

In this part I will discuss some questions, challenges, and new ideas about Quantum Measurement

1 - Decoherence and Quantum Measurement - Last Pass Summary

Introduction

Quantum mechanics, as we have seen, has two types of time evolution.

1. An isolated system evolves according to the Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi = H\Psi . \quad (1)$$

The solution satisfies $\Psi(t) = U(t)\Psi(0)$, and the time evolution operator $U(t) = \exp(-iHt)$ is unitary: $U^\dagger U = 1$.

2. However, when a measurement is made the state undergoes non-unitary von Neumann projection to an eigenstate corresponding to the observed eigenvalue.

Because the two types of time evolution are so radically different, one should demand a rigorous definition of exactly when each of them apply.

Under what conditions, exactly, does von Neumann projection occur?

When and why does the system deviate from ordinary Schrodinger evolution?

It is widely acknowledged as we have discussed that the conventional interpretation does not supply a satisfactory definition for when von Neumann projection (or “wavefunction collapse”) applies.

The modern formulation of quantum mechanics, as we have seen, does not require non-unitary wavefunction collapse.

The phenomenology of von Neumann projection can be reproduced under unitary evolution of the system as a whole, due to a phenomenon known as decoherence, as we have seen.

What is conventionally referred to as measurement is actually a continuous process that results from entanglement between the measured object and the many degrees of freedom in the measuring device or local environment.

Idealized Measurement of a Qubit

Let Q be a single qubit(state) and M a macroscopic device which measures the spin of the qubit along a particular axis.

The eigenstates of spin along this axis are denoted $|\pm\rangle$.

We define the operation of M as follows, where the combined system is $S = Q + M$.

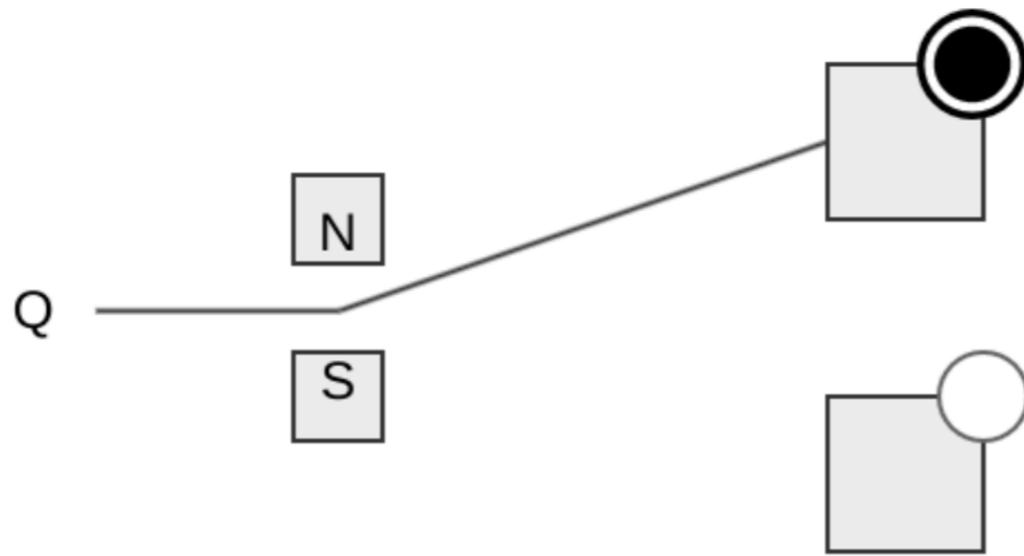
Note S includes both the measuring device and the qubit Q .

$$|+\rangle \otimes |M\rangle \longrightarrow |S_+\rangle, \quad (2)$$

$$|-\rangle \otimes |M\rangle \longrightarrow |S_-\rangle, \quad (3)$$

where S_+ denotes a state of the total system in which the apparatus M has recorded a $+$ outcome (i.e., is in state M_+), and similarly with S_- .

See figure below.



The figure shows an idealized measurement of a single qubit (spin): $|\pm\rangle$ states are deflected up or down, entering one of the two detectors.

Each detector has an indicator light which releases a macroscopic number of photons when activated.

In the figure, a $|+\rangle$ spin is deflected upwards and enters the top detector, causing its indicator light to emit photons.

The resulting state of the system is $|S_+\rangle$.

A $|-\rangle$ spin state results in the system state $|S_-\rangle$ (not shown), in which the photons are emitted from the bottom detector.

The states $|S_+\rangle$ and $|S_-\rangle$ differ macroscopically and have almost zero overlap $\langle S_+ | S_- \rangle \approx 0$.

However, if the combined system $S = Q + M$ evolves according to the Schrodinger equation (in particular, linearly), we obtain a superposition of measurement device states:

$$(c_+|+\rangle + c_-|-\rangle) \otimes |M\rangle \longrightarrow c_+ |S_+\rangle + c_- |S_-\rangle. \quad (4)$$

This seems counter to actual experience: measurements produce a single outcome, not a superposition state.

However, it is almost impossible for an observer in the state S_+ to be aware of the second branch of the wave function in state S_- .

Any object sufficiently complex to be considered either a measuring device or observer (for example, which can be regarded as semi-classical) will have many degrees of freedom.

A measurement can only be said to have occurred if the states M_+ and M_- are very different: the outcome of the measurement must be stored in a redundant and macroscopically accessible way in the device (or, equivalently, in the local environment).

In the figure, M_+ corresponds to the activation of the top detector, and a flash of its light. M_- corresponds to the activation of the bottom detector and its light.

Typically, the overlap of M_+ with M_- is effectively zero: of order $\exp(-N)$, where N is a macroscopic number of degrees of freedom.

This tiny overlap generally persists under further dynamical evolution.

For our purposes the phenomenology described above can be taken as a definition of what we mean by decoherence: two components of a given superposition state interact with, and become entangled with, environmental or measuring device degrees of freedom.

The resulting environmental components of the superposition state are radically different and have almost zero overlap.

For All Practical Purposes, to use Bell's terminology, an observer on one branch can ignore the existence of the other: they are said to have decohered.

Each of the two observers will perceive a collapse to have occurred, although the evolution of the overall system S has continued to obey the Schrodinger equation.

We can formalize this analysis using a density matrix.

Taking c_{\pm} equal for simplicity, the post-measurement state is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|+, M_+\rangle + |-, M_-\rangle \right) \quad (5)$$

$$\rho = |\Psi\rangle\langle\Psi| \quad . \quad (6)$$

Digression: Reminder about Density Matrices and Reduced Density Matrices

A density matrix ρ describes the quantum state of a physical system.

It is a generalization of the state vector ψ which represents a pure state.

For a pure state $\rho = |\psi\rangle\langle\psi|$, and satisfies $\text{tr } \rho^2 = 1$.

However, density matrices can also represent mixed states.

One example of a mixed state arises due to incomplete information: suppose we are told that ψ is randomly selected from an ensemble with equal probability of being in state $|+\rangle$ and $|-\rangle$.

Then the density matrix is

$$\rho = \frac{1}{2} \left(|+\rangle\langle+| + |-\rangle\langle-| \right)$$

Note for this mixed state $\text{tr} \rho^2 = 1/2$.

Another situation in which a mixed state arises is in the description of a degree of freedom A which is entangled with other degrees of freedom B.

While the combined state Ψ_{AB} of A and B may be a pure state, we cannot describe A by itself as such – it is in a mixed state.

The *reduced* density matrix describing A is obtained by tracing over B (i.e., summing over the basis vectors of the B Hilbert space):

$$\rho_A = \text{tr}_B \rho_{AB} = \text{tr}_B |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

Expectations values of operators O_A acting on the A subspace are given by $\langle O_A \rangle = \text{tr} O_A \rho_A$.

Under the assumption of entanglement between A and B, ρ_A describes a mixed state.

Returning to our discussion

We obtain a reduced density matrix ρ_Q , describing only the qubit degrees of freedom, by tracing over the M degrees of freedom as we showed earlier.

$$\rho_Q = \frac{1}{2} \left(|+\rangle\langle+| + |-\rangle\langle-| + \langle M_- | M_+ \rangle |+\rangle\langle-| + \langle M_+ | M_- \rangle |-\rangle\langle+| \right) \quad (7)$$

In a properly designed measurement, the states M_{\pm} are macroscopically distinct, with very small overlap.

Therefore, we have, as we saw earlier,

$$\rho_Q \approx \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \quad (8)$$

which describes a mixed state.

A mixed state is not a superposition, but rather can be interpreted as a classical ensemble of outcomes with certain probabilities.

In this case, there are two outcomes (spin up or down), each with probability 1/2.

Recall, the density matrix describing a pure state satisfies $\text{tr } \rho^2 = 1$, which no longer holds here. We see that decoherence can, For All Practical Purposes, reproduce the phenomenology of von Neumann projection:

there are two possible outcomes,

and only one is perceived by a semiclassical observer.

Macroscopic Superposition and Environmental Decoherence

Decoherence is also responsible for the emergence of a semiclassical reality from the quantum realm, due to interactions with the environment.

Consider a macroscopic object localized at spatial position x .

Now form a superposition state with support at two different locations x_1 and x_2 – i.e., a superposition of the object localized at x_1 with another state in which it is localized at x_2 .

What happens when this superposition state interacts with the environment?

Assume for example that the environment contains air molecules and photons which scatter from the object.

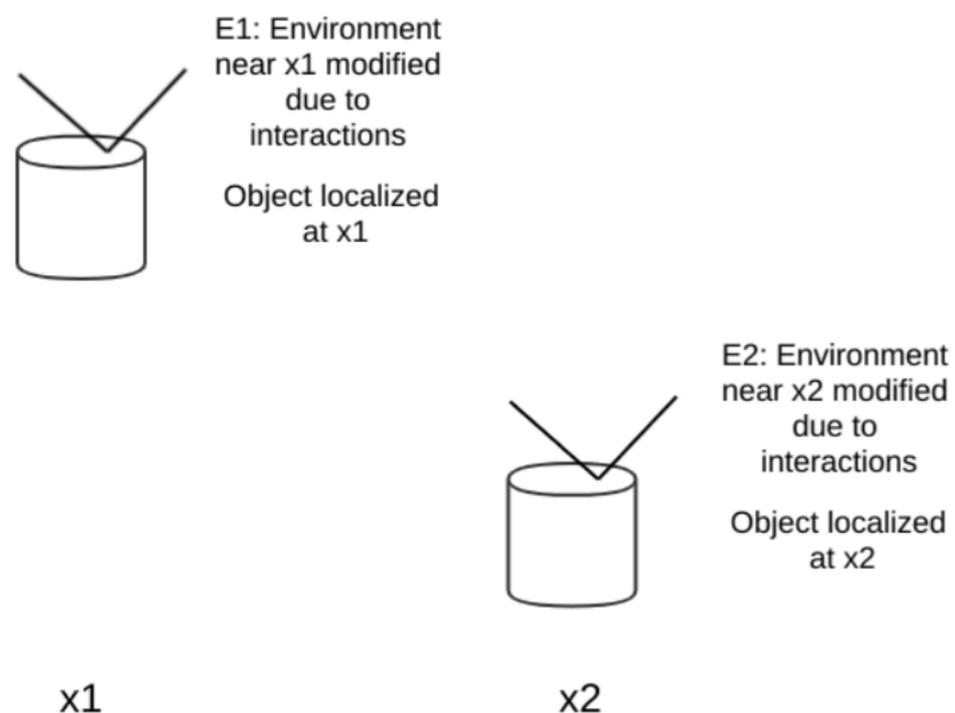
The environment near the object is altered by its presence.

Let E_1 be the environment state which results from interactions with the object at x_1 and E_2 be the environment state after interaction with the object at x_2 .

For example, when the object is at x_1 the air molecules near x_1 scatter from it, but near x_2 the air molecules are undisturbed.

Placing the object at x_2 rather than at x_1 reverses this pattern, so clearly E_1 and E_2 , describing the air molecules, are very different environmental states.

See figure below.



Interactions between the environment (e.g., air molecules) and a macroscopic object localized at x_1 lead to a modified environmental state $|E_1\rangle$, with scattered air molecules in the region near x_1 .

If the object is placed at x_2 the resulting modified environmental state $|E_2\rangle$ describes scattered air molecules in the region near x_2 .

$|E_1\rangle$ and $|E_2\rangle$ differ macroscopically and have nearly zero overlap.

