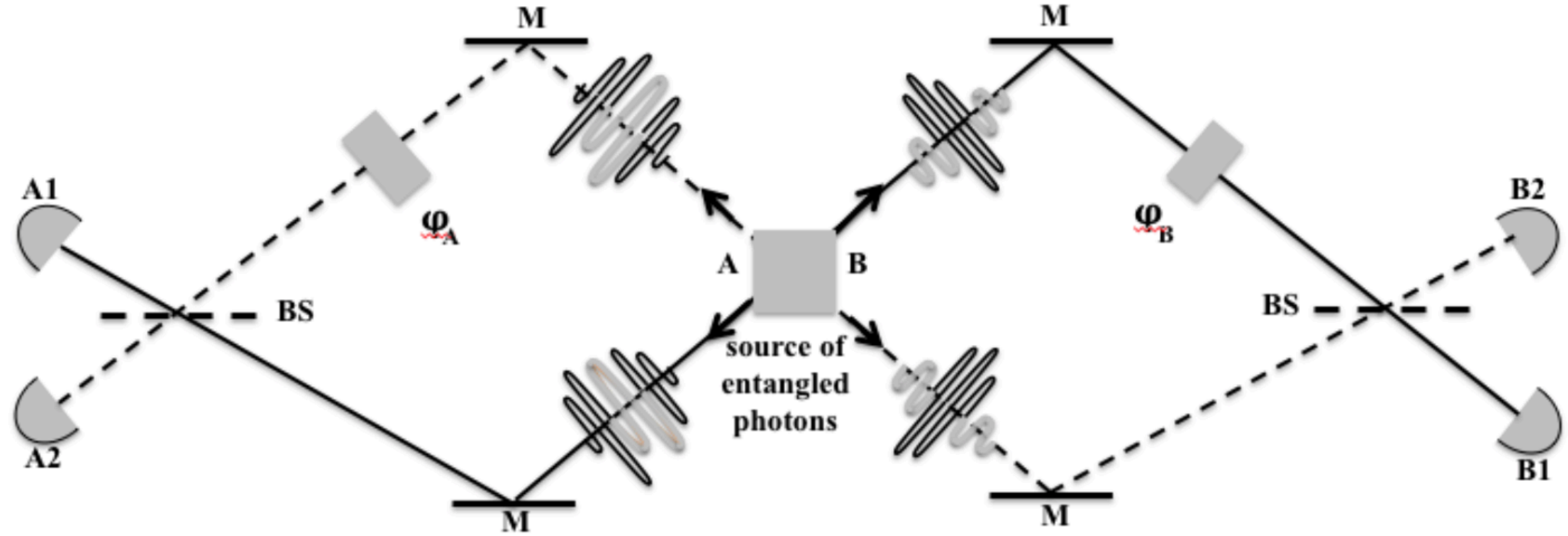


Figure 4. The RTO experiments. In each trial, each of two entangled photons travels two phase-shifted paths (one solid, the other dashed) to beam splitters and coincidence detectors. Think of one biphoton spreading from the source along both the solid and the dashed paths.

The source creates entangled pairs of photons *A* (moving leftward) and *B* (moving rightward) by laser down-conversion in a non-linear crystal.

The down-converted photons are prepared in the entirely microscopic state represented by $|\Psi_{AB}\rangle$ (Equation (2)) by selecting four single-photon beams, each a plane wave having a distinct momentum (i.e. wave vector), from the output of the crystal.

Figure 4 resembles two back-to-back Mach- Zehnder interferometer experiments (Figure 2) with BS1 located effectively inside the source.

For simplicity and clarity, Figure 4 differs from the layout shown in the published papers.

In Figure 4, paired photons are directed oppositely.

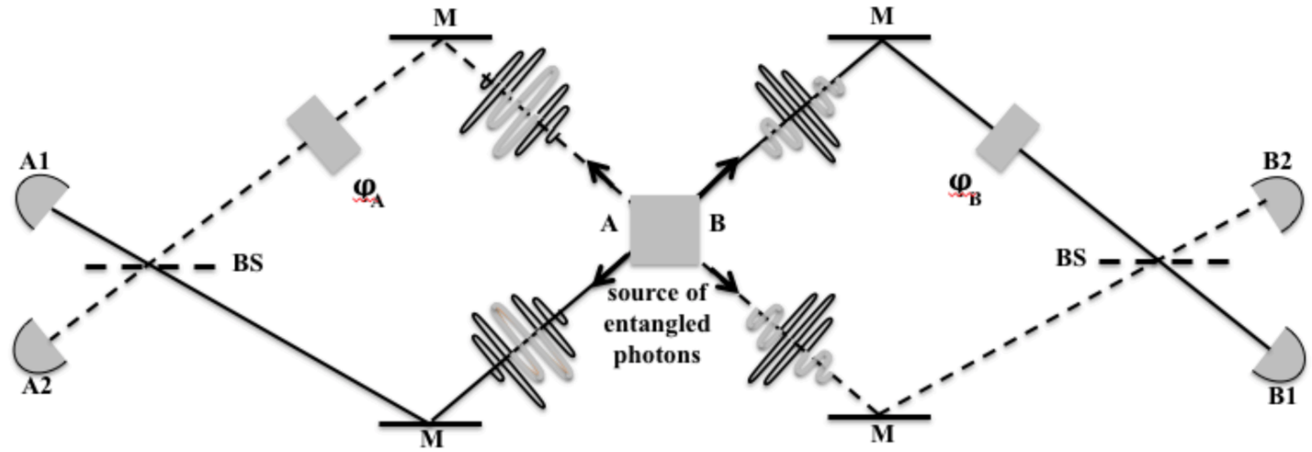


Figure 4. The RTO experiments. In each trial, each of two entangled photons travels two phase-shifted paths (one solid, the other dashed) to beam splitters and coincidence detectors. Think of one biphoton spreading from the source along both the solid and the dashed paths.

This arrangement would result if the entanglement were prepared by the cascade decay of an atom.

In the actual experiments, however, down-converted photon pairs are emitted into two angular cones, resulting in photons that are not oppositely directed.

Figure 4's simpler geometry is pedagogically useful and has no effect on our arguments.

Although Figure 4 represents each photon as a wave packet spreading along two paths directed leftward or rightward, the composite system *AB* should be regarded as a *single* object, a **"biphoton".**

In each trial, a biphoton spreads outward from the source along two superposed branches.

One branch, represented by the first term $|A1\rangle\,|B1\rangle$ in Equation (2), spreads along the solid path and the other branch, $|A2\rangle\,|B2\rangle$, spreads along the dashed path.

As the biphoton *AB* moves outward along the solid path, *A* encounters a mirror *M,* then a beam splitter *BS* where it transmits and reflects to detectors *A1* and *A2*; photon *B* encounters a mirror *M,* a phase shifter ϕ_B , and a beam splitter *BS* where it transmits/reflects to detectors *B1*/*B2*.

The other half of the entanglement, namely the dashed path, has a similar description.

The experiments record outcomes at four photon detectors equipped with coincidence timers.

Published papers predict the experimental results theoretically and we **follow** their optical-path analysis here(slightly different but equivalent to our earlier arguments).

They begin by calculating the two-point nonlocal quantum field amplitudes $\Psi(Ai, Bj)$ at the four coincidence detectors *(Ai, Bj)*, and from these results they predict single-photon results.

For example, $\Psi(A1, B2)$ has two contributions, one from phase shifts in the beam following the solid path (the first term in $|\Psi_{AB}\rangle$) and the other from the dashed path (the second term).

From Equation (2), assuming distinct plane waves $\exp(i\mathbf{k}\cdot\mathbf{x})$ for each single-photon beam,

$$\Psi(A1, B2) = \frac{1}{2\sqrt{2}}\{\exp(i\phi_w)\exp[i(\phi_x + \phi_B)] + \exp[i(\phi_y + \phi^A)]\exp(i\phi_z)\} \tag{9}$$

where $\phi_w, \phi_x, \phi_y, \phi_z$ are fixed phase-shifts resulting from mirrors and beam splitters, and the additional factor of 1/2 comes from the superpositions created at the two beam splitters.

Using the Born rule, Equation (9) implies the coincidence probability

$$P(A1, B2) = |\Psi(A1, B2)|^2 = \frac{1}{4}[1 + \cos(\phi_B - \phi_A + \phi_v)] \tag{10}$$

where ϕ_v is a fixed phase arising from $\phi_w, \phi_x, \phi_y, \phi_z$.

Similarly,

$$P(A1, B1) = \frac{1}{4}[1 + \cos(\phi_B - \phi_A + \phi_u)] \tag{11}$$

where ϕ_u is another fixed phase.

Remarkably, the sinusoidal terms predict coherent (phase-dependent) nonlocal interference between A and B, **regardless of their separation**.

There are similar expressions for *P(A2, B1)* and *P(A2, B2)*.

Single-photon predictions then follow. For example, from simple probability theory

$$P(A1) = P(A1, B1) + P(A1, B2)$$
$$= [1+\cos(\phi_B-\phi_A + \phi_u)]/4 + [1+\cos(\phi_B-\phi_A + \phi_v)]/4. \quad (12)$$

The theory papers then show the two fixed phase factors ϕ_u and ϕ_v differ by π:

$$\phi_v = \phi_u + \pi \pmod{2\pi}. \quad (13)$$

Thus the sinusoidal terms in Equation (12) *interfere destructively*, and *P(A1) = 1/2 regardless of phase*.

Equations (12) and (13) show this remarkable result to arise from destructive interference of two phase-dependent nonlocal contributions from the distant ***other*** photon *B*!

The result at all four single-photon detectors is the same:

$$P(A1) = P(A2) = P(B1) = P(B2) = 1/2. \quad (14)$$

Unlike the non-entangled single-photon superposition $|\Psi_A\rangle$, where the superposed photon is coherent (phase-dependent) as shown by Figure 3, each entangled photon is "**decohered**" and cannot interfere with itself.

Instead the two photons interfere with each other despite being separated by an arbitrary distance.

More accurately, ***each biphoton interferes with itself***.

Thus no single-photon interference fringes are associated with the state represented by $|\Psi_{AB}\rangle$.

Special relativity requires that this must be the case:

Since single-photon phase dependence could be used to establish an instantaneous communication channel between *A* and *B*, entanglement ***must*** deprive individual photons of their phase.

The result is **nonlocal coherence of the biphoton, AND decoherence of individual photons**.

The decoherence is required by special relativity.

