On the measurement problem in quantum mechanics: a simple proposal

1 Introduction

Quantum mechanics is the theory of atoms and molecules: it explains why atoms absorb and emit light of definite frequencies, how atoms can combine to generate molecules.

For instance, it explains not only how hydrogen and oxygen atoms can combine to form water, but also the shape of its molecules.

Moreover, since matter is made of atoms and molecules, we can say that every object is as such thanks to the rules of quantum mechanics.

The peculiarity of quantum mechanics is that, in contrast to the classical theory, it is not deterministic: given the state of a system, i.e., knowing how it has been prepared, though it is true that – thanks to the Schrodinger equation – its evolution at any later time is well determined, it is not possible, in general, to predict the value of the physical quantities, such as energy, momentum, position etc., related to the system.

If one wants to know the value of a specific observable, the system must be subjected to a measurement process and only the probabilities of the results of such measurements can be predicted.

It is clear therefore, that there is a twofold evolution underlying any quantum system.

The first one is governed by the Schrodinger equation and is strictly deterministic; the second one reveals itself when a measurement is performed: the state of the system collapses, with a given probability p_i, into one of the eigenstates $|\xi_i\rangle$ of the measured observable ξ.

Ever since the work of Born, which constituted a milestone in the interpretation of quantum mechanics, this situation has been considered a weak point of the theory.

Indeed, the measuring apparatus must be a classical object, i.e., not subject to the laws of quantum mechanics: its 'pointers' are classical because they must be recordable by the observer.

This led to the conclusion that there is some limit beyond which quantum mechanics must give way to classical physics: Bohr's opinion was that there must be a dividing line, not necessarily a fixed one, between the quantum and classical domains.

At this point, however, Robert Griffiths' words(Consistent Quantum Theory) sound really appropriate:

> "... after all, what is special about a quantum measurement? All real
> measurement apparatus is constructed out of aggregates of particles to which
> the laws of quantum mechanics apply, so the apparatus ought to be described by
> those laws, and not used to provide an excuse for their breakdown."

The natural consequence of Griffiths' words is that the so called measuring apparatus must be included as part of the system, giving rise to a so-called Grand-System.

Nonetheless, this idea is not free from difficulties either: for instance, what operates the measurements on the Grand-System?

This is an endless question that led Wigner to suggest that the wavefunction collapse is ultimately due to our consciousness, even if later he distanced himself from this idea.

This following discussion will merge and synthesize many ideas results mentioned by many physicists and, above all, present all such contributions in a systematic way.

This discussion is organized as follows: in the next section the problems of the measurement in quantum mechanics are summarized, then some of the paradoxes, as the "de Broglie box" and the "Schrodinger cat", are considered.

In Section 3 the so called *Laboratory Physics Assumption* (LPA) is introduced and its consequences are discussed in Section 4, where it is shown how the aforementioned problems can be considered under a new perspective and the wavefunction collapse postulate can be replaced by the LPA.

In Section 5 the main points of this discussion are highlighted.

2 Interpretative challenges of quantum mechanics

2.1 The measurement problem

While the wavefunction collapse postulate had a place within a theory where there is a clear distinction between the quantum and the classical domains, it is now difficult, with the Grand-System, to pinpoint how it can be replaced.

Indeed, let the state of the quantum system be $\sum_i a_i |\xi_i\rangle$, Ξ the apparatus which measures the observable ξ and $|\Xi_i\rangle$ the "pointer states", i.e., the mutually orthogonal states of the apparatus (detector) when the display exhibits the value ξ_i; the pointer states generate the Hilbert space $\mathscr{H}_D$ of the detector.

For the Grand-System, prior to the interaction between the quantum system and the measuring device, the state is the product

$$\sum_i a_i \, |\xi_i\rangle \, |\Xi_0\rangle \tag{1}$$

where $|\Xi_0\rangle$ is the state of the untriggered measuring device.

The final state is

$$U(t_{\text{fin}}) \sum_i a_i \, |\xi_i\rangle \, |\Xi_0\rangle = a_i \, |\xi_i\rangle \, |\Xi_i\rangle. \tag{2}$$

This should be a pure state of the Grand-System but it is not clear at which point the measurement takes place.