Under (linear) Schrodinger evolution, the superposition state evolves as

$$\frac{1}{\sqrt{2}} (|x_1\rangle + |x_2\rangle) \otimes |E\rangle \longrightarrow \frac{1}{\sqrt{2}} (|x_1 E_1\rangle + |x_2 E_2\rangle) . \quad (9)$$

We can express the content of (9) as follows.

The macroscopic object becomes entangled, through interaction, with the many degrees of freedom in the environment.

The two branches of the superposition state $|x_1\rangle + |x_2\rangle$ decohere from each other due to interactions with the environment E , specifically because E evolves into very different states $E_{1,2}$ depending on the location of the object.

The reduced density matrix describing the object position is

$$\rho_x = \frac{1}{2} \left(|x_1\rangle\langle x_1| + |x_2\rangle\langle x_2| + \langle E_2|E_1\rangle |x_1\rangle\langle x_2| + \langle E_1|E_2\rangle |x_2\rangle\langle x_1| \right) . \quad (10)$$

Because E_1 and E_2 have very small overlap, we obtain

$$\rho_x \approx \frac{1}{2} \left(|x_1\rangle\langle x_1| + |x_2\rangle\langle x_2| \right) . \quad (11)$$

This is again a mixed state, not a pure state.

At this level of approximation, For All Practical Purposes, an observer either perceives the object at x_1 or x_2 , but not both.

Macroscopic superposition states of this kind can now be created in the lab: for example, drum-like mechanical resonators measuring around 10 microns across have been placed into superpositions of different vibrational modes.

The state of the macroscopic drum membrane, at a given instant in time, is a superposition with support in different positions, analogous to the setup analyzed above.

To maintain the superposition state (avoid decoherence) the drum system has to be isolated from environmental interactions.

The End

The purpose of this last summary was to remind you of the modern perspective on quantum measurement, taking into account decoherence.

Because of decoherence observers inside the system perceive outcomes consistent with the von Neumann projection postulate even if the system as a whole never deviates from Schrodinger time evolution.

That is, we can simply assume that the quantum state of the system (observer, measurement device, environment, etc.) evolves according to the Schrodinger equation at all times.

The projection postulate is not needed for quantum mechanics to describe the experiences of observers inside the system.

What is conventionally referred to as measurement is actually a continuous process that results from entanglement between the measured object and the many degrees of freedom in the measuring device or local environment.

Distinct branches of macroscopic superposition states lose contact with each other as they become entangled with environmental degrees of freedom.

The branches are said to decohere from each other.

Under Schrodinger evolution, which is unitary, the branches never fully disappear from the theory.

In fact, isolated quantum systems described by Schrodinger evolution can be shown to spend most of their time in macroscopic superposition states.

Macroscopic superposition states have been realized in the laboratory – there is no experimental evidence against the possibility that you might be in a superposition state as you read this.

The main open problem is to explain the Born rule, which associates probabilities $|c_{\pm}|^2$ with outcomes of measurements on superposition states as in equation (4).

In traditional formulations of quantum mechanics the Born rule is introduced ad hoc together with the projection postulate.

Under pure Schrodinger evolution it requires further justification.

Everything I have done with the measurement problem hinges on an implicit assumption, namely, the Born Rule for calculating probabilities, which do not have a proof - let me try to fix that!

2 - How Do the Probabilities Arise in Quantum Measurement?

Introduction

The quantum measurement problem as we have seen is considered as one of the important unresolved problems in physics although its origin goes back to the very inception of quantum mechanics nearly a century ago.

In spite of an abundance of ideas that have been pursued throughout the decades resulting in countless articles, absence of a satisfactory explanation of the processes involved in the quantum to classical transition, also known as quantum measurement problem, has tenaciously persisted as a frustrating feature of quantum physics.

This is very possibly because it involves the most distinctive characteristics of superposition of states in the quantum arena.

A quantum state that has not yet been measured is in a superposition of two or more possible states of definite eigenvalue, the superposition differing qualitatively from any one of those states.

There is no apparent manifestation of superposition in our familiar daily classical world, where tangible measurements of the quantum states are accomplished.

In fact when we try to extrapolate the quantum superposition to classical domain in its entirety, we end up with such absurdity as the existence of a simultaneously dead and alive Schrodinger's cat.

Yet it is an undeniable fact that the simultaneous existence of both the microscopic quantum world and the macroscopic classical world is essential for reality in a rather inseparably intertwined manner.

For example, we humans are large and as such belong to the macroscopic classical world.

However, everyone of us consists of about 7×10^{27} atoms each containing additional elementary particles, all of which are in the microscopic quantum domain.

Thus, we and everything else around us inevitably belong simultaneously both to the microscopic quantum as well as the macroscopic classical domain of the universe without ever paying much attention to this momentous reality.

Indubitably the quantum realm does not exist somewhere out there.

It is an essential part of our very existence.

This inevitable transition from the quantum to classical domain taking place every moment of our life represents the basic premise of the quantum measurement problem.

Significantly in recent times, experimental evidences conspicuously demonstrate that the distinctive phenomenon like quantum superposition just does not entirely disappear but inevitably get masked by interactions with the plethora of particles manifestly operational in the classical domain.

In a recent experiment, using a rather sophisticated procedure, coherent superposition has been demonstrated in a macroscopic object containing an estimated ten trillion atoms.

For this purpose, the investigators used a 40 micron long mechanical resonator, just large enough to be visible with the naked eye.

The resonator with a resonant frequency of 6.175 GHz to its first excited phonon state was cooled to a temperature of merely 25 mK over absolute zero and put in a very high vacuum to minimize environmental effects.

Under these circumstances, the resonator was confirmed to be in its ground state.

Then a signal from a coupled qubit possessing the resonance frequency of 6.175 GHz was injected into the resonator thereby transferring the superposition feature of the qubit to the macroscopic object.

Superposition of the ground and the first excited phonon state of the macroscopic resonator lasted for the resonator relaxation time of 6.1 ns.

The above demonstration provides strong evidence that quantum mechanics and its attendant aspect of superposition applies to macroscopic objects and can be revealed under appropriate circumstances provided that it was sufficiently decoupled from its environment.

Can it apply to Schrodinger's cat?

The answer in principle should be yes.

But to prove it, the cat will surely perish for other reasons!

Because in order to conduct the experiment, it would be necessary to remove all sources of environmental disturbances exposing the cat to exceptionally low temperatures and high vacuum that would stop the metabolic processes for its survival.

More recently, the quantum phenomena essentially arising from quantum fluctuations and superposition has been demonstrated in an as large an object as a man size 40 kg mirror in a gravitational detector.

The was concluded,

“It is remarkable that quantum vacuum fluctuations can influence the motion of these macroscopic, human-scale objects, and that the effect is measured.”

These experiments strongly point toward the fact that the quantum effects of the microscopic world is indeed present in the macroscopic domain but substantially veiled in their existence by the effects of some processes for the disappearance of the distinct quantum characteristics and the appearance of the classical world where we deal with an innumerable number of particles.

Although substantial progress has been made, exactly how this is accomplished still comprises a subject of an overabundance of investigations with some intense debates.

However, one particular aspect that is common to all these investigations is the scarcity of comprehension about where do the probabilities, rather than a certainty, in quantum measurement come from.

So far, only some *ad hoc* propositions such as Born's rule have allowed the physicists to predict experimental results with uncanny accuracy of better than a part in trillion.

But the basic cause of this essential rule has remained shrouded in a veil of mystery.

Some investigators have attempted to provide a derivation of the Born rule.

But they faced a stiff resistance from some foremost investigators including one of the giants of physics of our time, Nobel laureate Steven Weinberg.

In his classic textbook, *Lectures on Quantum Mechanics*, Weinberg states,

“There seems to be a wide spread impression that decoherence solves all obstacles to the class of interpretations of quantum mechanics, which take seriously the dynamical assumptions of quantum mechanics as applied to everything, including measurement.”

Weinberg goes on to characterize his objection by asserting that the problem with derivations of the Born's rule is that,

“they are clearly circular, because they rely on the formula for expectation values as matrix elements of operators, which is itself derived from the Born rule.”

He questions,

“If physical states, including observers and their instruments, evolve deterministically, where do the probabilities come from? “

Again in his recent book, Weinberg questions,

“So if we regard the whole process of measurement as being governed by the equations of quantum mechanics, and these equations are perfectly deterministic, how do probabilities get into quantum mechanics?”

Maximilian Schlosshauer and Arthur Fine remark,

“Certainly Zurek's approach improves our understanding of the probabilistic character of quantum theory over that sort of proposal by at least one quantum leap.”

However, they also criticize the derivation of the Born's rule for obvious circularity, stating:

“We cannot derive probabilities from a theory that does not already contain some probabilistic concept; at some stage, we need to “put probabilities in to get probabilities out.”

In this discussion, I present a plausible solution of the mysterious appearance of probabilities from some basic aspects of the well-established Quantum Field Theory (QFT) of the Standard Model of particle physics.

Our argument relies on some characteristics of the universal quantum fields that appear to predetermine the values of the complex coefficients involved in the inherent superposition of eigenstates before measurement.

Thus, it seems the century old mystery of quantum to classical transition could get a necessary boost from some recently revealed fundamental properties of the universe through the advent of QFT.

Significant characteristics of quantum fields

The ultimate ingredient of reality, uncovered by science so far, consists of fields, which are distinctively non-material in nature.

Some perception of a field can be gained from our daily experience with the classical field of gravity that pervades us.

The field that we do experience is stable everywhere in our vicinity but varies from place to place around its origin, the Earth.

However, the ultimate reality of quantum fields also pervade all space including the one in which we exist, although we have no perception of them what so ever.

Unlike the stable classical fields, however, the quantum fields are distinctly different in that they are incessantly teeming with intrinsic, spontaneous, and totally random activity all taking place locally in all space time elements, from the infinitesimal to the infinite everywhere in this unimaginably vast universe.

Even though, we do not perceive its lively reality, indisputable evidence of its existence can be found everywhere in nature with the help of appropriate equipment.

An outline of the salient features of the universal quantum fields are summarized below:

- Quantum fields are the primary ingredients of reality, from which all else is formed, fills all space and time.
- Every fragment, each spacetime element of the universe, has the same basic properties as every other fragment.
- “The deeper properties of the quantum field theory [...] arise from the need to introduce infinitely many degrees of freedom, and the possibility that all these degrees of freedom are excited as quantum mechanical fluctuations.”
- Thus the quantum fields are indeed alive with eternal, incessant, innately spontaneous, totally unpredictable activity of the quantum fields locally at each space time element even in perfect vacuum at absolute zero temperature.
- “Loosely speaking, energy can be borrowed to make evanescent virtual particles. Each pair passes away soon after it comes into being, but new pairs are constantly boiling up, to establish an equilibrium distribution.”
- One of the most notable aspects of the liveliness of the quantum fields is the fact that the expectation value or the average value of the quantum fields has remained immutable almost since the beginning of time in spite of its unique spontaneous random activities up to infinite dynamism.

Any reasonable concept of physical reality should then owe its eventual origin to the fundamental reality of quantum fields and their characteristic attributes.

Of particular interest to us is to explore how the incessant, innately spontaneous and totally unpredictable activity of the primary reality of quantum fields comprising the overabundance of quantum fluctuations could foster the probabilistic nature of quantum states.

All fundamental particles are inseparably intertwined in their existence with the quantum fields.

Effects of the quantum fluctuations appear to have been generally underestimated even though matter would not have certain exceptional properties like the anomalous g factor and the Lamb shift without them.

Wave function of an electron

The elementary particles like electrons, one of the initial products of material formation from the abstract but physical quantum fields, are quanta of the fields.

However in reality, the physical electron state is actually a superposition of states produced by the interactions with the other fields of the standard model.

But then the most significant question is, for a nonrelativistic single electron, what are these states that are superposed and how do they owe their existence to the interactions with the other fields?

Are these really typical quantum states or just irregular disturbances in the field?

In order to give a physical depiction of the disturbances of the fields and quantum fluctuations, quoting Wilczek:

“Here the electromagnetic field gets modified by its interaction with a spontaneous fluctuation in the electron field—or, in other words, by its interaction with a virtual electron-positron pair.

The virtual pair is a consequence of spontaneous activity in the electron field.

They lead to complicated, small but very specific modifications of the force you would calculate from Maxwell’s equations.

Those modifications have been observed, precisely, in accurate experiments.”

Emphasizing Wilczek’s critical observation again that in spite of the precipitous transitory characteristics of the virtual particles, there is an equilibrium distribution.