Equation (14) can also be derived by tracing the pure state density operator $|\Psi_{AB}\rangle\langle\Psi_{AB}|$ over one subsystem to obtain the density operator for the other subsystem, which is what we did earlier.

This yields two density operators that appear to be mixtures but are not really "ignorance mixtures" as the word "mixture" is usually understood because the biphoton is in fact not in a mixed state but rather in a pure state represented by $|\Psi_{AB}\rangle$.

That is why our earlier idea was incomplete!

The optical path analysis, above, derives Equation (14) while avoiding these controversial subtleties

A few definitions can put these predictions into more comprehensible form:

If one photon is detected in state 1 and the other in state 2, the two outcomes are said to be "different."

Otherwise, the outcomes are the "same."

Then from Equations (10) and (11), and similar expressions for *P(A2, B1)* and *P(A2, B2)*,

$$P(same) = P(A1,B1) + P(A2,B2) = 1/2[1 + \cos(\phi_B - \phi_A)] \quad (15)$$

$$P(different) = P(A1,B2) + P(A2,B1) = 1/2[1 - \cos(\phi_B - \phi_A)]. \quad (16)$$

Their difference, graphed in Figure 5, is called the "**degree of correlation**":

$$C = P(same) - P(different) = \cos(\phi_B - \phi_A). \quad (17)$$

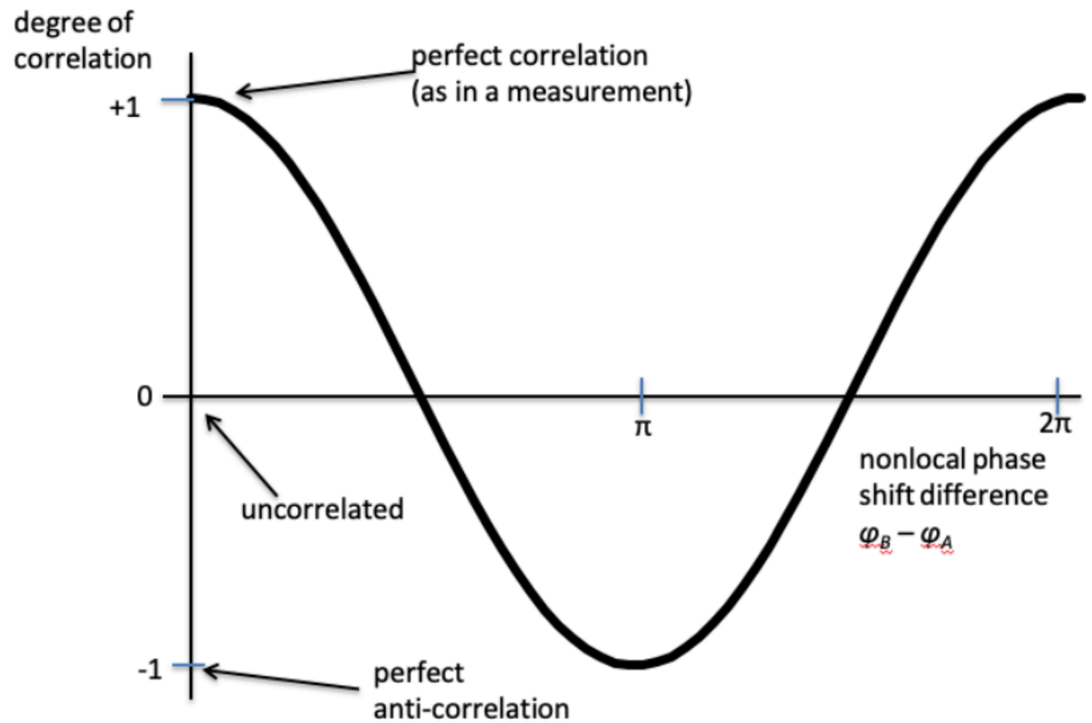


Figure 5. Results of the RTO experiment, demonstrating nonlocality (violation of Bell's inequality). The two photons interfere with each other across an arbitrary distance, i.e. each biphoton interferes with itself.

We now explore its physical significance.

5 - INTERPRETATION OF ENTANGLED MICROSCOPIC STATES

The original purpose of RTO's experiments was to demonstrate violations of Bell's inequality by comparing theoretical predictions, Figure 5, with experimental measurements.

The experimental results agreed with Figure 5 and violated Bell's inequality by 10 standard deviations, confirming the nonlocal nature of $|\Psi_{AB}\rangle$.

What does Figure 5 mean conceptually?

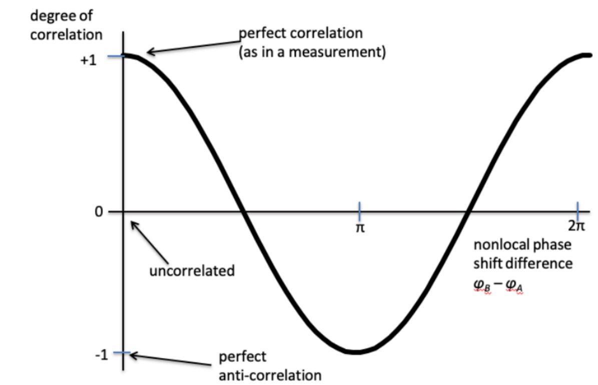


Figure 5. Results of the RTO experiment, demonstrating nonlocality (violation of Bell's inequality). The two photons interfere with each other across an arbitrary distance, i.e. each biphoton interferes with itself.

At zero phase difference, where the two phase shifters are set to equal phases, P(same) = 1 and C = P(same) - P(different) = +1.

Thus both stations always agree, despite the presence of beam splitters that randomize each photon prior to detection (see Figure 4).

It is as though coins were flipped at each station and they always came out either both heads or both tails!

Zero is the "measurement" phase angle where B's state is perfectly (and instantaneously) correlated with A's state.

The nonlocality is intuitively obvious:

Each photon acts like a detector of the state of the other photon regardless of separation!

Simply based on this conclusion, we can make an important observation about the entangled MS Equation (4):

Nonlocality is a central feature of quantum measurements.

For small non-zero phase differences, there is a small probability P(different) that results at the two stations will differ, i.e. observation of B no longer provides reliable information about A.

With increasing phase difference, this unreliability increases until, at $\pi/2$, the two detector pairs are entirely uncorrelated, and $C = 0$.

As the phase further increases from $\pi/2$ to π, P(different) increases while P(same) decreases, making C more and more negative.

Finally, $C = -1$ at phase difference π, implying perfect anti-correlation.

Thus C is aptly called the **"degree of correlation"**.

This description gives us a clear sense of the physical meaning of the fully entangled state Equation (2), indicating precisely which entities are superposed.

The biphoton's phase controls the degree to which the fixed phase-independent 50-50 states of its two spatially separated subsystems are ***statistically correlated***.

Compare this with the phase of the simple superposition $|\Psi_A\rangle$ (Equation (1)), which controls the degree to which the single system *A* is represented by one or the other *state*.

The entities before and after the plus signs in Equations (1) and (2) are conceptually quite different:

Equation (1) sums two *states* while Equation (2) sums two *correlations between states*.

This **distinction is crucial**.

A ***state*** is a situation (or configuration or path) of a single quantum object, but a ***correlation*** is a statistical relationship between two (or more) quantum objects.

A ***superposition*** is the simultaneous existence of two or more states of a single quantum object.

An ***entanglement*** is the simultaneous existence of two or more relationships (specifically, correlations) between the states of two or more quantum objects.

Creating an entanglement is **quite a different matter** from creating a superposition.

Simple superposition of 1 photon.		Entangled superposition of 2 photons		
$\phi_2 - \phi_1$	State of photon	$\phi_B - \phi_A$	State of each photon	Correlation between photons
0	100% 1, 0% 2	0	50-50 1 or 2	100% corr, 0% anticorr
$\pi/4$	71% 1, 29% 2	$\pi/4$	50-50 1 or 2	71% corr, 29% anticorr
$\pi/2$	50% 1, 50%2	$\pi/2$	50-50 1 or 2	50% corr, 50% anticorr
$3\pi/4$	29% 1, 71% 2	$3\pi/4$	50-50 1 or 2	29% corr, 71% anticorr
π	0% 1, 100% 2	π	50-50 1 or 2	0% corr, 100% anticorr

Table 1. Comparison between a simple superposition (Fig, 2) and an entangled superposition (Fig. 4). In Fig. 2, single-photon states vary with phase. In Fig. 4, only the *correlation between single-photon states* varies with phase while single-photon states have no phase. Thus *each biphoton is coherent* but its subsystems are incoherent. That is, entanglement *decoheres* each photon while transferring coherence to the biphoton.

To elaborate, Table 1 compares the superposition represented by $|\Psi_A\rangle$ (columns 1-2) with the entanglement represented by $|\Psi_{AB}\rangle$ (columns 3-5) at five different phases.

Column 2 demonstrates interference between the states represented by $|A1\rangle$ and $|A2\rangle$, implying the photon is in a superposition of following both paths and that the *state of A* varies with phase.

The phase dependence in column 2 shows that *A* interferes with itself.

Simple superposition of 1 photon.		Entangled superposition of 2 photons		
$\phi_2-\phi_1$	State of photon	$\phi_B-\phi_A$	State of each photon	Correlation between photons
0	100% 1, 0% 2	0	50-50 1 or 2	100% corr, 0% anticorr
π/4	71% 1, 29% 2	π/4	50-50 1 or 2	71% corr, 29% anticorr
π/2	50% 1, 50%2	π/2	50-50 1 or 2	50% corr, 50% anticorr
3π/4	29% 1, 71% 2	3π/4	50-50 1 or 2	29% corr, 71% anticorr
π	0% 1, 100% 2	π	50-50 1 or 2	0% corr, 100% anticorr

Table 1. Comparison between a simple superposition (Fig, 2) and an entangled superposition (Fig. 4). In Fig. 2, single-photon states vary with phase. In Fig. 4, only the *correlation between single-photon states* varies with phase while single-photon states have no phase. Thus *each biphoton is coherent* but its subsystems are incoherent. That is, entanglement *decoheres* each photon while transferring coherence to the biphoton.

In contrast, column 4 shows that, when the subsystems are represented by the pure state $|\Psi_{AB}\rangle$, neither photon has a phase.

Thus neither photon can interfere with itself, so **neither photon** can be represented by a superposition state.

They are decohered.

Both photons are **represented** by fixed, phase-independent, 50-50 states at all phase angles, **just as though they were in ignorance mixtures (which, however, they are not)**.

But phase dependence has not vanished, it has only been **transferred** to the composite system.

As column 5 reveals, the *degree of correlation between the fixed states of A and B* now varies with phase.