In any case, since a set of ξ_i values, each with frequency $p_i = |a_i|^2$, are somehow registered in correspondence with the states $|\xi_i\rangle|\Xi_i\rangle$ of the Grand-System, then the Grand-System is only *formally* described by Eq. (2) but it actually behaves as the statistical mixture

$$\{|\xi_i\rangle|\Xi_i\rangle, \, |a_i|^2; \, i = 1, \cdots\}. \tag{3}$$

However, this result is clearly incompatible with the rules of quantum mechanics: the deterministic unitary evolution governed by Schrodinger's equation *cannot* transform pure states into statistical mixtures.

2.2 The paradoxes of quantum mechanics

The uneasiness produced by the interpretation of quantum mechanics is witnessed by the many paradoxes that were put forward in the early years of quantum mechanics.

The most popular ones are the "de Broglie box" and "Schrodinger's cat".

Both have in common the problems raised by the superposition principle when applied at the macroscopic scale.

We shall eventually see how they can be reconciled with our common sense.

Let us begin with de Broglie's box.

An electron is in a stationary state within a box.

By taking all necessary precautions, the box is split into two equal boxes by means of a sliding diaphragm, then the two boxes – let us say the right one and the left one – are taken far apart from each other.[1]

According to quantum mechanics, the electron is in a state $|A\rangle$ that is a superposition of the state $|r\rangle$, corresponding to the electron being in the right box, and of the state $|l\rangle$, in which the electron is in the left box.

Since one only knows that the probabilities of finding the electron in any one of the boxes is 1/2, the state $|A\rangle$ can be expressed as

$$|A\rangle = \frac{1}{\sqrt{2}} \left(|r\rangle + e^{i\phi} |l\rangle \right) \tag{4}$$

How can the phase ϕ be determined?

This is possible only if one can measure observables that have nonvanishing matrix elements between $|r\rangle$ and $|l\rangle$, since only in that case the two states can be made to interfere.

Certainly, there are infinitely many self-adjoint operators in the Hilbert space of the system having this property, but are there actually any real measurement instruments corresponding to those operators?

If the two boxes are sufficiently far apart, we are sure that in no laboratory such instruments do exist.

All the same, according to the principles of quantum mechanics, the state of the system is a pure state, even if in practice it behaves as the statistical mixture

$$\{|r\rangle, \tfrac{1}{2} \; ; |l\rangle, \tfrac{1}{2} \; \}. \tag{5}$$

Also in the case of Schrodinger's cat paradox we are sure that there do not exist observables (i.e., measurement instruments) with nonvanishing matrix elements between the dead and the alive cat: indeed, if such instruments existed, then with a couple of measurements it would be possible, with nonvanishing probability, to resuscitate a dead cat.

Therefore, in this case too, the state of the system (cat + the ampule of cyanide), although pure, behaves as a statistical mixture.

As already noticed, the paradoxical aspects of the above examples stem from the superposition principle being applied to macroscopically distant states in the first case, and to a macroscopic object as the cat in the second case.

Should we then accept that quantum mechanics applies only to the microscopic world?

Certainly not, since we know several cases of macroscopic systems that exhibit quantum properties, such as helium at very low temperatures or the superconductors.

Therefore, given that with Griffiths quantum mechanics applies to every system, how can we know if a given system can be described and treated with the laws of classical physics?

It is widely accepted that the postulates, or rather the interpretation, of quantum mechanics need some revision.

In this regard, interesting discussions of the various problems and proposals that have been put forward can be found in many articles and books.

The LPA proposal discussed in the subsequent section offers an different contribution to the subject that allows one to deal with most of the critical issues raised in the literature in a straightforward way, without resorting to hypotheses that may sound rather artificial or in contrast to the commonly agreed principles of quantum mechanics.

3 LPA

I now present a proposal called Laboratory Physics Assumption (LPA) as a substitute for the wavefunction collapse postulate.

It starts from the observation that in quantum mechanics it is assumed that to every self-adjoint operator ξ in the Hilbert space of any system there corresponds an instrument Ξ which measures the observable ξ.

The LPA considers as "observables" only the self-adjoint operators corresponding to existing measuring instruments in some laboratory in the world.[2]

This requirement may seem vague if not unviable: we shall see, however, that no problem arises in any of the situations in which it will be applied.

This assumption has strong effects on the very concept of "state": as it has been shown in the examples of the previous section, according to the classical rules of quantum mechanics pure states may behave as statistical mixtures.