Paraphrasing for further clarification, it turns out that the innately spontaneous activity of the electron field disturbs the electromagnetic field around them, and so electrons spend some of their time as a combination of two disturbances, one in the electron field and one in the electromagnetic field.

The disturbance in the electron field is not an electron particle, and the disturbance in the photon field is not a photon particle.

However, the combination of the two is just such as to be a nice ripple, with a well-defined energy and momentum, and with an electron’s mass.

This continues on and on, with a ripple in any field disturbing, to a greater or lesser degree, all of the fields with which it directly or even indirectly has an interaction.

So we ascertain that particles are just not simple objects, and although we often naively describe them as simple ripples in a single field, that is far from true.

Only in a universe with no spontaneous activities — with no interactions among particles at all — are particles merely ripple in a single field!

Would not it then be cogent to pronounce that the “states” being superposed here are the irregular disturbances of the fields originating from the incessant, innately spontaneous, and totally unpredictable quantum fluctuations?

In fact we know quite explicitly what the states are out of which the physical electron is built, at least order by order in perturbation theory.

The irregular disturbances of the fields indeed correspond to virtual particles.

In particular, their respective energy-momentum does not correspond to the physical mass of a particle.

One says that these particles are off-shell.

However, in the process, the total energy-momentum is exactly conserved at all times.

Because of the self-interaction of the quantum fields, such an energy-momentum eigenstate of the field can be expressed as a specific Lorentz covariant superposition of field shapes of the electron field along with all the other quantum fields of the Standard Model of particle physics.

It is particularly important to emphasize again that the quantum fluctuations are transitory but new ones are constantly boiling up to establish an equilibrium distribution so stable that their contribution to the screening of the bare charge provided the measured charge of the electron to be stable up to nine decimal places (noteworthy, the elementary charge is no longer a measurable quantity because it is exactly defined since 20 May 2019 by the International System of Units) and the electron g-factor results in a measurement accuracy of better than a part in a trillion.

The Lorentz covariant superposition of fluctuations of all the quantum fields in the one-particle quantum state can be conveniently depicted leading to a well behaved smooth wave packet.

A fairly rigorous underpinning of the wave packet function for a single particle QFT state in position space for a scalar quantum field has been provided by Klauber.

Since particles of all quantum field are invariably an admixture of contributions from essentially all the fields of the Standard Model, the wave packet function of a single particle of a scalar quantum field can be considered to be qualitatively representative of those of the spinor and vector quantum fields as well.

Following Klauber, the wave function $\psi(x)$, for an electron in one dimension, can then be given by the Fourier integral

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{\psi}(k) e^{ikx} dk \quad (1)$$

where $\tilde{\psi}(k)$ is a function that quantifies the amount of each wave number component $k = 2\pi/\lambda$ that gets added to the combination.

From Fourier analysis, we also know that the spatial wave function $\psi(x)$ and the wave number function $\tilde{\psi}(k)$ are a Fourier transform pair.

Therefore, we can find the wave number function through the Fourier transform of $\psi(x)$ as

$$\tilde{\psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x) e^{-ikx} dx \quad (2)$$

Thus the Fourier transform relationship between $\psi(x)$ and $\tilde{\psi}(k)$, where x and k are known as conjugate variables, can help us determine the frequency or the wave number content of any spatial wave function.

A plot of the wave function $\psi(x)$ in equation (1) gives us the familiar wave function of a quantum particle like electron (Fig. 1).

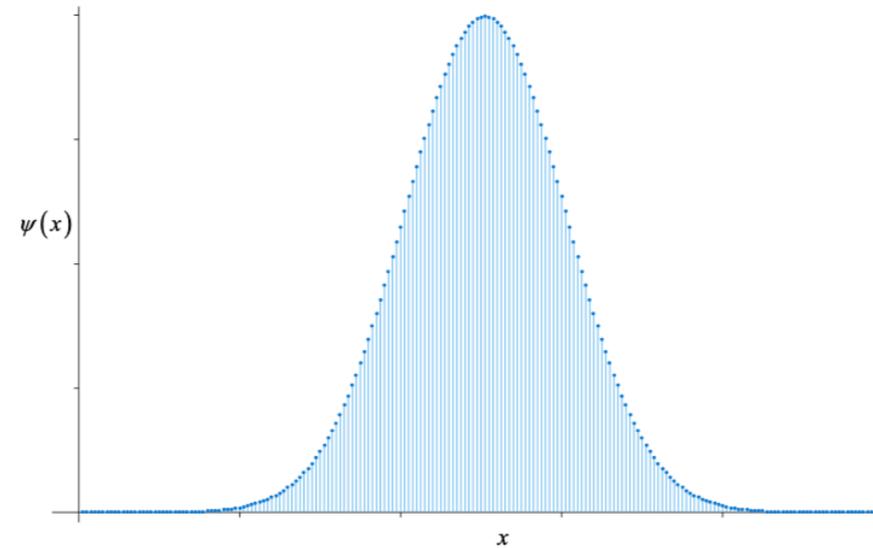


Figure 1: Plot of Gaussian wave packet function of an electron portrayed in equation (1) in position space. The amplitudes represent genuine reality as they correspond to the contributions from various quantum fields, the verified ultimate reality uncovered by science so far.

A few unique aspects of this depiction should be noted:

- First of all, the entire wave function as a whole represents all the requisite properties of the single electron. Therefore, in measurement, the entire wave packet should be acquired holistically or nothing at all. Experimental results demonstrate [_____](#) that the entire extended wave packet can be reduced to the position of measurement instantaneously quite possibly because of the entanglement of the wave packet with the wave function of the quantum vacuum.
- The plot (Fig. 1) is a superposition of amplitudes as a function of position x . But what do these amplitudes represent? As we have repeatedly emphasized, these amplitudes arise from a mixture of different quantum fields even for a single quantum of the electron field. Consequently, they have nothing in common except energy, since the attributes of energy is the same irrespective of which quantum field they belong to. We are aware that these amplitudes do not constitute charge distribution of an electron as originally proposed by Schrödinger and shown to be incorrect by Born from his electron scattering experiments. But then what could these amplitudes mean?

Probability amplitudes

Following Einstein's intuition, it would be cogent to consider that in measurement, a quantum particle would have the highest probability of being found where the intensity or the energy density of the quantum particle is the highest inside the wave packet.

Energy density of a wave is given by the square of its amplitude.

Therefore, to get the probability density, we have to take the square of the amplitude of the wave function, which usually involves a complex quantity.

Consequently, the absolute square amplitude $|\psi(x)|^2 = \psi^*(x)\psi(x)$, which is the probability density function $p(x)$, should represent the probability density for finding a particle in position space.

Thus

$$p(x) = |\psi(x)|^2 \quad (3)$$

Since the total probability is 1, the integral

$$\int_{-\infty}^{+\infty} \psi^*(x)\psi(x)dx = 1 \quad (4)$$

Max Born did something similar in formulating his famous Born's rule.

Quoting Born from his Nobel Lecture:

“Again an idea of Einstein's gave me the lead. He had tried to make the duality of particles —light quanta or photons—and waves comprehensible by interpreting the square of the optical wave amplitudes as probability density for the occurrence of photons.

This concept could at once be carried over to the ψ -function: $|\psi(x)|^2$ ought to represent the probability density for electrons (or other particles).”

Since then it is known as the Born's rule.

However, Born could not have realized at his time that the wave function of the electron is derived from real quantum fields and therefore actually is real and so are the energy density amplitudes that can be described as probability amplitude, which consequently are real as well.

It appears to be an erudite guess from Born's part, especially judging from the fact that the single particle wave function was considered a fictitious mathematical construct at the time and the square of the wave function was added after submission of the original manuscript in 1926 without any mention of energy density or intensity involved with wave functions.

It is hard to imagine a fictitious mathematical construct having any energy density or intensity!

By now it should be evident that a quantum particle wave packet function is indeed real and far from being fictitious.

Also Born's rule can be reasonably derived following Einstein's intuition and does not need to be a mere postulate.

The fact that the position of the electron is given by a probability instead of certainty should not be surprising either.

It is inevitable since the wave packet function is real.

The Fourier transform correlations between conjugate variable pairs of any real wave packet have powerful consequences since these variables obey the uncertainty relation

$$\Delta x \Delta k \geq \frac{1}{2} \quad (5)$$

where Δx and Δk relate to the standard deviations σ_x and σ_k of the wave packet.

This is a completely general property of a wave packet with a reality of its own and is in fact inherent in the properties of all wave-like systems.

It becomes important in quantum mechanics because of the real wave nature of particles having the relationship $p = \hbar k$, where p is the momentum of the particle.

Substituting this in the general uncertainty relationship of a wave packet, the intrinsic uncertainty relation in quantum mechanics becomes

$$\Delta x \Delta p \geq \frac{1}{2} \hbar \quad (6)$$

It is thus evident that a particular fixed value of the position x is not compatible with other measurable quantities like momentum.

However, it is critically important to note that whichever position x turns up in the measurement process, its probability amplitude is predetermined from the complex interactions of the various quantum fields and encoded in the wave packet function.

Again emphasizing from our earlier rather elaborate discussions, it should be aptly highlighted that the wave function is real and the computable value of the complex amplitudes $\psi(x)$ in the wave packet function is preordained from the indispensable interactions of the various quantum fields and their quantum fluctuations involved in the formation of the electron wave packet function.

For clarification, the wave number k in the argument of the wave function for a massive particle like electron obeys the relativistic relation $p = \hbar k$.

Therefore, for electrons k in the exponent should be replaced by p/\hbar .

Probability in Hilbert space

It is of immense interest to emphasize again that the amplitudes in the wave packet of a particle resulting from the contributions of the diverse quantum fields are already predetermined.

Hence the probability distribution of a quantum observable is already preordained before as well as after the unitary evolution of the Schrodinger equation.

Can this comprehension be cogently extended to the observables in the customary Hilbert space formalism?

In Hilbert space, use of Dirac's abstract algebraic model of bras and kets, from the bracket notation for the inner product, proved to be of great computational value.

However, there were serious difficulties in finding a mathematical justification for using them in observables that have continuous spectrum as in the wave packet function of an electron we have been exploring so far.

These difficulties were circumvented by the advent of the *rigged Hilbert space* (RHS) in 1960s.

The RHS is neither an extension nor an interpretation of the physical principles of Quantum Mechanics, but is simply a mathematical tool to extract and process the information contained in observables that have both continuous as well as discrete spectrum.

Therefore, in spite of not using the Dirac's bras and kets for the single particle, the inferences derived for the single particle wave packet can still be reasonably extended to the Hilbert space formalism. Observables with discrete spectrum and a finite number of eigenvectors (e.g., spin) do not need the RHS. For such observables, the Hilbert space is sufficient.

The Hilbert space is a square integrable, complex, linear, abstract space of vectors possessing a positive definite inner product assured to be a number.

The states of a quantum mechanical system are vectors in a multi-dimensional Hilbert space containing an orthonormal basis set of eigenfunctions.

The observables are Hermitian operators on that space, and measurements are orthogonal projections.

The quantum wave functions, for example, the solutions of the Schrödinger equation describing physical states in wave mechanics are considered as the set of components $\psi(x)$ of the abstract vector Ψ , the state vector.

However, the state vector does not depend upon any particular choice of coordinates.

The same state vector can be described in terms of the wave function in position or momentum state or written as an expansion in wave functions ψ_n of definite energy

$$|\Psi\rangle = \sum_n \psi_n |E_n\rangle \quad (7)$$

suggesting that every linear combination of vectors in a Hilbert space is again a vector in the Hilbert space.

In general, the energy eigenstates $|E_n\rangle$ may not commute with the position eigenstates $|x\rangle$.

The normalized square moduli $|\psi_n|^2$ of the energy complex coefficients are then interpreted as the probability for the system to be in the energy state $|E_n\rangle$ analogous to the single particle wave function where $|\psi(x)|^2$ is interpreted as the probability density for the particle to be at $|x\rangle$.

It is worthy to note that the coefficients $\psi_n(t)$ change during unitary time evolution but the probability for measuring outcome in a given energy eigenstate does not.

The solution of the time-dependent Schrodinger equation is given by:

$$|\Psi(t)\rangle = \sum_n \psi_n(0) e^{-i\frac{E_n}{\hbar}t} |E_n\rangle \quad (8)$$

where E_n is the eigenvalue of the corresponding energy eigenvector $|E_n\rangle$.