A photon represented by $|\Psi_A\rangle$ is in a coherent (phase-dependent) superposition of being in two states (i.e. of following two *paths)*.

$|\Psi_{AB}\rangle$, on the other hand, represents the coherent superposition of two *correlations between fixed states*.

Instead of two coherent *states* existing simultaneously, two coherent *relationships between states* exist simultaneously.

Neither subsystem is "*smeared*" (as Schrodinger apparently believed);

instead, *only the relationship between subsystems is smeared*.

Briefly, $|\Psi_A\rangle$ is a superposition of *states* and $|\Psi_{AB}\rangle$ is a superposition of *correlations*.

Thus $|\Psi_{AB}\rangle$ is qualitatively different from $|\Psi_A\rangle$.

$|\Psi_A\rangle$ exhibits properties of $|A1\rangle$ AND $|A2\rangle$, where "AND" indicates the superposition.

If you amplify *A* to macroscopic dimensions, you will get a macroscopic superposition.

$|\Psi_{AB}\rangle$ exhibits properties of correlations between $|A1\rangle$ and $|B1\rangle$ AND correlations between $|A2\rangle$ and $|B2\rangle$.

If you amplify *A* and *B* to macroscopic dimensions, you will *not* get a macroscopic superposition, you will simply get **correlations between macroscopic objects**.

The entanglement process transfers the coherence (phase dependence) of each photon to correlations between the two photons, leaving individual photons in mixtures that are incoherent but that are **not** ignorance mixtures.

$|\Psi_{AB}\rangle$ is a "superposition of correlations between properties," in contrast to $|\Psi_{A}\rangle$ which is a "superposition of properties."

There is a even better way to think about all this:

Regard *AB* as a single object, a biphoton.

Then Equation (2) describes a superposition of this object.

In the experiment (Figure 4), the two superposed states are represented by the solid line and the dashed line.

In this context, it makes no sense to speak of the superposition of a single subsystem, but it does make sense to speak of the superposition of the biphoton.

It is the biphoton that goes through the phases graphed in Figure 5 and indicated in Table 1 column 5.

Both branches (solid and dashed) of the biphoton exist simultaneously.

6 - INTERPRETATION OF THE MEASUREMENT STATE

Earlier we analyzed the microscopic state represented by Equation (2) mathematically, and then interpreted this state physically.

In order for *B* to be a reliable detector, its states must be perfectly correlated with *A*'s states--it must exhibit $|Bi\rangle$ when and only when *A* is represented by $|Ai\rangle$ (*i* = *1*, *2*).

Thus Figure 5 implies the MS must be established at zero non-local phase: $\phi_B - \phi_A = 0$.

At this phase, two nonlocal perfect statistical correlations between a phase-independent 50-50 state of *A* and the corresponding phase-independent 50-50 state of *B* exist simultaneously.

As shown earlier, contrary to Schrodinger's description, *neither subsystem state can be "smeared out" (superposed) because neither subsystem has a phase*.

Instead, *correlations between* fixed states of *A* and *B* are smeared as shown in Table 1, while the detector indicates a single definite outcome.

Applying Table 1 to Schrodinger's example, the cat is predicted to be alive in 50% of trials, dead in the other 50%, and never in both states simultaneously.

Phase alterations would not smear the cat, they would smear only the correlations between the cat and the nucleus leading not to a smeared cat but **only** to imperfect detection.

There is no paradoxical macroscopic superposition.

But if neither *A* nor *B* is superposed, what *is* superposed?

What does the MS's "plus" sign really mean?

The answer, from Table 1 at zero phase:

Simple superposition of 1 photon.		Entangled superposition of 2 photons		
$\phi_2-\phi_1$	State of photon	$\phi_B-\phi_A$	State of each photon	Correlation between photons
0	100% 1, 0% 2	0	50-50 1 or 2	100% corr, 0% anticorr
$\pi/4$	71% 1, 29% 2	$\pi/4$	50-50 1 or 2	71% corr, 29% anticorr
$\pi/2$	50% 1, 50%2	$\pi/2$	50-50 1 or 2	50% corr, 50% anticorr
$3\pi/4$	29% 1, 71% 2	$3\pi/4$	50-50 1 or 2	29% corr, 71% anticorr
π	0% 1, 100% 2	π	50-50 1 or 2	0% corr, 100% anticorr

Table 1. Comparison between a simple superposition (Fig, 2) and an entangled superposition (Fig. 4). In Fig. 2, single-photon states vary with phase. In Fig. 4, only the *correlation between single-photon states* varies with phase while single-photon states have no phase. Thus *each biphoton is coherent* but its subsystems are incoherent. That is, entanglement *decoheres* each photon while transferring coherence to the biphoton.

$|A1\rangle$ is perfectly correlated with $|B1\rangle$ AND $|A2\rangle$ is perfectly correlated with $|B2\rangle$, where "AND" represents the superposition.

This simply says *both correlations exist simultaneously:*

$|A1\rangle$ if and only if $|B1\rangle$ AND $|A2\rangle$ if and only if $|B2\rangle$.

Again, there is no paradoxical macroscopic superposition. It's only the *correlations (relationships) between states,* not the states themselves, that are superposed.

Entanglement transforms a superposition of 2 *states* into a superposition of *two correlations between states*.

This makes quantum measurements possible because subsystem states can then be amplified to macroscopic dimensions **without requiring the creation of a macroscopic superposition**.

Neither subsystem is in a macroscopic superposition.

Since neither subsystem is superposed, only a single outcome occurs--a conclusion that also follows from Equation (14).

This single definite outcome occurs instantly upon entanglement, as facilitated by the nonlocal properties of the entangled MS.

Thus we have derived the collapse as an inevitable consequence of entanglement, and have **no need to postulate** such a process.

The MS **is** the collapsed state.

Our conclusion follows merely from standard principles of quantum theory with no other assumptions.

So von Neumann's enigmatic measurement state, Equation (4), is just what we want.

This entangled pure state provides the desired correlations, a single outcome, and the nonlocality required by Einstein's argument.

Note that the collapse is established at the microscopic level, prior to macroscopic amplification.

Next, we provide an example of the sequence of events.

7 - EXAMPLE

The following simple example typifies quantum measurements and illustrates the preceding insights in terms of a specific measurement process.

Consider the set-up in Figure 6.

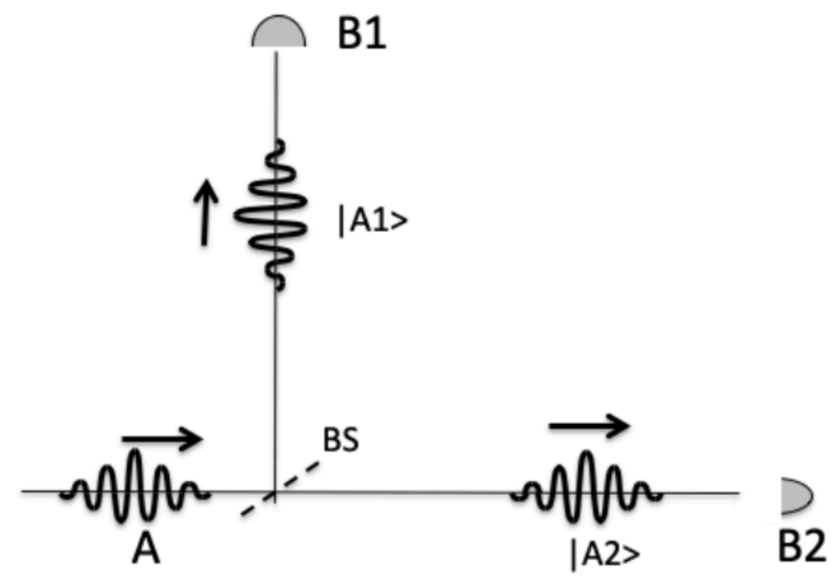


Figure 6

A single photon traverses a beam splitter, creating the superposition represented by Equation (1) whose branches correspond to separate paths toward widely separated photon detectors.

Analogously to Figure 1, we assume the two detectors are equidistant from the beam splitter.

Each detector contains a photo-sensitive plate that, upon absorbing a photon, **releases** an electron.

Von Neumann's argument implies that, as the two branches of the superposition "approach" the detectors, at some point the branches overlap the detectors sufficiently that the entanglement process represented by Equation (4) occurs, where $|B_{ready}\rangle$ denotes the microscopic state of the detectors prior to entanglement while $|B1\rangle$ and $|B2\rangle$ denote their states following entanglement but prior to amplification and macroscopic recording.

At the instant of interaction, the state jumps from a superposition of two paths of A (Equation (1)) to a superposition of two correlations between A and B (right-hand side of the process in Equation (4)).

This entangled state is not paradoxical.

The right-hand side of Equation (4) entails precisely the proper correlations:

$$|A1\rangle \text{ if and only if } |B1\rangle \quad \text{AND} \quad |A2\rangle \text{ if and only if } |B2\rangle \ .$$

The excitation is transferred to only *one* detector while the other detector remains unexcited.

More correctly, either the solid branch or the dashed branch of the superposed biphoton (Figure 4) is randomly selected.

In fact, it has been shown experimentally and theoretically that the set-up shown in Figure 6 leads to entanglement and that the predicted nonlocal collapse occurs.

The nonlocality of the collapse is verified quantitatively by the experimental violation of a mathematical inequality derived based on QM.

Thus the collapse, a non-linear and irreversible process, occurs at the microscopic level.

Once one photoelectron is released, the process is thermodynamically irreversible because the electron is released into a vast number--a continuum--of free electron states and cannot feasibly be reversed.

This electron triggers an avalanche of other electrons leading to a macroscopic mark at one detector.

Other measurement set-ups follow the same general principles.

For example, in the measurement described by Einstein (discussed earlier), each small region of the detection screen acts as a single detector, and the diffracted electron's quantum state entangles with all these many regions.

Thus the argument above, involving only two detectors, must be extended to N detectors.

8 - SUMMARY AND CONCLUSION

Using only the standard principles of quantum physics, but minus the collapse postulate, we have shown that quantum state collapse occurs as a consequence of the entanglement that occurs upon measurement as described in 1932 by the von Neumann measurement state MS (Equation (4)).

The entangled "measurement state" of a quantum system and its detector is the collapsed state:

It incorporates the required perfect correlations between the system and its detector, it predicts precisely one definite outcome, and it incorporates the nonlocal properties--the instantaneous collapse across all branches of the superposition--that Einstein showed to be required in quantum measurements.