The LPA asserts that in all such situations, i.e., in the absence of observables with non-vanishing matrix elements between the various component of the mixture, those states are actually statistical mixtures.

Thus, for instance, in the case of the de Broglie box the electron is, prior to observation, either in the $|r\rangle$ or in the $|l\rangle$ state, each with probability 1/2, and Schrodinger's cat is either dead or alive even before opening the box.

In other words, the superposition principle can be applied only to those states to which it makes sense, i.e., to states such that instruments capable of measuring the relative phases do exist.

Thus, even in the Wigner's friend paradox – where Wigner's friend belongs to the laboratory where the box with the cat and the ampule of cyanide are present – it does not make sense for Wigner, while outside the laboratory, to think of a coherent superposition between $|$ cat alive, friend happy $\rangle$ and $|$ cat dead, friend sad $\rangle$: a relative phase factor could not be measured.

Therefore, the properties ("pure" or "not pure") of a state are not intrinsic but depend on which observables (i.e., measurement instruments) do exist today.

Indeed, we can say that, while according to the rules of quantum mechanics a dead cat can in principle be resuscitated, according to the LPA today a dead cat cannot be brought to life again.

We have emphasized the word today because we cannot exclude that the observables that are not available today may be available in the future and states that today are statistical mixtures tomorrow will be pure.

The literature already offers examples of this kind: a few decades ago the two photons emitted in the cascade $^1S_0 \rightarrow {}^1P_1 \rightarrow {}^1S_0$ of the Calcium atom used by Aspect and collaborators to prove the violation of Bell's inequalities, could be considered a pure state only for distances between the two photons of a few tens of meters, today the experiment by Yin et al. shows that coherence can be maintained for more than one thousand kilometers

4 Consequences

As for the paradoxes of quantum mechanics, the LPA allows one to be freed from the embarrassing necessity of applying the superposition principle in situations where its application is indeed not appropriate.

However, the relevant consequence of the LPA is concerned with the measurement problem.

The first unavoidable question is about the status of a measurement instrument: with Griffiths, we take for granted that a measurement instrument is a quantum system.

It is quite obvious that the only pure states of the instrument are the pointer states: this means that coherent superpositions of the pointer states will never be observed as the result of a measurement.

According to the LPA, this is equivalent to stating the non-existence in $\mathcal{H}_D$ of observables with nonvanishing matrix elements between the pointer states, and this is taken as an inalienable part of the definition of a measurement instrument.

Therefore not only all the states as in Eq. (2) behave as statistical mixtures, but they are, indeed.

What was not possible according to the classical rules of quantum mechanics, i.e., the transition from pure states to statistical mixtures, is now possible thanks to the LPA.

The difference between the LPA and the postulate of the wavefunction collapse is only conceptual: while the second is an ad hoc postulate, the LPA reproduces the same results on the basis of a precise physical fact (the absence of observables in $\mathcal{H}_D$ with nonvanshing matrix elements between the pointer states) and is responsible for the transition from an initial pure state as Eq. (1) to a statistical mixture as Eq. (3).

Hence, it should be clear that, in the LPA perspective, the phenomenon commonly referred to as the wavefunction collapse must be ascribed to the measurement apparatus.

5 Conclusions

The main points of this discussion are

1. the properties of the states (pure, not pure) are determined only by the observables to which there correspond existing measuring instruments;

2. a different, but equivalent way of formulating the LPA is the following: the superposition principle can only be applied to states such that instruments capable of measuring the relative phases do exist (thus, a coherent superposition principle can be applied to particular states only).

Specifically, the superposition principle cannot be applied to the pointer states of a measuring instrument;

3. the state of the Grand-System, although starting as a pure state and having its time evolution formally determined by a unitary operator, ends up as a statistical mixture.

At this point the measurement has taken place, irrespective of whether observed or not.

A point that in this paper has only been touched upon (but is widely discussed in the literature) is that with the LPA nothing is definitive: since all depends on the actually existing observables, what today is a statistical mixture, tomorrow may be a pure state; a detector behaves as a classical object as long as no observable is available (in $\mathcal{H}_D$) which connects different pointer states.

I don't know how this perspective may be considered from a philosophical viewpoint, anyhow it is worth noting that in the measuring process the LPA reproduces the same results as the wavefunction collapse postulate: what changes is only the interpretation of the process.