From (8), it is seen that for measurements in the energy basis the time dependence of $\psi_n(t)$ during unitary evolution drops out of the square modulus of the wave vector for computing the probability

$$|\psi_n(0)|^2 e^{-i\frac{E_n}{\hbar}t} e^{+i\frac{E_n}{\hbar}t} = |\psi_n(0)|^2 e^0 = 1 \times |\psi_n(0)|^2 \quad (9)$$

and the effective coefficient of superposition remains unchanged.

Measurement in any non-commuting basis, however, leads to quantum interference effects of the $e^{-i\frac{E_n}{\hbar}t}$ terms .

Thus, it seems plausible that the gist of the ideas regarding the eventual origin of the probabilities from the incessant spontaneous activities of the ultimate reality of the quantum fields can be extended to the Hilbert space formalism.

After all, Born's rule was first derived historically for the single quantum particle and subsequently extended to the Hilbert space.

Projective measurement

Every vector in the Hilbert space, can be expressed in Dirac's notation as a linear combination (7) of the energy basis vectors $|E_n\rangle$ with complex coefficients ψ_n .

Multiplying both sides of (7) by $\langle E_m|$ gives

$$\langle E_m|\Psi\rangle = \sum_n \psi_n \langle E_m|E_n\rangle \quad (10)$$

Since

$$\langle E_m|E_n\rangle = \delta_{mn} = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \quad (11)$$

it follows that

$$\psi_n = \langle E_n|\Psi\rangle \quad (12)$$

which is the transition amplitude of state $|\Psi\rangle$ to state $|E_n\rangle$.

The energy basis vectors are then superposition of quantum states with complex coefficients if viewed in a different non-commuting basis.

Inserting (12) into (7) gives

$$|\Psi\rangle = \sum_n |E_n\rangle \langle E_n|\Psi\rangle \quad (13)$$

Further defining a projection operator $\hat{P}_n = |E_n\rangle \langle E_n|$ transforms (13) into

$$|\Psi\rangle = \sum_n \hat{P}_n |\Psi\rangle \quad (14)$$

leading to $\sum_n \hat{P}_n |\Psi\rangle$ signifying that the sum of all the projection operators is unity.

The outer product $|\psi\rangle\langle\psi|$ is called the projection operator since it projects an input ket vector $|\phi\rangle$ into a ray defined by the ket $|\psi\rangle$ as follows

$$|\psi\rangle\langle\psi| |\phi\rangle = |\psi\rangle\langle\psi|\phi\rangle \quad (15)$$

with a probability $|\langle\psi|\phi\rangle|^2$, as the inner product between two state vectors is a complex number recognized as the probability amplitude.

This is usually known as projective measurement and we should notice that it is important for the measurement of a mixed state consisting of an ensemble of pure states in a density matrix.

Operator valued observables

In a quantum system, what can be measured in an experiment are the eigenvalues of various observable physical quantities like position, momentum, energy, etc.

These observables are represented by linear, self-adjoint Hermitian operators acting on Hilbert space.

Each eigenstate of an observable corresponds to eigen-vectors $|\psi_i\rangle$ of the operator \hat{A} , and the associated eigenvalue λ_i corresponds to the value of the observable in that eigenstate

$$\hat{A}|\psi_i\rangle = \lambda_i|\psi_i\rangle \quad (16)$$

For a Hermitian operator \hat{A} , the quantum states associated with different eigenvalues are orthogonal to one another

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} \quad (17)$$

The possible results of a measurement are the eigenvalues of the operator, which explains the choice of self-adjoint operators for all the eigenvalues to be real.

The probability distribution of an observable in a given state can be found by computing the spectral decomposition of the corresponding operator.

For a Hermitian operator \hat{A} on an n -dimensional Hilbert space, this can be expressed in terms of its eigenvalues following (16) as

$$\hat{A} = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i| \quad (18)$$

If the observable \hat{A} , with eigenstates $\{|\psi_i\rangle\}$ and spectrum $\{\lambda_i\}$ is measured on a system described by the state vector $|\Psi\rangle$, the probability for the measurement to yield the value λ_i is given by

$$p(\lambda_i) = |\langle\psi_i|\Psi\rangle|^2 \quad (19)$$

This again is the famous Born's rule and we can see that it can be derived by extending the concepts discussed in the case of the single particle.

After the measurement the system is in the eigenstate $|\psi_i\rangle$ corresponding to the eigenvalue λ_i found in the measurement, which is called the reduction of state.

Recalling the discussions of the probability amplitudes in the one particle wave function, the probability amplitudes of the quantum states involved in superposition in Hilbert space has likewise been predetermined very possibly again because of the characteristic ceaseless activity and mutual interactions of the quantum fields.

A quantum state in superposition generally has non-zero values for all states in superposition [9].

This is why we assert that the energy of the states is not on the mass shell.

Again, this could be possible because of the interactions of the quantum states in superposition with the ceaseless quantum fluctuations just as in the case for the superposed components making up the composition of the structure of the non relativistic single electron.

It is of paramount importance to reemphasize that matter and particularly the elementary particles comprising them would not have some behavior in the absence of the special characteristics of the quantum fields listed earlier.

These distinct activities are well recognized in the Lamb shift, anomalous electron g-factor, etc.

Quantum superposition with definite complex amplitudes can therefore be also an example of such behavior.

Stern–Gerlach experiment

The Stern–Gerlach experiment is the most striking illustration of the experimental implementation of quantum measurements.

It is as simple as it is persuasive.

The following Stern–Gerlach experiment carried out using neutrons, each having a spin $1/2$, reinforces the fact that the probabilities of the eigenstates in superposition is present from the beginning, very likely because of the incessant activities of the quantum fields.

The results of a series of Stern–Gerlach setups in tandem show that the precise probabilities in superposition are indeed restored repeatedly following each projective measurement.

After examining the results of these simple experiments, it is hard not be convinced about our assertion that the coefficients of probabilities are preexistent in superposition of states and their effect in the measurement of probability do not change during the unitary evolution of the superposed system.

For the purpose of presentation clarity, we will assume that before entering the first Stern–Gerlach magnet (Fig. 2), the direction of the neutron spin magnetic moment is in a definite superposed state of two states referred to as spin-up and spin-down

$$|\Psi_0\rangle = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle \quad (20)$$

with unknown complex coefficients α and β that are constrained by $|\alpha|^2 + |\beta|^2 = 1$.

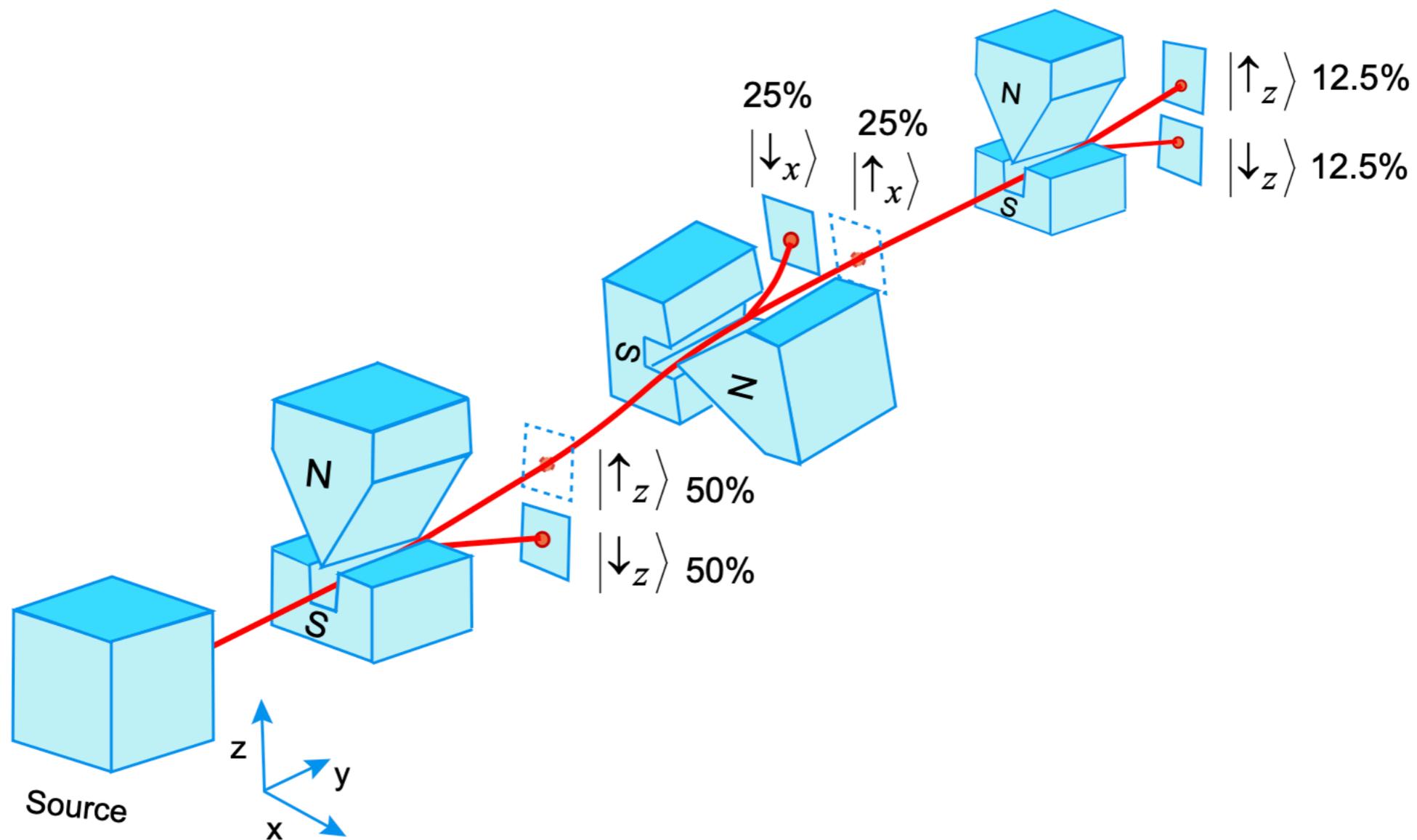


Figure 2: Serial Stern–Gerlach experiment in which neutrons are fired from a source into the non-homogeneous magnetic fields of three sequential Stern–Gerlach magnets oriented along the z-axis, the x-axis and the z-axis, respectively.

The superposed state (20) reduces (or collapses) as soon as the neutron enters the magnets of the analyzer to just one spin- z direction by immediate momentum and energy transfer with the magnet, rather than by subsequent determination at the screen.

After exiting the magnet aligned in the z direction, the trajectory of the neutron spin that can take only two equal but opposite values, will be deflected in either the $z+$ or $z-$ directions.

If we denote these states by $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$ respectively, we could say that the initial state performs one out of two equally probable quantum jumps

$$\begin{cases} |\Psi_0\rangle \rightarrow |\uparrow_z\rangle & \text{with } p = \frac{1}{2} \\ |\Psi_0\rangle \rightarrow |\downarrow_z\rangle & \text{with } p = \frac{1}{2} \end{cases} \quad (21)$$

When the beam of neutrons hits a detector screen, two spatially separated spots will appear corresponding to the two distinct trajectories.

Each of the two spots would show equal number of neutrons following $|\alpha|^2 = |\beta|^2 = 1/2$.

If we now choose to send only the $|\uparrow_z\rangle$ state through second Stern–Gerlach magnet aligned in the z direction, all the neutrons will be found, consistent with its preparation, in the upper region only.

However, if the state $|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$ faces Stern–Gerlach magnet aligned along the noncommuting orthogonal x -axis, any previous information about $|\uparrow_z\rangle$ will be completely destroyed and the direction of the spin magnetic moment will no longer be in an eigenstate of $\hat{\sigma}_z$ due to occurrence of one out of two equally probable quantum jumps

$$\begin{cases} |\uparrow_z\rangle \rightarrow |\uparrow_x\rangle & \text{with } p = \frac{1}{2} \\ |\uparrow_z\rangle \rightarrow |\downarrow_x\rangle & \text{with } p = \frac{1}{2} \end{cases} \quad (22)$$

Therefore, if only the $z+$ neutrons are passed through a second Stern–Gerlach magnet, which measures the neutron’s x -spins, the neutrons are deflected either right or left, labeled $x+$ and $x-$, and the number of neutrons with $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$ spin is split even as expected.