The measurement state Equation (4) does not describe a detector in a paradoxical superposition of displaying multiple outcomes, as had been supposed by Schrodinger and others.

Instead, quantum theory concludes that this state entails just what we expect following a measurement:

The states represented by $|A1\rangle$ and $|B1\rangle$ are perfectly correlated, AND the states represented by $|A2\rangle$ and $|B2\rangle$ are perfectly correlated, where "AND" represents the plus sign in the mathematical representation of the state.

Entanglement entails merely the simultaneous occurrence of two correlations between subsystems, not the simultaneous occurrence of two individual states of either subsystem.

There can be no paradoxical superposition of different detector states or of different system states, because the entanglement has removed the quantum phases from both the detector and the quantum system - just as Feynmann supposed in the quote I gave earlier!

The phase has been transferred from the individual subsystems to the degree of correlation between subsystems.

To put all of this more directly, the single quantum object *AB* (the biphoton) collapses from a superposition to one of its members.

The measurement state's entanglement and its nonlocal properties, far from being paradoxical, are required in order to guarantee that the collapse occurs simultaneously across all branches of the superposition.

Eight previous insolubility proofs failed because they did not incorporate this required nonlocality.

Nonlocality is a central feature of quantum measurement.

There is no need for a special collapse postulate because the entangled state is the collapsed state.

Collapse occurs instantly upon entanglement.

Let me repeat this argument to make sure I am clear:

A “quantum measurement” means any quantum process that results in a macroscopic effect, regardless of whether humans or laboratories are involved.

All of this suggests that measurements affect superposed quantum states via entanglement of the superposed quantum with a detector.

Precisely what is superposed and what interferes in this measurement state?

The answer is surprisingly simple: Only the correlations between S and M are superposed.

Details:

The quantum system has two possible state, |S1> and |S2>.

When the quantum state |S> is created the states |S1> and |S2> are in a superposition i.e.,
|S> = a|S1> + b|S2>

When the quantum state S is measured by some macroscopic device that has two state |M1> and |M2>, such that if the quantum system were in state |S1>, then the measuring device ends up in the state |M1> after a measurement and if the quantum system were in state |S2>, then the measuring device ends up in the state |M2> after a measurement

This is expressed mathematically in quantum theory by saying the measurement state is |SM> = a|S1>|M1> + b|S2>|M2>. This is called entanglement.

It is at this point that most textbooks get the interpretation wrong!

The measurement state should be read as:

The state |S1> is positively correlated with the state |M1>, and the state |S2> is positively correlated with the state |M2>.

ONLY correlations are superposed, NOT states.

When the superposition |S> = a |S1> + b |S2> of S entangles with states of M, the superposition shifts, from a superposition of states of S to superposition of correlations between S and M, so S can be in an incoherent mixture while maintaining unitary global dynamics.

Only the correlations between S and M are superposed.

This is how nature resolves problem of definite outcomes.

A so-called collapse mechanism is NOT needed!

This analysis should not be regarded as one more interpretation of quantum physics.

It is instead a **correction** of the previous misunderstandings of von Neumann's entangled measurement state.

It is not surprising that this misunderstanding has persisted for nearly 90 years.

After all, entanglement and nonlocality are deeply involved in the measurement problem's proper resolution but they only began to be understood in 1964, leading to a long period of gradual acceptance with confirmation only in 2015.

The delay in understanding measurement stemmed from this delay in understanding nonlocality.

For more than a century, much has been made of the odd and supposedly paradoxical nature of the quantum.

This presumed quantum spookiness has led to an excess of attempted fixes and interpretations.

Many experts have even declared the theory to be not a description of reality at all, but only a mathematical recipe that helps humans predict the results of experiments.

As Niels Bohr put it, **“There is no quantum world. There is only an abstract quantum description.”**

According to this hypothesis, quantum theory doesn’t describe anything real at all, so there’s no cause for concern about collapse of the quantum state and other odd quantum behaviors.

Such an easy resolution of the quantum quandaries amounts to giving up on science’s project of understanding the realities of the natural world.

It’s an extraordinary claim, requiring extraordinary proof. But there is no such proof, and there are no grounds for regarding quanta as any less real than rocks.

Indeed, rocks are made of quanta.

Although it has long been a conceit of humankind to imagine the universe to be centered around us, there is no reason to think that reality comprises only the kinds of things we experience in our own daily lives.

The real world does not fade from existence, nor does it become incomprehensible, at distances that happen to be several powers of ten smaller than teapots.

Atomic and subatomic processes are just as real as teapots and, with the help of technology, accessible to human experimentation and understanding.

From the viewpoint of the macroscopic and classical world that we are pleased to call “normal", there is certainly oddness in wave-particle duality, indeterminacy, quantum states, superposition, nonlocality, measurement, and quantum jumps.

But there are no logical contradictions here, no disagreements with experiment, and nothing that should persuade us that quantum physics is about anything other than the real world.

Quantum physics is either charmingly counterintuitive or maddeningly puzzling, depending on your taste, but it is entirely self-consistent and experimentally accurate.

It's time to accept it with all its charms and puzzles, and stop trying to repair or reinterpret it.

It's time, in other words, to relax and admit the world is not as we had thought.

Nature is far more creative than we could have conceived.

Our most fundamental theory is in better shape than its detractors suppose.

Quantum physics is a remarkable treasure trove of far-reaching phenomena and ideas whose surface we have probably only begun to scratch.

It’s time to fully embrace these ideas, incorporating them into our ways of thinking about the universe, about our planet, and about ourselves.

This is a process that will engage our minds and stretch our imaginations far into the future, for quantum physics is, indeed, not what anybody could have imagined.

Remembering one statement from Richard Feynman…………….

“Since we are not quantum objects we may never be able to see the details of this process, but it clearly happens. “

So let me just say.....

Don’t believe all the hype.

There is no need for many-worlds, multiverses and hidden variables, and so on.

There are no problems with standard quantum mechanics!

I have been thinking about all of this stuff for many years.

I am convinced that the arguments that I have presented are correct.

Let me show you some of my other ways of thinking about this stuff.

Quantum Mechanics via Postulates or Some Things Everyone Knows(as I taught you)
(Even if not everyone believes them)

(i) The state of a physical system at a fixed time is a vector in a Hilbert space, $|\psi\rangle$, normalized such that $\langle\psi|\psi\rangle = 1$.

(ii) The state evolves in time according to the equation

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle \quad \text{or} \quad |\psi(t+t')\rangle = U(t')|\psi(t)\rangle = e^{-iH(t-t)}|\psi(t)\rangle$$

where *H* is "the Hamiltonian", some Hermitian linear operator. This is causal(deterministic or unitary) time evolution

(iii) Some (maybe all) Hermitian operators are "observables". If $|a_j\rangle$is an eigenstate of the observable *A* with eigenvalue a_j,

$$A|a_j\rangle = a_j|a_j\rangle$$

then we say "the value of *A* is certain to be observed to be a_j".

(iv) Every measurement of *A* yields one of the eigenvalues of *A*. The probability of finding a particular eigenvalue, a, if the system is in the state $|\psi\rangle$ is

$$Prob(a|\psi) = |\langle a|\psi\rangle|^2 = \langle\psi|a\rangle\langle a|\psi\rangle = \langle\psi|P(a)|\psi\rangle$$

where *P*(a) is the projection operator onto the state with eigenvalue a. (We assume, for notational simplicity, that *A* has a discrete spectrum.) If a_7 has been measured, then the state of the system after the measurement is

$$|a_7\rangle$$

Let us fill in some details.

The state of a physical system at a fixed time is a vector in Hilbert space. Following Dirac we call it $|\psi\rangle$. We normalize it to unit norm. It evolves in time according to the time-development operator, where the Hamiltonian is some Hermitian linear operator - a simple one if we're talking about a singleatom, and a complicated one if we're talking about a quantum field theory.

Now some, maybe all, Hermitian operators are "observables". If the state is an eigenstate of an observable A, with eigenvalue a, then we say the value of A is a - is certain to be observed to be a.

Now, strictly speaking, this is just a definition of what is meant by "observable" and "observed".

Of course, that's like saying Newton's second law F= ma, as it appears in textbooks on mechanics, is just a definition of what you mean by “force". That's true, strictly speaking, but we live in a landscape where there is an implicit promise that if someone writes that down, then when they begin talking about particular dynamical systems that they will give laws for the force, and not, say, for some quantity involving the 17th time derivative of the position!

Likewise, the words "observable" and "observed" have a history before quantum mechanics. People like to say all these things have a meaning in classical mechanics, but really it goes way earlier than classical mechanics. I'm sure the pre-Columbian inhabitants of Georgia were capable of saying, in their language, "I observe a deer", despite their scanty knowledge of Newtonian mechanics. Indeed, I even suspect that the deer was capable of observing the Native Americans despite its even weaker grasp on action and angle variables.

So there's an implicit promise in here that, when you put the whole theory together and start calculating things, that the words "observes" and "observable" will correspond to entities that act in the same way as those entities do in the language of everyday speech under the circumstances in which the language of everyday speech is applicable.

Now we come to the fourth item: every measurement that happens when the state $|\psi\rangle$ is not an eigenstate of the observable yields one of the eigenvalues, with the probability of finding a particular eigenvalue a proportional to the magnitude of the projection of the wave function onto the state with eigenvalue a. (I'm assuming here just for notational simplicity that the eigenvalue spectrum is discrete.) If a has been measured, then the state of the system after the measurement is just that part of the wave function - all the rest of it has been annihilated. And, of course, it has to be rescaled so it has unit norm again. This is the famous **projection** or **collapse** postulate. It is sometimes called "the reduction of the state vector".

It's very different from the previous three statements because it contradicts one of them: causal time evolution using the time-development operator is totally causal: given the initial state vector - given the initial state of the system - the final state is completely determined. Furthermore, this causality is time reversal invariant: given the final state the initial state is completely determined.

This operation is something other than unitary time development. It is not deterministic. It is probabilistic. It isn't just that you cannot predict the future from the past. Even when you know the future, you don't know what the past was. If I measure an electron and discover it is an eigenstate of σ_z with $\sigma_z = +1$, I have no way of knowing what its initial state was. Maybe it was $\sigma_z = +1$, maybe it was $\sigma_x = +1$, and it turned out that I was in the 50% probability branch that got the measurement $\sigma_z = +1$.

That ends the review of Quantum Mechanics as we have taught it to you.

Better than Bell: the GHZM effect

We talked a lot about Einstein's EPR ideas, Bells reaction to them and Bell's theorem and the relationships to hidden variables, locality, etc.

