Repeat of Time Evolution Derivation - it is clearly so important

Energy eigenvectors evolve in time with exponential factor $e^{-iEt/\hbar}$

—> for energy eigenvector $|E\rangle$ have $\hat{U}(t)\,|E\rangle = e^{-iEt/\hbar}\,|E\rangle$

Thus, if want to find time evolution of arbitrary state,

then follow procedure stated earlier:

1. Write arbitrary initial (t = 0) state $|\psi\rangle$ in terms of energy eigenvectors,

   i.e., use energy basis.

$$|\psi(0)\rangle = \sum_n a_n\,|E_n\rangle$$

where $a_n = \langle E_n | \psi\rangle$.

2. Operate with time development operator

$$|\psi(t)\rangle = \hat{U}(t)\,|\psi\rangle = \hat{U}(t)\sum_n a_n\,|E_n\rangle \;=\sum_n a_n\hat{U}(t)\,|E_n\rangle \;=\sum_n a_n e^{-iE_nt/\hbar}\,|E_n\rangle$$

3. Do measurement of observable, $\hat{B}$

   —> must change basis to eigenvectors of $\hat{B}$ (go to HOME space). Have

$$|E_n\rangle = \sum_k c_{kn}\,|b_k\rangle$$ where $\hat{B}\,|b_k\rangle = b_k\,|b_k\rangle$ and $c_{kn} = \langle b_k\,|\,E_n\rangle$

4. Finally, probability amplitude for measuring b_k if in state $|\psi(t)\rangle$ given by

$$\langle b_k \mid \psi(t)\rangle = \sum_n a_n e^{-iE_n t/\hbar} \langle b_k \mid E_n\rangle \quad = \sum_n a_n e^{-iE_n t/\hbar} c_{kn}$$

Have, formally, answered question,

although computation might be difficult.

Another way of thinking about time evolution

is to work directly with time-evolution operator $\hat{U}(t)$.

Makes above discussion more formal.

Most of operators have we been discussing correspond to observables.

In mathematics —> Hermitian operators.

Since their eigenvalues

are possible results of measurements of observables and eigenvalues

must be real numbers

—> always true for Hermitian operators.

Time-evolution operator representative of 2nd class of operators in quantum theory.

These operators do not represent observables,

but instead transform kets (states) into different kets (states).

Because coefficients of basis states in representation of arbitrary state

are related to probability amplitudes

and therefore sum of their absolute squares must equal 1,

these transformation-type operators, in mathematics —> unitary operators.

Unitary operators in mathematics have eigenvalues whose absolute value always equals 1.

From our study of complex numbers,

know that if $z = e^{i\alpha} = \cos\alpha + i\sin\alpha$, then $|z|^2 = \cos^2\alpha + \sin^2\alpha = 1$

or $|z| = 1$ always.

Thus, eigenvalues of any unitary operator

can always be represented by a complex exponential.

If eigenvectors/eigenvalues of $\hat{U}(t)$ represented by equation $\hat{U}(t)\,|\beta_n\rangle = e^{i\beta_n}\,|\beta_n\rangle$

then can write $$\hat{U}(t) = \sum_n e^{i\beta_n}\,|\beta_n\rangle\,\langle\beta_n|$$

In many physical systems,

energy operator does not change with time.

In quantum theory

—> both energy operator

and time-evolution operator have same eigenvectors.

If have $\hat{H}\,|E_k\rangle = E_k\,|E_k\rangle$, then —> can write $\hat{U}(t) = \sum_n e^{-iE_n t/\hbar}\,|E_n\rangle\,\langle E_n|$

—> for energy eigenvector have

$$\hat{U}(t)\,|E_k\rangle = \left[\sum_n e^{-iE_n t/\hbar}\,|E_n\rangle\,\langle E_n|\right]|E_k\rangle = e^{iE_k t/\hbar}\,|E_k\rangle$$

i.e., only change by phase factor.

—> no probabilities changes during time evolution of these states

and called **stationary states**.

More importantly, property gives method for finding time evolution for arbitrary state —>

1. Write initial arbitrary state in energy basis $|\psi(0)\rangle = \sum_n a_n\,|E_n\rangle$ where $a_n = \langle E_n\,|\,\psi\rangle$

2. Operate with time development operator

$$|\psi(t)\rangle = \hat{U}(t)\,|\psi\rangle = \hat{U}(t)\sum_n a_n\,|E_n\rangle \quad = \sum_n a_n \hat{U}(t)\,|E_n\rangle \quad = \sum_n a_n e^{-iE_n t/\hbar}\,|E_n\rangle$$

3. and so on as before......

If you wanted to do it, you are now able to to do QM calculations!!