Subsequently, if we pass only the $|\uparrow_x\rangle$ neutrons through a third Stern–Gerlach magnet oriented along the orthogonal z -direction, we observe that their previous z -spin value has been reset, and they are again split evenly between $z+$ and $z-$.

This is despite the fact that we selected only the $z+$ neutrons from the first Stern–Gerlach magnet.

When the second one is measured, it resets the state of the first one.

Thus, there is another clear indication that the complex coefficients of superposition are predetermined, again very possibly by the quantum fluctuations.

We can hence infer that the ceaseless quantum fluctuations as well as the mutual interactions of the quantum fields preordain the probability of detection of a quantum state.

In a recent investigation, by performing an ingenious experiment involving superposition of three eigenstates of a state vector given by

$$|\Psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle \quad (23)$$

it was strikingly demonstrated that the complex coefficient α_0 governing the probability of the particular quantum state $|0\rangle$ in a superposition of three states can be measured without affecting the superposition of the two other remaining states in superposition.

This further reinforces the fact that the coefficients of superposition that determines the probability outcome of measurement are predetermined.

Conclusion

History of the development of quantum mechanics is replete with a notable trend.

Because of the sheer novelty of the subject so remarkably different from the established classical physics, the pioneers of the development of quantum physics utilized a procedure quite frequently with notable success.

Due to a thorough lack of experience with the precepts of the newly emerging subject of quantum mechanics, an empirical model was fashioned first to accommodate the observed information.

The experiential model was then amended to accommodate a more realistic version from a deeper understanding gained from subsequent revelations.

This successful procedure started almost from the beginning with the proposal of a quantum by Max Planck.

Out of sheer frustration of not being able to match the characteristics of blackbody radiation to his equation, Planck introduced the indivisible radiation quantum believing it was just a necessary mathematical oddity without having any reality whatsoever.

Five years later, Einstein persuasively demonstrated the reality of the quantum from the results of photoelectric effect.

Citing another example, Neils Bohr crafted the first atomic model with discrete electron orbits merely to fit the observed spectral data.

With the proposal and subsequent verification of matter wave, Schrodinger demonstrated Bohr's discrete atomic orbits to be real standing wave patterns of matter wave and the list continues.

A similar situation presented itself with the fabrication of the wave packet function to accommodate the observed wave particle duality.

It has essentially been considered to be a rather fictitious mathematical construct giving the probability density amplitude following Max Born's educated guess.

From our current deeper understanding based on contemporary knowledge of the primary reality of the quantum fields and their incessant, innately spontaneous, totally unpredictable activities identified as quantum fluctuations, we now realize that the wave packet function for a single nonrelativistic electron is in fact real.

The wave function represents among others, real energy density amplitudes of the electron and consequently the probability density amplitude following Einstein's intuition.

Since the amplitudes of the wave function in position space are computable, the probability of the electron at a particular position is predetermined as a distinct result of the specific activities of the quantum fields.

This phenomenon can be cogently extended to the superposition of quantum states in Hilbert space.

We, therefore believe that a plausible answer has now been provided to the question, where do the probabilities in the measurement come from, thus removing one of the hurdles in resolving the century old measurement problem.

Now for an interesting idea about the actual measurement process.

3 - Quantum measurement: a game between observer and nature?

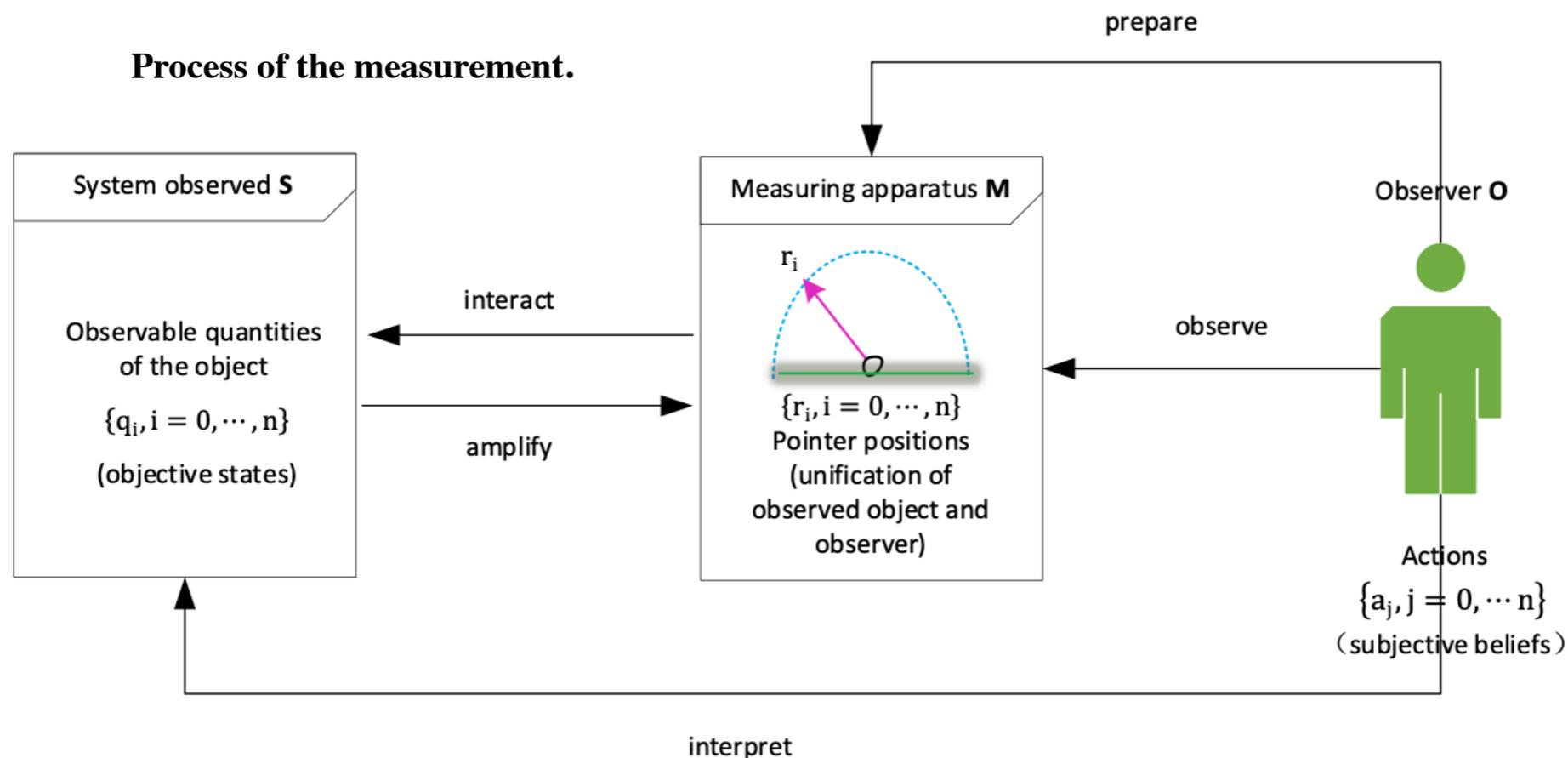
Introduction

In physics, measurement is an act to collect numerical data that describes a property of an object.

Usually, a measurement is made by comparing a quantity of an object with a standard unit so that a measuring device can obtain a value that matches the property of the measured object; in other words, measurement is an act to deliver a value by correlating the state of observed object with the pointer state of apparatus.

Usually apparatus are set up by observer, and when the measurement is done, the measured results need to be observed and interpreted by an observer.

The process of the measurement is shown as in the figure.



A measurement has three parts:

(1) System observed $\mathbf{S} \rightarrow q_i = \{q_1, \dots, q_i, \dots, q_n\}$ observable quantities to be measured.

(2) Measuring device $\mathbf{M} \rightarrow r_i = \{r_1, \dots, r_i, \dots, r_n\}$ pointer's states to be correlated $\left(1 \xrightarrow{t_0} 1\right)$ with the states of object $q_i = \{q_1, \dots, q_i, \dots, q_n\}$ by interacting with the system observed and delivers (amplify) a value r_j (pointer position).

(3) Observer $\mathbf{O} \rightarrow$ prepare the apparatus, observe the measured results $r_i = \{r_1, \dots, r_i, \dots, r_n\}$, and interpret the observed results with physical meanings.

If observer doesn't "look at" the apparatus, he or she can subjectively take an action

$a_i = \{a_1, \dots, a_i, \dots, a_n\}$ to "compute" the "theoretical" results.

In other words, observer "gambles" with nature to see if the "guessing" results are right (compared with the measured results), so that the observer can learn about nature and build-up his or her own experience of nature.

The process of a measurement is as follows:

(1) Prepare ($t = t_0$): the pointer state of the apparatus is set to initial state by an observer.

$$|\psi^M(t_0)\rangle = |r_0\rangle \quad (1)$$

(2) Measure ($t_0 < t < t_1$): apparatus interacts with the measured system and amplifies the signal.

(a) Classical measurement:

$$|r_0\rangle \otimes |q_i\rangle \rightarrow |r_i\rangle |q_i\rangle \quad (2a)$$

(b) Quantum measurement:

$$|\psi^{S+M}(t_0)\rangle = |r_0\rangle \otimes \left(\sum_{i=1}^n c_i |q_i\rangle \right) \rightarrow \sum_{i=1}^n c_i |r_i\rangle |q_i\rangle = |\psi^{S+M}(t_0 < t < t_1)\rangle \quad (2b)$$

(3) Observe ($t = t_1$): an observer observes and interprets the measured results.

$$|\psi^M(t_1)\rangle = |r_i\rangle, i = 1, \dots, i, \dots, n \quad (3)$$

For the observer, the final measurement result is a definite pointer state $|r_i\rangle$ for both classical and quantum measurement. The difference is that the results of the classical measurements are consistent with the properties of the observed system (except for a negligible error), while the results of the quantum measurement are inconsistent with the properties of the observed system (the pointer state r_i for each measurement is indeterminate and can only occur in Born probabilities $\omega_i = |c_i|^2$).

Classical measurement postulate: the objective properties of the observed system are independent of the measurement and are not interfered with by the apparatus. The observed system has a definite state q_i , which corresponds to the pointer state r_i with a definite one-to-one mapping, and it can be verified by the system measurement. After the measurement, the observed system's q_i can be inferred from the result r_i . The classical theory does not have an interpretation problem, because measured results are consistent with the predictions of the classical theory. There is no problem at all with the classical measurements from (2) to (3).

Quantum measurement postulate: there are multiple possible states $\{q_1, \dots, q_i, \dots, q_n\}$ of the system observed, and prior information regarding the system is incomplete. Because the interaction of the observed system and the apparatus “disturbs” , via the other states, the pointer state r_i of an apparatus can only be pointed to q_i with a certain probability $\omega_i = |c_i|^2$ (Born rule), so observer cannot accurately infer which state $q_i \in \{q_1, \dots, q_i, \dots, q_n\}$ the system was “in” before the measurement. The core of the quantum measurement problem is : how does it happen from (2) to (3)?

Based on the above analysis, there are three problems in quantum measurement:

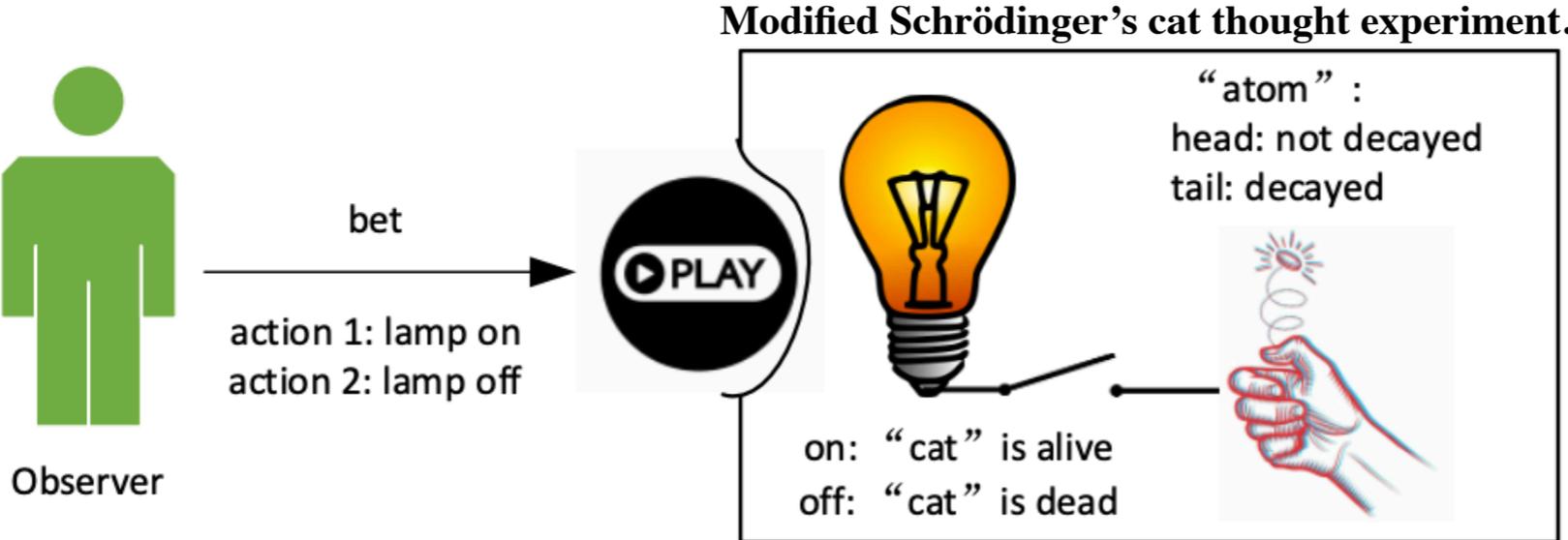
(1) S Reality problem: does state (wave function) describe the reality of quantum entity or just a mathematical tool. Is the wave function a complete description of physical reality?