Now I will review of a pedagogical improvement on John Bell's famous analysis of hidden variables in quantum mechanics, which deserves to be widely publicized. It is easier to explain than Bell's original argument. It was created by David Mermin. The way we will think of this analysis is by imagining a physicist, whom we call "Albert", who was around at the time of the discovery of quantum mechanics in the late 1920s, and didn't believe it. Although some time has passed since then, he's still around - quite old but intellectually vigorous, and he still doesn't believe in it. Our task is to convince him that quantum mechanics is right and classical ideas are wrong, or as I say even primitive pre-classical ideas.

There's no point in trying to wow him with the any of the many quantum mechanical explanations of 1000s of experiments that has occurred or anything like that, because he is so deeply opposed to quantum mechanics and so old and stubborn that as soon as you start putting a particular quantum mechanical equation on the board his brain turns off, rather like my brain in a seminar on string theory. So the only way to convince him is on very general grounds - not by doing particular calculations

At first thought you say: "It's easy - quantum mechanics is probabilistic, classical mechanics is deterministic. If I have that electron in an eigenstate of σ_x and choose to measure σ_z, I can't tell whether I'm going to get +1 or −1. There's no way anyone can tell. That's very different from classical mechanics, and it seems to describe the real world".

But Albert is not convinced for a second by that. He says.

> "Probability has nothing to do with this fancy quantum mechanics. Jerome Cardan was writing down the rules of probability when he analyzed games of chance in the late Renaissance. When I flip a coin or go to Las Vegas and have a spin on the roulette wheel, the results seem to be perfectly probabilistic. But I don't see Planck's constant playing any significant role there" "The reason the roulette wheel gives me a probabilistic result is that there are all sorts of sensitive initial conditions which I can't measure well enough - initial conditions to which the final state of the ball is sensitive - there are alls orts of degrees of freedom of the system which I cannot control, and because of my ignorance, not because of any fundamental physics, I get a probabilistic result."

This is sometimes called the **hidden variable position**.

> "Really, you don't know everything about the state of the electron when you measure its momentum and its spin along the x-axis. There are zillions of unknown hidden variables which you can't control; maybe they are also in the system that is measuring the electron. (There's no separation in this viewpoint between the observer observing the system and the quantity being observed.) If you knew those quantities exactly, then you know exactly what the electron was going to do in any future experiment. But since you only know them probabilistically, you only have a probabilistic distribution."

Here I’ve written it down in somewhat fancy-shmancy mathematical notation.

Albert neither believes in nor understand quantum mechanics. "Deep down, it’s all classical!”

Probabilistic? "Just classical probability!"

$$A = A(\alpha)$$

where α = "subquantum" or "hidden" variables; maybe very many; may involve "apparatus" as well as "system".

$$Prob\{A \leq a\} = \int \theta(a - A(\alpha))d\mu(\alpha)$$

where $\mu(\alpha)$ = probability distribution for the hidden variables - "a result of our ignorance not some quantum nonsense!”

In fact, Albert is not wrong - you can get probability from classical mechanics.

John von Neumann way back was aware of this. He said: "No, that’s not the real difference between classical mechanics and quantum mechanics. The real difference is that in quantum mechanics you have non-commuting observables: If you measure σ_x repeatedly for an electron and take care to keep it isolated from the external world, you always get the same result. But if you then measure σ_z and get a probabilistic result, when you measure σ_x again you will again get a probabilistic result the first time - the first measurement of σ_z has interfered with the measurement of σ_x. That’s because you have non-commuting observables, and those are characteristic of quantum mechanics.

Noncommuting Observables? "Just interfering measurements!" according to Albert.

Albert continues

> "Absolute nonsense! We're big, clumsy guys. When we think we're doing a nice clean measurement of σ_x we might be messing up all of those hidden observables. When we measure your σ_z we then get a different result because we've messed things up. My friends the anthropologists talk about this a lot when they discuss how an anthropologist can affect an isolated society he or she believes they're observing. And, for some reason I don't understand, they call it the uncertainty principle".

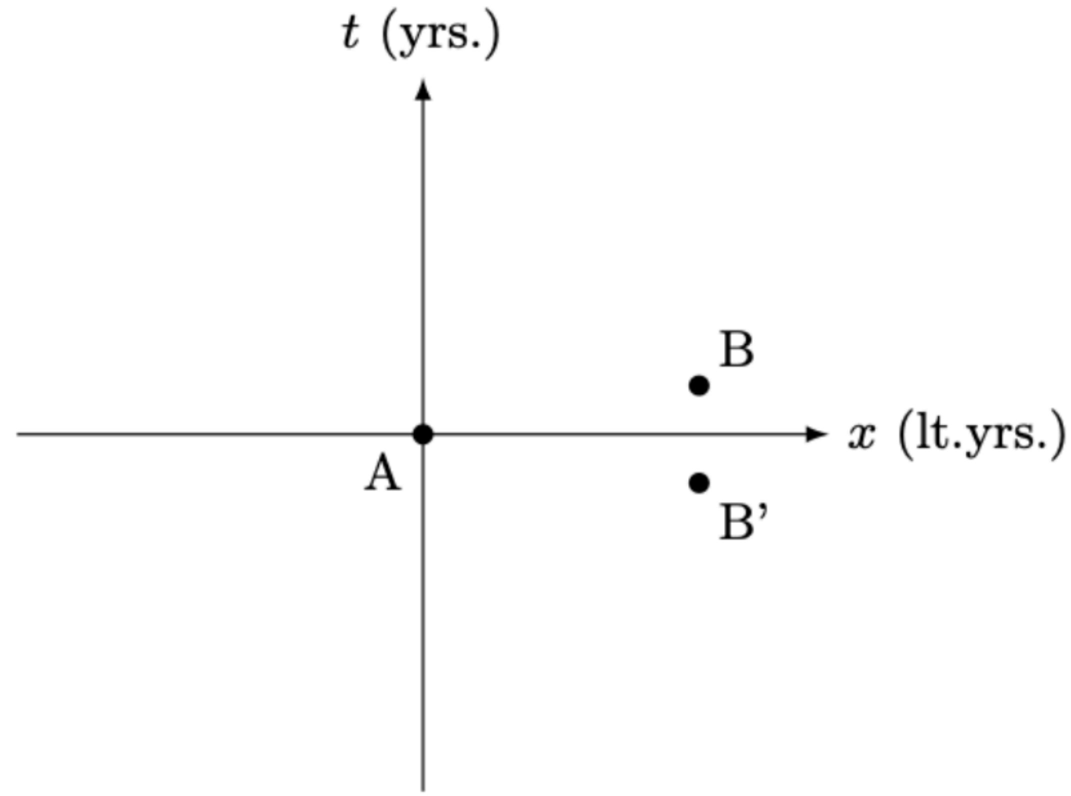
> "My friends the social psychologists tell me that if you do an opinion survey, unless you construct it very carefully, the answers you will get to the questions will depend upon the order in which they are asked?.

(This is true, by the way.) He doesn't see any difference between that and measurements of σ_x and σ_z. That's Albert's position.

As John Bell pointed, this is in fact an irrefutable position, despite all the stuff to the contrary that has been said in the literature. On this level there is no way of refuting it. He gave a specific example of a classical theory that on this level reproduced all the results of quantum mechanics - the de Broglie pilot wave theory.

However, if Albert admits one more thing, we can trap him. I will now explain what that one thing is.

Here we have a drawing of space-time. It's really four dimensional, but due to budgetary constraints I've had to represent it as a two-dimensional object. The scale has been chosen so that time t is measured in years and x in light-years, therefore the paths of light rays are 45-degree lines.



Simple Spacetime

Now let's consider two measurements on possibly two different systems done in two regions A and B - forget B' for the moment, its role will emerge later. Thus these black dots represent actually substantial regions in space time, during which an experiment has been conducted.

Now one thing Albert will have to admit is that although the results of an experiment in A may interfere with an experiment in B, the results of an experiment in B can hardly interfere with the results of an experiment in A unless information can travel backwards in time, which we will assume he does not accept. That's because A is over and done with and its results recorded in the log book before B occurs.

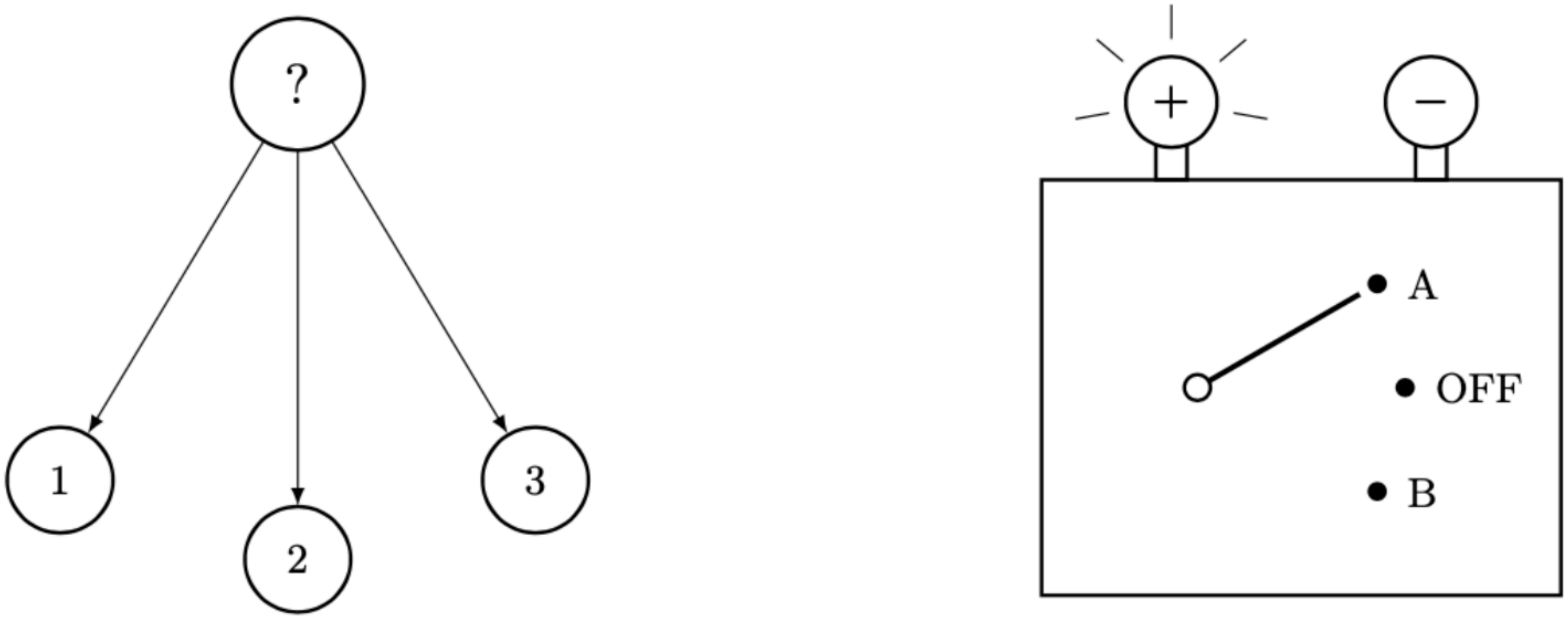
On the other hand, if we imagine another Lorentz observer with another coordinate system, B will appear as B' here. B and B', as you can see by eyeball, are on the same space-like hyperbola - there is a Lorentz transformation that leaves A at the origin of coordinates unchanged and turns B into B'. B and B' are space-like separated from A. A light signal cannot get from A to B, and nothing traveling slower than the speed of light can get from A to B.

Now that second Lorentz observer would give the same argument we gave, except she would interchange the roles of A and B'. She would say the results of an experiment at A cannot interfere with the act of doing an experiment at B' because B' is earlier than A. **But B' is B, just B seen by a different observer.**

Therefore, if you believe in the principal of Lorentz invariance, and if you believe you cannot send information backwards in time, you have to conclude that experiments done at space-like separated locations sufficiently far apart from each other cannot interfere with each other. It can't matter what order you ask the questions if this question is being asked of someone on Earth and the other of an inhabitant of the Andromeda Nebula, and they're both being asked today.

On everything else we accept the Albert position. Now here is the experimental proposal - this is a drawing from an imaginary proposal to the Department of Energy for the Albert experiment.

This amazing argument was obtained from an old friend - Sidney Coleman - it was to clever for me on the first reading, so I read in several times and eventually saw that the results gave the same conclusions as Bell's arguments.



The Albert Proposal

Three of Albert's graduate students are assigned to experimental stations, we assume they are several light-minutes from each other. The graduate students, with lack of imagination, are called numbers 1, 2, and 3. They're almost as old as Albert - it's difficult to get a thesis under him. They are informed that once a minute something will be sent from a mysterious central station to each of the three teams - what something is, they don't know. However, they're armed with measuring devices whose structure they again do not know. They are called dual cryptometers because they can measure each of two things, but what those two things are nobody knows - at least the grad students don't know. They can turn a switch to either measure A or measure B. They make this decision once a minute shortly before the announced time of the signal, and sure enough, a light bulb lights up that says either A is +1 or A is -1 if they are measuring A, or the same thing for B.

They have no idea what A or B is. It's possible the central station is sending them elementary particles. It's possible the central station is sending them blood samples, which they have the choice of analyzing for either high blood cholesterol or high blood glucose. It is possible the whole thing is a hoax, there is no central station, and a small digital computer inside the cryptometer is making the lights go on and off. *They do not know*.

In this way, however, they obtain a sequence of measurements, which they record as this.

$$
\begin{array}{lll}
A_1 = 1 & B_2 = -1 & B_3 = -1 \\
A_1 = 1 & A_2 = -1 & B_3 = -1 \\
B_1 = 1 & B_2 = 1 & A_3 = 1 \\
 & \cdots &
\end{array}
$$

The Data

The first line means observer 1 has decided to measure A and obtained the result +1; observer 2 has decided to measure B and obtained the result -1; and observer 3 has decided to measure B and obtained the result -1. They have obtained in this way zillions of measurements on a long tape. They record them in this way because they really believe that whatever this thing is doing, $A_1 = 1$, that is to say, the value of quantity A that would be measured at station 1 is +1 independent of what is going on on stations 2 and 3, because these three measurements are space-like separated. That's what they have to believe if they believe Albert. They have to believe there's really some predictable value of this thing which they would know if they knew all the hidden variables. In this particular case, they don't know what B_1 is but they know what A_1 is.

Now as they go through their measurements, they find in that roughly 3/8 of the measurements - they're making random decisions about which things they measure - whenever they measure one A and two B's the result of the product of the measurements is +1. Now they're making their choices at random and since they believe that these things have well-defined meanings independent of their measurements, they have to believe, if they believe in normal empirical principles, that all the time the value of one A and two B's - the value that would be obtained if they had done the measurement - the product is +1.

Sometimes all three of these numbers are +1. Sometimes one of them is +1 and two are -1. But the product is always +1. It's as if I gave you a zillion boxes and you turned up 3/8 of them and discovered each of them had a penny in it, you would assume within 1 over the square root of N - negligible error -that if you opened up all the other boxes, they would also have pennies in them. By the miracle of modern arithmetic - that is to say by multiplying these three numbers together and using the fact that each B squared is 1 - they deduce that if they look on their tape for those experiments in which they've chosen to measure the product of three A's, they would obtain the answer +1, i.e., they find whenever they measure $A_1B_2B_3$ it is +1; likewise for $B_1A_2B_3$ and $B_1B_2A_3$, so that they must deduce that $A_1A_2A_3 = +1$, i.e, the only way is for all the A,s to be 1.

Now let's look behind the scenes and see what's actually going on

It's not blood samples we're sending to them after all, it's three spin one-half particles arranged in the following peculiar initial state: one over the square root of two all spins up minus all spins down:

$$\frac{1}{\sqrt{2}}[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle]$$

A is simply σ_x for the particle that arrives at the appropriate station, and B is σ_y.

Let's first check that $A_1B_2B_3$ acting on this state is +1. We have

$$A_1 = \sigma_x^{(1)} \quad , \quad B_1 = \sigma_y^{(1)}, etc$$

so that using

$$\sigma_x |\uparrow\rangle = |\downarrow\rangle \quad , \quad \sigma_x |\downarrow\rangle = |\uparrow\rangle$$
$$\sigma_y |\uparrow\rangle = i\,|\downarrow\rangle \quad , \quad \sigma_y |\downarrow\rangle = i\,|\uparrow\rangle$$

we get

$$A_1 B_2 B_3 |\psi\rangle = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)} |\psi\rangle = \frac{1}{\sqrt{2}}[\sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)} |\uparrow\uparrow\uparrow\rangle - \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)} |\downarrow\downarrow\downarrow\rangle]$$

$$= \frac{1}{\sqrt{2}}[- |\downarrow\downarrow\downarrow\rangle + |\uparrow\uparrow\uparrow\rangle] = + |\psi\rangle$$

and similarly for $B_1A_2B_3$ and $B_1B_2A_3$.

However, for $A_1A_2A_3$, using the same process we always get

$$A_1 A_2 A_3 |\psi\rangle = - |\psi\rangle$$

What "spooky" action-at-a-distance is going on to make this happen? Actually none - this is just the result of using quantum mechanics reasoning as based on the postulates!

The return of Schrodinger's cat

Now I will turn to the much vexed question that was the subject of our class, sometimes called "the interpretation of quantum mechanics", although, as I will argue, that's really a bad name for it. I want to stress that I have made no original contributions to this subject. There is nothing I will say in this section that cannot be found in the literature.

Of course, such is the nature of the subject that there is nothing I will say where the contradiction cannot also be found in the literature. So I claim a measure of responsibility, if no credit - the reverse of the usual scholarly procedure.

The Albert grad students using only these proto-classical ideas - they aren't even so well developed to be called classical physics, they're sort of the underpinnings of classical reasoning - deduce that they will always get $A_1A_2A_3 = +1$, sometimes a +1 and two -1's, but always +1. In fact, if quantum mechanics is right, they will always get -1. This is superior to the original Bell argument for two reasons: Firstly, it doesn't involve correlation coefficients - it's not that classical mechanics says this will happen 47% of the time and quantum mechanics says it happens 33% of the time. Secondly, it is easy to remember - whenever I lecture on the Bell inequality I have to look it up again because I can never remember the derivation. This thing - the ingredients in it are so simple that if someone awakens you in the middle of the night four years from now, and puts a gun to your head, and says: "show me the GHZM argument", you should be able to do it.

We have shown that there are quantum mechanical experiments where the conclusions cannot be explained by classical mechanics - even the most general sense of classical mechanics - unless, of course, the classical mechanical person is willing to assume transmission of information faster than the speed of light, which, with the relativity principle, is tantamount to transmission of information backwards in time.

This is, of course, also John Bell's conclusion. This is, I must say, much misrepresented in the popular literature and even in some of the technical literature, where people talk about quantum mechanics necessarily implying connections between space-like separated regions of space and time. That's getting it absolutely backwards. There are no connections between space-like separated regions of space and time in this experiment. In fact, there's no interaction Hamiltonian, let alone one that transmits information faster than the speed of light, except maybe an interaction Hamiltonian between the individual cryptometers and the particles. But, otherwise, it's either quantum mechanics or superluminal transmission of information, not both.

Why on earth do people - I'm trying to see inside other people's heads, which is always a dangerous operation, but let me do it - why, why on earth do people get so confused, so wrong about such a simple point? Why do they write long books about quantum mechanics and non-locality full of funny arrows pointing in different directions? Okay, that's the technical philosophers. They really - well, I'll avoid the laws of libel - so, anyway, why do they do this? It's because, I think, secretly in their heart of hearts they believe it's really classical mechanics - that we're really putting something over on them - deep, deep down it's really classical mechanics.

People get things backwards and they shouldn't - it has been said, and wisely said, that every successful physical theory swallows its predecessors alive. By that we mean that in the appropriate domain - for example the way statistical mechanics swallowed thermodynamics - in the appropriate domain of experience, the fundamental concepts of thermodynamics - entropy for example, or heat - were explained in terms of molecular motions, and then we showed that if you defined heat in terms of molecular motion it acted under appropriate conditions pretty much the way it acted in thermodynamics. It's not the other way around. The thing you want to do is not to interpret the new theory in terms of the old, but the old theory in terms of the new.

There is a videotape of Feynman explaining elementary concepts in science to producer Christopher Sykes. He asked Feynman to explain the force between magnets. Feynman hemmed and hawed for a while, and then he got on the right track, and he said something that's dead on the nail. He said:

> "You've got it all backwards, because you're not asking me to explain the force between your pants and the seat of your chair. You want me, when you say the force between magnets, to explain the force between magnets in terms of the kinds of forces you think of as being fundamental - those between bodies in contact".

Obviously, I'm not phrasing it as wonderfully as Feynman. But, well, as Picasso said in other circumstances, it doesn't have to be a masterpiece for you to get the idea.