(2) M Entangle problem: how pointer states $\{q_1, \dots, q_i, \dots, q_n\}$ “entangle” the observed states $\{r_1, \dots, r_i, \dots, r_n\}$ with the defined $\left(1 \xrightarrow{t_0} 1\right)$ mapping? Where is the sharp boundary between the quantum world and the classical world?

(3) Q Interpretation problem: for a single measurement there is no so-called objective probability of repeated measurements; and in this case for the observer, how to interpret the inherent uncertain state of quantum systems? Does human consciousness cause reducing of wave packet? Or does the measurement process not require the existence of an observer?

We don't believe human consciousness causes the reduction of the wave packet; also we believe an observer is required to interpret the measured results. We propose a quantum decision approach for quantum measurement. For an observer to measure the properties of a quantum entity is like to play a game with the nature: nature makes his “choice”, and the observer “bet” on it.

In other words, an observer has to make a decision under uncertainty with incomplete information regarding nature’s “choice”; through this learning progress, the observer gradually built-up his (her) own experience of the nature in his (her) memory for future decision-making. The figure shows a modified Schrödinger’s cat thought experiment for an observer to play game with nature (quantum system).

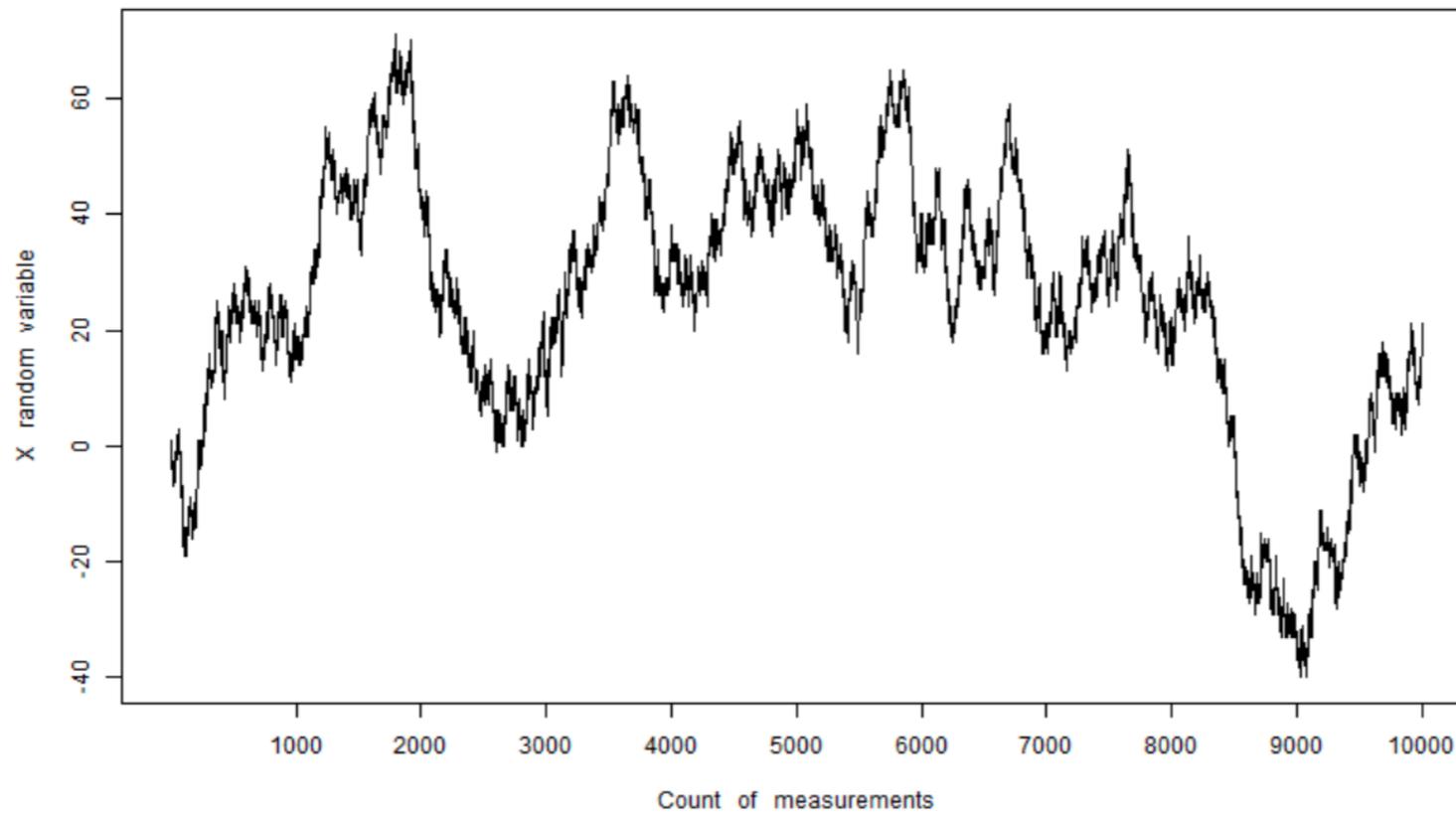


The rules of the game are as follows: $\{(q_1, x_1), \dots, (q_k, x_k), (q_{k+1}, x_{k+1}), \dots, (q_N, x_N)\}$

(1) A digital “coin” will be thrown within an hour, if heads is up (0), nothing happens, the lamp is still on; if the tails is up (1) then a switch will be closed to cut the lamp off.

(2) If the lamp is on and an observer bets the lamp is on, the observer wins the game, otherwise the observer loses; if the lamp is off and an observer bets the lamp is off, the observer wins the game, otherwise the observer loses.

The modified Schrodinger’s cat thought experiment can be simulated by a computer program which randomly generates N (=10000) results. We can define a random variable x to represent the fluctuation of the digital cat's state(4). The generated data series $\{(q_1, x_1), \dots, (q_k, x_k), (q_{k+1}, x_{k+1}), \dots, (q_N, x_N)\}$ is shown as Table1; q_k denotes the state of “atom“: 0 (not decayed) and 1 (decayed); if the “atom” has not decayed then variable x increases by 1, else variable x decreases by 1. The uncertainty of the cat’s state is represented by the volatility of variable x as shown in the figure.



Generated data series (10000 results) simulated by a computer program for the modified Schrödinger's cat thought experiment.

$$x_{k=0} = 0; \tag{4a}$$

$$x_{k+1} = x_k + \begin{cases} -1, & \text{if "decayed"} \\ 1, & \text{if not "decayed"} \end{cases} \quad k = 1, \dots, N \tag{4b}$$

number	state (q _k)	value (x _k)
1	0	1
2	1	0
3	1	-1
⋮	⋮	⋮
⋮	⋮	⋮
5000	0	56
⋮	⋮	⋮
⋮	⋮	⋮
9998	0	20
9999	0	21
N=10000	1	20

10000 generated results of digital "cat" experiment.

Now if we ask this question: is it possible that the observer can “beat” the nature (quantum system) in the game? In other words, is it possible that the observer can have a reasonable expectation of the natural state by learning historical results of repeated measurements on copies of the same system (“atom” + “cat”)?

For a large amount of repeated measurements, there is an objective frequency (50% the lamp is on); but for a single game there is no so-called objective probability, the lamp is either on or off, and the observer either wins or loses; in this case, only the observer has subjective degrees of belief that the lamp is on or off before the box is opened.

Because of nature's inherent uncertainty (atom decayed or not), we can only get answers (“cat” is alive or dead) by asking questions and play a game with nature. Before opening the “black box” to have a right answer, the observer can “compute” what the answer is, and can improve the successful rate of having the right answer by learning historical data based on the maximization of expected value. We propose a quantum expected value decision theory for observers, and quantum genetic programming is applied to evolve “satisfactory” strategies for observers to “guess” (with degrees of belief) the natural state as best possible based on quantum expected value.

Quantum expected value (qEV)

An observer subjectively chooses an action $a_i \in \{a_1, a_2\}$ where the atom's objective state is in $\varphi_j \in \{q_1, q_2\}$ when a bet is made, and the result of bets depends on both the state of the atom and choice of observer's brain shown in the table .

State Action	q_1	q_2
a_1	r_{11}	r_{12}
a_2	r_{21}	r_{22}

A pay-off table of a game between an observer and the quantum system observed (nature).

The state of the atom describes the objective world; it can be represented by the superposition of all possible states in terms of the Hilbert state space as in (5).

$$|\psi\rangle_{atom} = c_1 |q_1\rangle + c_2 |q_2\rangle \quad |c_1|^2 + |c_2|^2 = 1 \quad (5)$$

where $|q_1\rangle$ denotes a state in which the atom has not decayed and $|q_2\rangle$ denotes a state in which the atom has decayed. $|c_1|^2$ is the objective frequency of the decayed atom; $|c_2|^2$ is the objective frequency of the atom without decay.

The observer's mental state describes the subjective world; we postulate that when the observer is undecided in making a bet, it can be represented by superposition of all possible actions as in (6).

$$|\psi\rangle_{mental} = \mu_1 |a_1\rangle + \mu_2 |a_2\rangle \quad |\mu_1|^2 + |\mu_2|^2 = 1 \quad (6)$$

Where $|a_1\rangle$ denotes the observer's action to believe the "cat" is alive and $|a_2\rangle$ denotes the observer's action to believe the "cat" is dead. $|\mu_1|^2$ is observer's subjective probability (degrees of belief) in betting "cat" is alive; $|\mu_2|^2$ is observer's subjective probability in betting "cat" is dead.

The prior information of the quantum world is incomplete; the result of the observer's decision is uncertain and it can be denoted by a mixed state's density operator as a value operator in (7). Value operator is a sum of projection operators which projects the observer's degrees of belief onto an action of choice.

$$\hat{V} = p_1 |a_1\rangle \langle a_1| + p_2 |a_2\rangle \langle a_2| \quad p_1 + p_2 = 1 \quad (7)$$

The observer's mental state is transformed from a pure state into a mixed state, and then one of available actions is selected by the brain (choose an action a_i with probability p_i) as in (8). Based on the information that the brain has before a bet is made, there is only the possibility of selecting an action with a subjective probability (degrees of belief), and it is the decision-making process that makes the potential possibility a reality. The decision process of an observer can be simulated by the continuous evolution of the value operator according to the environment (information). Information is the essence of people's subjective beliefs just like energy is the essence of the objective world. Valuable information can reduce uncertainty.

$$D: \hat{\rho} = |\phi\rangle \langle \phi| \rightarrow \hat{V} = p_1 |a_1\rangle \langle a_1| + p_2 |a_2\rangle \langle a_2| \xrightarrow{\text{decision}} |a_i\rangle \langle a_i|, \quad i = 1, 2 \quad (8)$$

Quantum expected value qEV can be represented as in (9).

$$\begin{aligned} \text{qEV} &= \langle \psi | \hat{V} | \psi \rangle = (c_1^* \langle q_1 | + c_2^* \langle q_2 |)(p_1 |a_1\rangle \langle a_1| + p_2 |a_2\rangle \langle a_2|)(c_1 |q_1\rangle + c_2 |q_2\rangle) \\ &= p_1 \omega_1 |\langle a_1 | q_1 \rangle|^2 + p_1 \omega_2 |\langle a_1 | q_2 \rangle|^2 + p_2 \omega_1 |\langle a_2 | q_1 \rangle|^2 + p_2 \omega_2 |\langle a_2 | q_2 \rangle|^2 \\ &= p_1 \omega_1 r_{11} + p_1 \omega_2 r_{12} + p_2 \omega_1 r_{21} + p_2 \omega_2 r_{22} \\ &= \sum_{i=1,2} p_i \sum_{j=1,2} \omega_j r_{ij} \end{aligned} \quad (9)$$

where $p_i = |\mu_i|^2$ is an observer's subjective probability in choosing an action a_i , subjective probability represents the observer's degrees of belief in a single event; $\omega_j = |c_j|^2$ is the objective frequency at which the state of the atom is in q_j , objective frequency represents the statistical results of multiple occurrences of objective states; matrix $r_{ij} = |\langle a_i | q_j \rangle|^2$ is the value when the decision was made, in which the observer chooses an action a_i where the atom's state is in q_j as in (10). The different actions that the observers took lead to different values; in other words, the value is "created" based on both observers' subjective beliefs and objective natural states.

$$r_{ij} = |\langle a_i | q_j \rangle|^2 = \begin{cases} 1, & i = j \\ -1, & i \neq j \end{cases} \quad (10)$$

Quantum decision tree (qDT)

The value operator is a 2 x 2 matrix, and the value operator needs to be diagonalized first and then normalized to get probabilities p_1 and p_2 as in (11).

$$\begin{aligned} \hat{V} &= \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \xrightarrow{\text{diagonalization}} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \xrightarrow{\text{normalization}} \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} \\ &= p_1 |a_1\rangle \langle a_1| + p_2 |a_2\rangle \langle a_2| \end{aligned} \quad (11a)$$

$$|a_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |a_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; |a_1\rangle \langle a_1| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, |a_2\rangle \langle a_2| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (11b)$$

A value operator \hat{V} , as a qDT, can be constructed from basic quantum gates with logic operations. The qDT composes of different nodes and branches. There are two types of nodes, non-leaf nodes and leaf nodes. The non-leaf nodes are composed of the operation set F as in (12); the leaf nodes are composed of the data set T (quantum gates) as in (13). The construction process of a qDT is to randomly select a logic symbol from the operation set F as the root of the qDT, and then grows corresponding branches according to the nature of the operation symbol and so on until a leaf node is reached.

$$F = \{+(ADD), *(MULTIPLY), //(OR)\} \quad (12)$$

$$T = \{H,X,Y,Z,S,D,T,I\} \quad (13a)$$

$$\left\{ \begin{array}{l} H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \right\} \quad (13b)$$

Quantum genetic programming (qGP)

Basically, an observer will try to maximize the qEV guided by qDT (nest of hierarchy yes/no logic) to “beat” the nature in the game. A qDT can be optimized by the qGP. The purpose of qGP iterative evolution is to find a satisfactory qDT through learning historical data. The learning rule is as follows:

(a) If the “cat” is alive (q_1) and an observer bets the “cat” is alive (a_1), the observer wins a game; if the observer bets the “cat” is dead (a_2), the observer loses a game.

(b) If the “cat” is dead (q_2) and an observer bets the “cat” is dead (a_2), the observer wins a game; if the observer bets the “cat” is alive (a_1), the observer loses a game.

An optimization problem mainly includes the selection of evaluation function and the acquisition of optimal solution. The evaluation function of qDT is a fitness function f_{fitness} (15) based on observed value V_k (14), and the optimal solution is obtained through continuous evolution by using selection, crossover, mutation as in (16) and implemented by qGP algorithm. The quantum expected value of the k^{th} bet is as follows:

$$\langle V_k \rangle = \begin{cases} p_1\omega_1 |\langle a_1 | q_1 \rangle|^2 = p_1\omega_1 r_{11} = p_1\omega_1, & |\phi\rangle_{\text{mental}} = |a_1\rangle \text{ and } |\phi\rangle_{\text{atom}} = |q_1\rangle \\ p_1\omega_2 |\langle a_1 | q_2 \rangle|^2 = p_1\omega_2 r_{12} = -p_1\omega_2, & |\phi\rangle_{\text{mental}} = |a_1\rangle \text{ and } |\phi\rangle_{\text{atom}} = |q_2\rangle \\ p_2\omega_1 |\langle a_2 | q_1 \rangle|^2 = p_2\omega_1 r_{21} = -p_2\omega_1, & |\phi\rangle_{\text{mental}} = |a_2\rangle \text{ and } |\phi\rangle_{\text{atom}} = |q_1\rangle \\ p_2\omega_2 |\langle a_2 | q_2 \rangle|^2 = p_2\omega_2 r_{22} = p_2\omega_2, & |\phi\rangle_{\text{mental}} = |a_2\rangle \text{ and } |\phi\rangle_{\text{atom}} = |q_2\rangle \end{cases} \quad (14)$$

$$f_{\text{fitness}} = \sum_{k=0}^N \langle V_k \rangle \quad (15)$$

$$qDT \xrightarrow{\text{evolution}} \underset{qDT \in (F \cup T)}{\text{argmax}} (f_{\text{fitness}}) \quad (16)$$

qGP algorithm

Input:

- Historical data set $d_k = (q_k, x_k)$, $k = 0, \dots, N$.
- Setting
 - (1) Operation set $F = +, *, //$
 - (2) Data set $T = H, X, Y, Z, S, D, T, I$, eight basic quantum gates
 - (3) Crossover probability = 60 ~ 90%; Mutation probability = 1 ~ 10%.

Initialization:

- Population: randomly create 100 ~ 500 qDTs.

Evolution:

- for $i = 0$ to n , $n = 50 \sim 100$ generations
 - (a) Calculate fitness for each qDT based on historical data set.
 - (b) According to the quality of fitness:
 - (i) Selection: selecting parent qDTs.
 - (ii) Crossover: generate a new offspring using the roulette algorithm based on crossover probability.
 - (iii) Mutation: randomly modify parent qDT based on mutation probability.

Output:

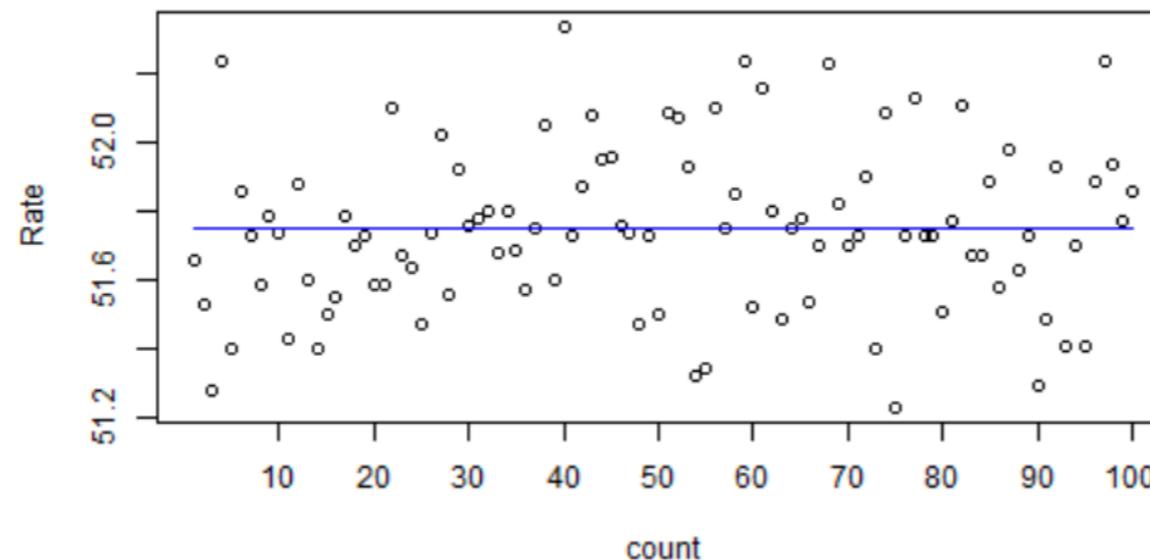
- A qDT of the best fitness.

Results

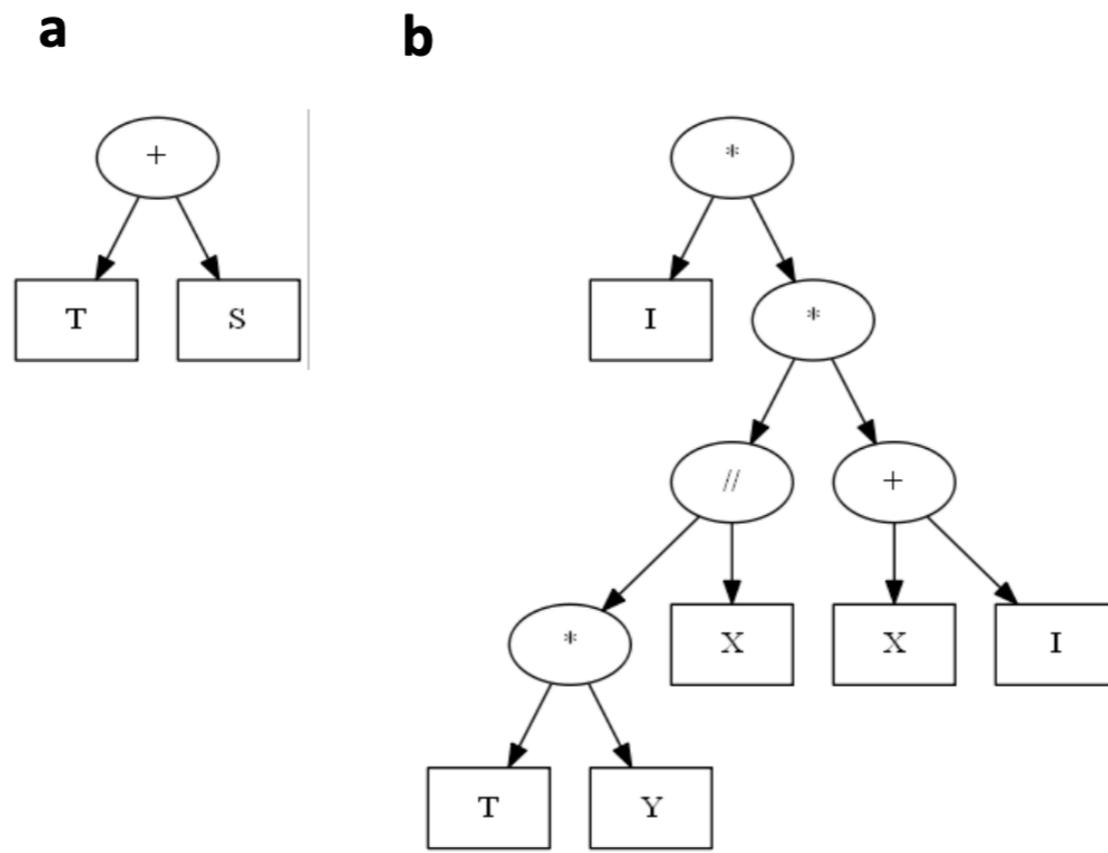
run	natural state	qDT ₁	qDT ₂	...	qDT ₄₇	...	qDT ₅₆	...	qDT ₁₀₀
1	0	1	0	...	0	...	0	...	0
2	1	0	0	...	1	...	0	...	1
3	1	0	1	...	1	...	0	...	0
⋮	⋮	⋮	⋮		⋮		⋮		⋮
⋮	⋮	⋮	⋮		⋮		⋮		⋮
5000	0	0	1	...	1	...	1	...	0
⋮	⋮	⋮	⋮		⋮		⋮		⋮
⋮	⋮	⋮	⋮		⋮		⋮		⋮
9998	0	0	1	...	0	...	0	...	0
9999	0	0	1	...	0	...	1	...	1
N=10000	1	1	1	...	0	...	1	...	1

100 qDTs of an observer are simulated and optimized by qGP.

Shown as in Table above, one hundred qDTs of an observer are simulated by computer and all 100 qDTs are optimized by qGP through learning historical records for this observer as his/her experiences stored in his/her memory. This observer will make decision based on these past experiences (100 qDTs). The average of winning rate is 51.7 and standard derivation of winning rate is 0.2 (see figure below). The winning rates for all qDTs are almost the same that means the experiences of this observer are very stable.