We physicists all know it's the other way around: the fundamental force between atoms is the electromagnetic force which does fall off as one over distance squared. Christopher Sykes was confused because he was asking something impossible. He should have asked to explain the pants-chair force in terms of the force between magnets. Instead he asked to derive the fundamental quantity in terms of the derived one! Likewise, a similar error is being made here. The problem is not the interpretation of quantum mechanics. That's getting things just backwards. The problem is the interpretation of classical mechanics.

"Every successful physical theory swallows its predecessor alive."

But it does so by interpreting the concepts of the old theory in terms of the new, NOT the other way around.

Thus our aim is NOT "the interpretations of quantum mechanics." It is the interpretation of classical mechanics.

Random Thoughts

Now, I'm going to address this, and in particular the famous, or infamous, projection postulate. The fundamental analysis is von Neumann's. I'd like to begin by recapitulating von Neumann's analysis of the measurement chain. I will summarize it first and then add many details.

The Measurement Chain (after von Neuman)

(1) Electron prepared in σ_x eigenstate:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle]$$

I measure σ_z:

$$|\psi\rangle = \begin{cases} |\uparrow\rangle \\ |\downarrow\rangle \end{cases} \quad \text{equal probabilities}$$

Non-deterministic "reduction of the wave function"

(2) Electron as before, measuring device in ground state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow, M_0\rangle + |\downarrow, M_0\rangle]$$

Electron interacts with the device:

$$|\psi\rangle \rightarrow \frac{1}{\sqrt{2}}[|\uparrow, M_+\rangle + |\downarrow, M_-\rangle]$$

(normal deterministic time evolution)

I observe device:

$$|\psi\rangle = \begin{cases} |\uparrow, M_+\rangle \\ |\downarrow, M_-\rangle \end{cases} \quad \text{equal probabilities}$$

Summary of von Neumann Approach

Now for some details. I prepare an electron in a σ_x eigenstate and I measure σ_z- the famous non-deterministic "reduction of the state vector" takes place, and with equal probabilities, I cannot tell which, the spin either goes up or down.

But this is rather unrealistic even for a highly idealized measurement. An electron is a little tiny thing, and I have bad eyes. I probably won't be able to see directly what it's spin is. There has to be an intervening measuring device. So we complicate the system.

The initial state is the same as before, as far as the electron goes, but the measuring device is in some neutral state $|M_0\rangle$. The electron interacts with the measuring device. Von Neumann showed us how to set things up with the interaction Hamiltonian so if the electron is spinning up the measuring device goes - maybe it's one of those dual cryptometers - the light bulb saying +1 flashes, if the electron is spinning down the light bulb saying -1 flashes. This is normal deterministic time evolution according to Schrodinger's equation or equivalently, the time-development operator.

Now I come by. I can't see the electron, but I observe the device. By the usual projection postulate, I either see it in state +1 or state -1. I make the observation, if I see the state +1, and the rest of the wave function is annihilated. I get with either probability these two things - note the state vectors $|\uparrow, M_+\rangle$and $|\downarrow, M_-\rangle$on the bottom figure. The result is the same as before because the electron is entangled with the device. I measure the device. The electron comes along for the ride!

Now let's complicate things a bit more. Let's suppose I cannot do the measurement because I'm giving a lecture. However, I have a colleague, a very clever experimentalist who has constructed an ingenious robot. I'll call him Gort. I say "Gort, during the lecture, I want you to go and see what the measuring device says about the electron". And so Gort goes and does this. Although he's an extremely ingenious and complicated robot, he's still just a big quantum mechanical system, like anything else. So it's the same story. Things starts out with electron in a superposition of up and down, measuring device neutral, a certain register and a RAM chip inside Gort's belly also has nothing written on it.

Then everything interacts and the state of this world is: electron "up", measuring devices says "up", Gort's RAM chip's register says "up", plus the same thing with "up" replaced by "down", all divided by the square root of two. And Gort comes rolling in the door there with his rollers, and I say: "Hey, Gort, which way is the electron spinning?" And he tells me. And wham-o, it either goes into one or the other of these states fifty percent probability.

I summarize Gort and all remaining actions below:

(3) Add robot

(4) Add colleague

The problem of death. Aharonov's question.

I will argue the there is

NO special measurement process

NO reduction of the wave function

NO indeterminancy

NOTHING probabilistic

in quantum mechanics.

ONLY deterministic evolution according to Schrödinger's Equation

Adding to the Summary of von Neumann Approach

Now Gort is very polite. He observes that I am lecturing. So rather than coming to me directly, he rolls up to my colleague sitting in the corner and hands him a clip of printout that says either up or down, and says: “Pass this on when the lecture is over". And he rolls away.

Well, of course, vitalism was an intellectually live position early in the 19th century. Proponents held that living creatures are not simply complicated mechanical systems. But it hasn't had many advocates this century.

I think most of us would admit that our colleague is just another quantum mechanical system, although perhaps more complicated than the electron and Gort, and certainly more likeable. Anyway, there he is.

So it's the same story as before: the state of the world after all this has happened is: electron "up", measuring device says "up", Gort's RAM chip says "up", my colleague's slip of paper says "up", plus the same thing with down, divided by the square root of two. After the lecture I go up to my colleague and say: "What's up?? Wham-o! He tells me. And the whole wave function collapses.

Now this is getting a little silly, especially if you consider the possibility that - after all, I'm getting on in years, I'm not in perfect health, here I am running around a lot - maybe I have a heart attack before the lecture is over and die. What happens then? Who reduces the state vector?

I had been reading von Neumann and thinking about this, and come to a conclusion which I did not like, which was solipsism: I was the only creature in the world that could reduce state vectors. Otherwise it didn't make any sense. I was not totally happy with this position, even though I was as egotistical as anyone - I was unhappy with the position. I was discussing this with a friend. Anyway, I explained this position to him, and he said: "I see. Tell me: before you were born, could your father reduce state vectors?"

After getting myself in this silly state of confusion, now I will argue that in fact there is no special measurement process, there is nor eduction of th estate vector in quantum mechanics, there is no indeterminacy, and nothing probabilistic - only deterministic evolution according to Schrodinger's equation. This is not a novel position.

In the famous paper on the cat, Schrodinger raised this position, the position that the cat is in fact in the coherent superposition of being dead and being alive, and instantly said it's ridiculous: "We reject the ridiculous possibility ...".

Some years later in the paper on Wigner's friend, where Wigner attempted to resolve the ancient mind-body problem through the quantum theory of measurement, he also raised this position, and said it was "absurd". Recently, Zurek added major contributions to the theory of decoherence - where instead of just saying it's ridiculous or absurd, he actually raised a question one can talk about. He said: "If this is so, why do I the observer perceive only one of the outcomes?" This is now the question I will attempt to address: Zurek's question. If there is no reduction of the state vector, why do I feel at the end of the day that I have observed a definite outcome, that the electron is spinning up or the electron is spinning down?

In order to ease into this, I'd like to begin with an analysis of Neville Mott summarized below:

N. Mott (1929) asked: "If an ionized particle is emitted in an s-wave state in the center of a cloud chamber, why is the ionization track a straight line rather than some spherical symmetric distribution?" [Of course, we must assume that particle momentum is unchanged (to within some small angle) when it scatters off an atom.]

Let $|\mathrm{C}\rangle$ be the state of the cloud chamber.

Define a "linearity operator" L, such that

$$L\,|\mathrm{C}\rangle = |\mathrm{C}\rangle \qquad \text{if track is straight (t.w.s.s.a.),}$$
$$L\,|\mathrm{C}\rangle = 0 \qquad \text{on states orthogonal to these.}$$

$$|\psi_\mathrm{i}\rangle = |\phi_{\boldsymbol{k}}, \mathrm{C_0}\rangle \;\rightarrow\; |\psi_{\mathrm{f},\boldsymbol{k}}\rangle$$

where $\phi_{\boldsymbol{k}}$ = state where the particle is concentrated near the center in position and near $\boldsymbol{k}$ in momentum.

$$L\,|\psi_{\mathrm{f},\boldsymbol{k}}\rangle = |\psi_{\mathrm{f},\boldsymbol{k}}\rangle$$

Now consider:

$$|\psi_\mathrm{i}\rangle = \int d\Omega_{\boldsymbol{k}}\,|\phi_{\boldsymbol{k}}, \mathrm{C_0}\rangle \;\rightarrow\; |\psi_\mathrm{f}\rangle = \int d\Omega_{\boldsymbol{k}}\,|\psi_{\mathrm{f},\boldsymbol{k}}\rangle$$

$$L\,|\psi_\mathrm{f}\rangle = |\psi_\mathrm{f}\rangle$$

Neville Mott Idea

Now for details. Neville Mott worried way back in 1929 about cloud chambers. He said:

> "Look, an atom releases an ionizing particle at the center of a cloud chamber in an s-wave(no angular momentum). And it makes a straight line track. Why should it make a straight line track? If I think about an s-wave, it is spherically symmetric. Why do they not get some spherically symmetric random distribution of sprinkles? Why should the track be a straight line?"

Now we're going to answer that question, and in a faster and slicker way then Neville Mott did. Of course, we have the advantage of 96 years of hindsight.

We must assume that the scattering between the particle and an atom when it ionizes it is unchanged or changed only within some small angle to begin with; otherwise, of course, even classically the particle would bounce around like a pinball on a pinball table. Let $|C\rangle$ be the state of the cloud chamber. We define a linearity operator L - a projection operator so that L on $|C\rangle$ equals $|C\rangle$if there is a track and it forms a straight line to within some small angle, and L on $|C\rangle$equals zero if the track is not a straight line, or there is no track for that matter. Now let's imagine we start out the problem in some initial state where the particle is concentrated near the center of the chamber and near some momentum **k**, and the cloud chamber in a neutral condition, all unionized ready to make tracks. This evolves into some final state. Now we all believe that if you started out with the particle in a narrow beam it would of course make a straight line track along that beam. The final state would be an eigenstate of this linearity operator and would have eigenvalue +1.

Now here comes the tricky part: not tricky to follow but tricky-clever. I consider an initial state that's an integral over the angles of **k** of this state. This is a state where the particle is initially in an s-wave, and the cloud chamber is still in a neutral state - that's independent of **k**. That state evolves by the linearity - the causal linearity of Schrodinger's equation - into the corresponding superposition of these final states here [see states $|\psi_f\rangle$ from earlier]. But if I have a linear superposition of eigenstates of the particle with respect to the operator L, each of which is an eigenstate with eigenvalue +1, then the combination is also an eigenstate with eigenvalue +1. So this also has straight line tracks in it.

That's the short version of Mott's argument. Mott said the problem is that people think of the Schrodinger equation as a wave in a three-dimensional space rather than a wave in a multi-dimensional space(configuration space). I would make a gloss on this and say: the problem is that people think of the particle as a quantum mechanical system but of the cloud chamber as a classical mechanical system. If you're willing to realize that both the particle and the cloud chamber are two interacting parts of one quantum mechanical system, then there's no problem. It's an s-wave not because the tracks are not straight lines but because there is a rotationally invariant correlation between the momentum of the particle and where the straight line points. But it's always an eigenstate of this linearity operator.

Nobody doubts it - the tracks in cloud chambers, or bubble chambers, etc, are straight lines, even if the initial state is an s-wave.

textNow I will give an argument with respect to Zurek's question. Zurek asked: "Why do I always have the perception that I have observed a definite outcome?" It is summarized below, with details following.

To answer Zurek's question, we must assume a (quantum) mechanical theory of consciousness.

$|S\rangle \in \mathcal{H}_S$ Hilbert space of states of observer **(Zurek)**

Introduce D, "the definiteness operator":

$D|S\rangle = |S\rangle$ if observer feels he has perceived only one of the outcomes

$D|S\rangle = 0$ on states orthogonal to these.

$$(1) \quad |\psi_i\rangle = |\uparrow, M_0, S_0\rangle \;\rightarrow\; |\psi_f\rangle = |\uparrow, M_+, S_+\rangle$$

$$D|\psi_f\rangle = |\psi_f\rangle$$

$$(2) \quad |\psi_i\rangle = |\downarrow, M_0, S_0\rangle \;\rightarrow\; |\psi_f\rangle = |\downarrow, M_-, S_-\rangle$$

$$D|\psi_f\rangle = |\psi_f\rangle$$

$$(3) \quad |\psi_i\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow, M_0, S_0\rangle + |\downarrow, M_0, S_0\rangle\right]$$

$$\rightarrow |\psi_f\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow, M_+, S_+\rangle + |\downarrow, M_-, S_-\rangle\right]$$

$$D|\psi_f\rangle = |\psi_f\rangle$$

Answering Zurek's Question

To answer this question, no cheating: we can't assume Zurek is some vitalistic spirit loaded with elan vital unobeying the laws of quantum mechanics.

We have to say the observer, say Zurek, has some Hilbert space of states, and some condition in Zurek's consciousness corresponds to the perception that he has observed a definite outcome, so there is some projection operator on it, the definiteness operator. If you want, we could give it an operational definition: the state where the definite-ness operator is +1 is one where a hypothetical polite interrogator asks Zurek: Have you observed a definite outcome?, and he says: "Yes". In the orthogonal states he would say: “No, gee, I was looking someplace else when that sign flashed" or "I forgot" or "Don't bother me, man, I'm stoned out of my mind" or, you know, any of those things.

Now let's begin. Our same old system as before: electron, measuring apparatus, and Zurek. If the electron is spinning up, the measuring apparatus measures spin in the up direction, and we get a definite state - no problem of superposition - and Zurek thinks: "I've observed a definite outcome". Same if everything is down. What if we start out with a superposition? Same story as Neville Mott's cloud chamber. The same reason the cloud chamber always shows the track to be a straight line is the reason Zurek always has the feeling he has observed a definite outcome.

Zurek didn't say: "It's a matter of common experience that in this experiment we always observe the electron spinning up", and Neville Mott didn't say: "It's a matter of common experience that in the cloud chamber the straight line is always pointing along the z-axis". The matter of common experience is that Zurek always has the perception that he has observed a definite outcome if you set up the initial conditions correctly. The matter of common experience is that the cloud chamber is always in a straight line. If you don't like this argument [the argument why Zurek perceives definite outcomes], you can't like that one [the argument why the cloud chamber detects straight lines]. If you like that one, you have to like this one.

The problem there - the confusion Neville Mott removed - was refusing to think of the cloud chamber as a quantum mechanical system. The problem here is refusing to think of Zurek as a quantum mechanical system.

We now go on to discuss the question of probability. A summary is given below with details following.

What about probability?

Classical probability theory reviewed:
Suppose we have an infinite sequence of coin flips, or, equivalently, a sequence $\sigma_r (r = 1, 2, \ldots)$ of plus and minus ones. We have a sequence of independent random flips of a fair coin if

$$\bar{\sigma} = \lim_{N\to\infty} \bar{\sigma}^N = \lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r = 0$$

and

$$\bar{\sigma}^a = \lim_{N\to\infty} \bar{\sigma}^{N,a} = \lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r\sigma_{r+a} = 0$$

for all a. Also

$$\lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r\sigma_{r+a}\sigma_{r+b} = 0$$

for all a, b. Etc.
(some mathematical niceties ignored.)

About Probability

Probability is a difficult question to discuss because it requires us to look at something counterfactual. If I ask whether a given sequence is or is not random, I can't do that even in classical probability theory for a finite sequence. For example, if I consider a binary sequence where the entries are either +1 or -1, and ask whether the sequence +1 is a random sequence, obviously there is no way of answering that question. But if I have an infinite sequence I can ask whether it's random. So let me talk about that.

Let me suppose I have an infinite sequence of +1 and -1's, which might represent heads and tails. I want to see if these sequences can be interpreted as a fair coin flip. Firstly, I want the average value of this quantity σ_r, which is of course the limit of the average of the first N terms as N goes to infinity, to converge to zero. Also, if I were an experimenter, I would probably look at correlations. I would take the r-th value σ_r times the (r+ a)-th value σ_{r+1} for some value of a, and look at the limit of this correlation, and ask that this quantity be also 0 for any value of a. That way I could reject sequences like +1,+1,-1,-1,+1,+1,-1,-1 . . . , which no one would call random. I could also look for triple and higher correlations. And if all those things were zero then I say there is a pretty good chance of a random sequence.

I would actually have to provide even further tests if I wanted the real definition of randomness, the Martin-Lof(Swedush logician) definition of randomness, but this will be good enough for now.

Now we want to ask the parallel question in quantum mechanics. We start out with an electron in the state I'll call sidewise - just our good old σ_x-eigenstate, the same x state I've used before. I consider an infinite sequence of electrons heading towards my σ_z-measuring apparatus, and I do the usual routine with the measuring system in my head and turn it into a sequence of memories in my head or maybe I have a notebook and I writes down +1,-1,+1,+1,-1...... I obtain a sequence of records correlated with the z-component of spin. I ask: "Does this observer observe this as a random sequence? That is to say, is this state here an eigenstate of the corresponding quantum observables with eigenvalue zero?"

Well, we know it's all correlated with σ_z. In order to keep the transparency from overflowing its boundaries, I just looked at σ_z , rather than the operator, for the records. I define the average value of σ_z exactly the same way as it is done up here(see below)

$$|\rightarrow\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

Consider

$$|\psi\rangle = |\rightarrow\rangle \otimes |\rightarrow\rangle \otimes |\rightarrow\rangle \cdots$$

This is an infinite sequence of electrons, each with $\sigma_x = 1$. Let these interact with a σ_z-measuring device and an observer, as before. Does the observer perceive a sequence of independent random flips? Equivalently, is $|\psi\rangle$ an eigenstate of

$$\bar{\sigma}_z = \lim_{N\to\infty} \bar{\sigma}_z^N = \lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_z^{(r)} = 0$$

with eigenvalue zero? (And likewise for σ_z^a etc.)

About Randomness

Then I ask: Is this an eigenstate of this operator with eigenvalue zero? If it is, we can say - despite the fact that there is nothing probabilistic in here - that the average value of σ_z is guaranteed to be observed to be zero.

Well, the calculation is sort of trivial. Let's compute the norm of the state obtained by applying this operator to this state (see below with details following):

$$\| \bar{\sigma}_z^N |\psi\rangle \|^2 = \frac{1}{N^2} \langle\psi| \sum_{r=1}^{N} \sum_{s=1}^{N} \sigma_z^{(r)} \sigma_z^{(s)} |\psi\rangle$$

$$\langle\psi| \sigma_z^{(r)} \sigma_z^{(s)} |\psi\rangle = \delta^{rs}$$

$$\Rightarrow \quad \lim_{N\to\infty} \| \bar{\sigma}_z^N |\psi\rangle \|^2 = \lim_{N\to\infty} \frac{1}{N^2} N = 0$$

Likewise for σ_z^a etc.

A definite deterministic state, definitely a random sequence. (An impossibility in classical physics—but this is not classical physics.)

Stoppard's Wittgenstein.

The Norm

It's two sums, and here I've written them out. Each of them is an individual thing, there's a 1 over N-squared, there's a sum on r, and a sum on s. Now in this particular state of course if r is not equal to s this "expectation value" is equal to zero, because you get just the product of the independent expectation values which are individually zero. On the other hand, if r is equal to s, then this is σ_z squared, which we all know is +1. Therefore, the limit of this thing up here is the limit of 1 over N-squared - the double sum collapses to a single sum, only the terms with r equals s contribute, and each entry with r equals s contributes 1, so you get N. Thus the result is N over N-squared, which is of course 0.

And the same thing happens for all those correlators, because each one is a sum of terms with a 1 over N-squared in front and only the entries that match perfectly will give you a nonzero contribution. So this definitely quantum mechanical state completely determined by the initial conditions nevertheless matches this experimenter's definition of randomness - something that would be impossible in classical mechanics, **but it's quantum mechanics, stupid.**

Now one final remark: In Tom Stoppard's play Jumpers, there's an anecdote about the philosopher Ludwig Wittgenstein. Someone is walking down the street in Cambridge and sees his friend Wittgenstein standing on a street corner lost in thought, and said: "What's bothering you, Ludwig?" Wittgenstein says: “I was just wondering why people said it was natural to believe the sun went around the earth rather than the other way around". The friend says: "Well, that's because it looks like the Sun goes around the earth”. Wittgenstein thinks for a moment and says: "Tell me: What would it have looked like if it had looked like if it was the other way around?"

Now people say the reduction of the state vector occurs because it looks like the reduction of the state vector occurs, and that is indeed true. What I'm asking you here is to consider seriously what it would look like if it were the other way around - if all that ever happened was causal evolution according to quantum mechanics. What I have tried to convince you is that what it looks like is ordinary everyday life. There is no need for "the reduction of the state vector".

Now you have some idea of what I am thinking about and working on in the area of Foundations of QM!

Hope you had fun……..

If you want more details I could continue……..(but it gets more technical)