Winning rates and average for the observer.



(a) qDT₄₇ with the smallest expected value (b) qDT₅₆ with the biggest expected value.

Of 100 qDTs, qDT₄₇ got the smallest expected value shown as figure above (a); qDT₅₆ got the biggest expected value shown as in figure above (b). Based on the qDT₄₇ as in (17) (figure part (a)), there are only one simple strategy (S_1) that the observer can take, which is 54% degrees of belief to believe the cat is alive and 46% degrees of belief to believe the cat is dead (figure below (a)). We can see this qDT's subjective probability (degrees of belief) close to 50/50 like randomly throw a coin to decide if the cat is alive or dead, so almost no valued information obtained, that's the reason qDT₄₇ obtained the smallest expected value. Based on the qDT₅₆ as in (18) (figure above (b)), there are two strategies ($S_i \in S_1, S_2$) that the observer can take, one is 100% degree of belief to believe the cat is alive and the other one is 96% degree of belief to believe the cat is dead and 4% degree of belief to believe the cat is alive (figure below (b)). We can see now the qDT₅₆'s subjective probability (degrees of belief) close to unity, so almost maximum information obtained, that's the reason observer₅₆ got the biggest expected value.

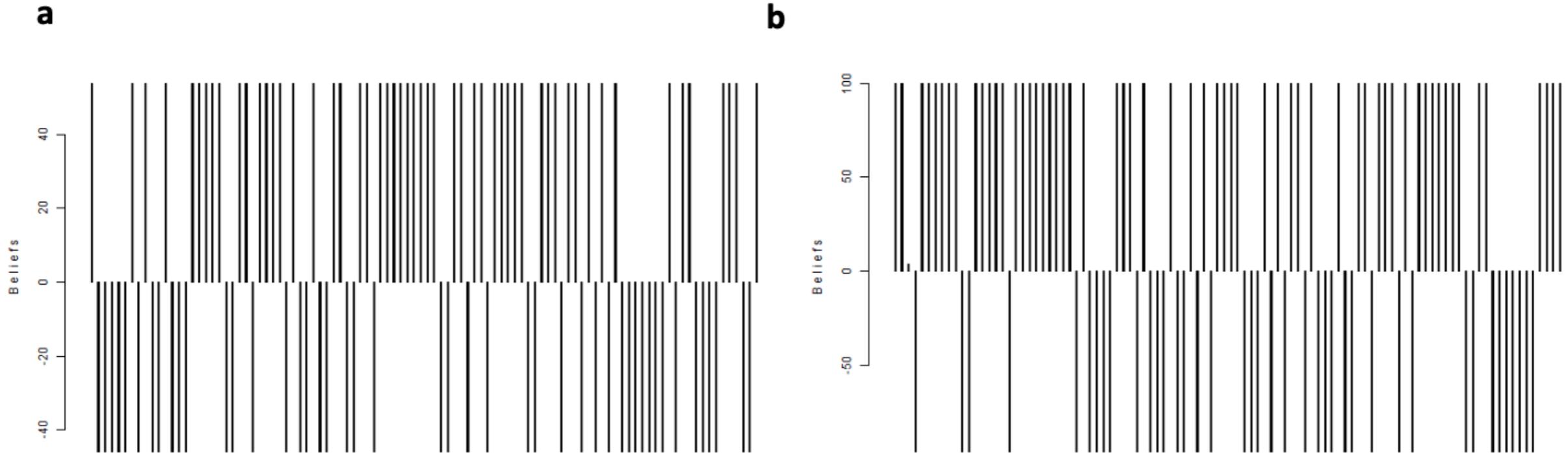
$$qDT_{47} = (T + S) \tag{17}$$

- $S_1 = (T + S) \rightarrow \hat{V} = 0.54 |a_1\rangle \langle a_1| + 0.46 |a_2\rangle \langle a_2|$

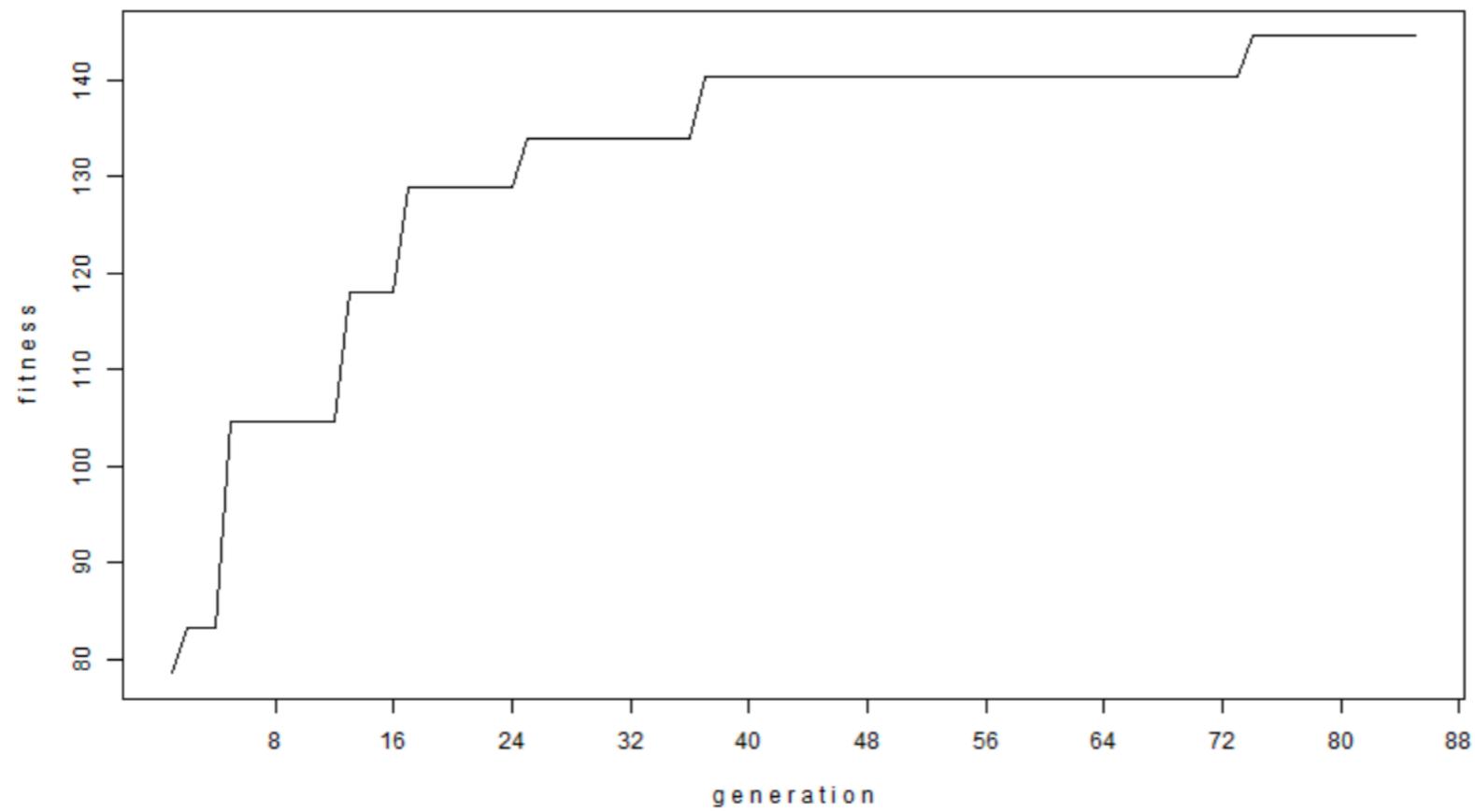
$$qDT_{56} = (1 * ((T * Y) // X) * (X + I)) \tag{18}$$

- $S_1 = (1 * (X * (X + I))) \rightarrow \hat{V} = |a_1\rangle \langle a_1|$

- $S_2 = (1 * ((T * Y) * (X + I))) \rightarrow \hat{V} = 0.04 |a_1\rangle \langle a_1| + 0.96 |a_2\rangle \langle a_2|$

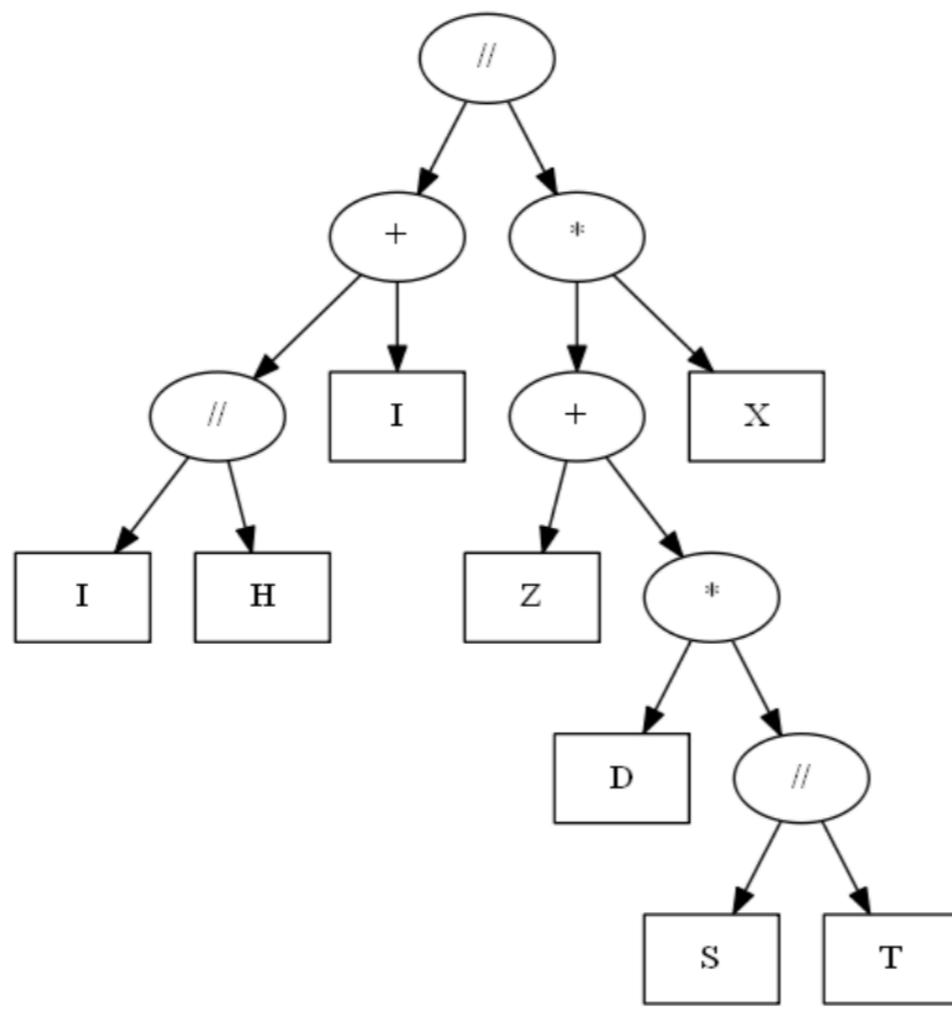


(a) subjective degrees of belief of qDT₄₇ (b) subjective degrees of belief of qDT₅₆



evolution of qDT_{33}

The expected value for qDT_{33} is between the biggest expected value and smallest expected value. The 88 generations of evolution of the qDT_{33} are shown in the above figure; the nest of hierarchy structure of qDT_{33} is shown in the figure below.



A nest of hierarchy structure of qDT₃₃

Based on the qDT₃₃ as in (19) (figure above), there are four strategies with different subjective degrees of belief that the observer can take.

$$qDT_{33} = (((I//H) + I)//((Z + (D * (S//T)))) * X) \quad (19)$$

$$S_1 = (I + I) \rightarrow \hat{V} = 0.5 |a_1\rangle \langle a_1| + 0.5 |a_2\rangle \langle a_2| \quad (50\% \text{ belief cat is alive, } 50\% \text{ belief cat is dead})$$

$$S_2 = (H + I) \rightarrow \hat{V} = |a_1\rangle \langle a_1| \quad (100\% \text{ belief cat is alive})$$

$$S_3 = ((Z + (D * S)) * X) \rightarrow \hat{V} = 0.05 |a_1\rangle \langle a_1| + 0.95 |a_2\rangle \langle a_2|$$

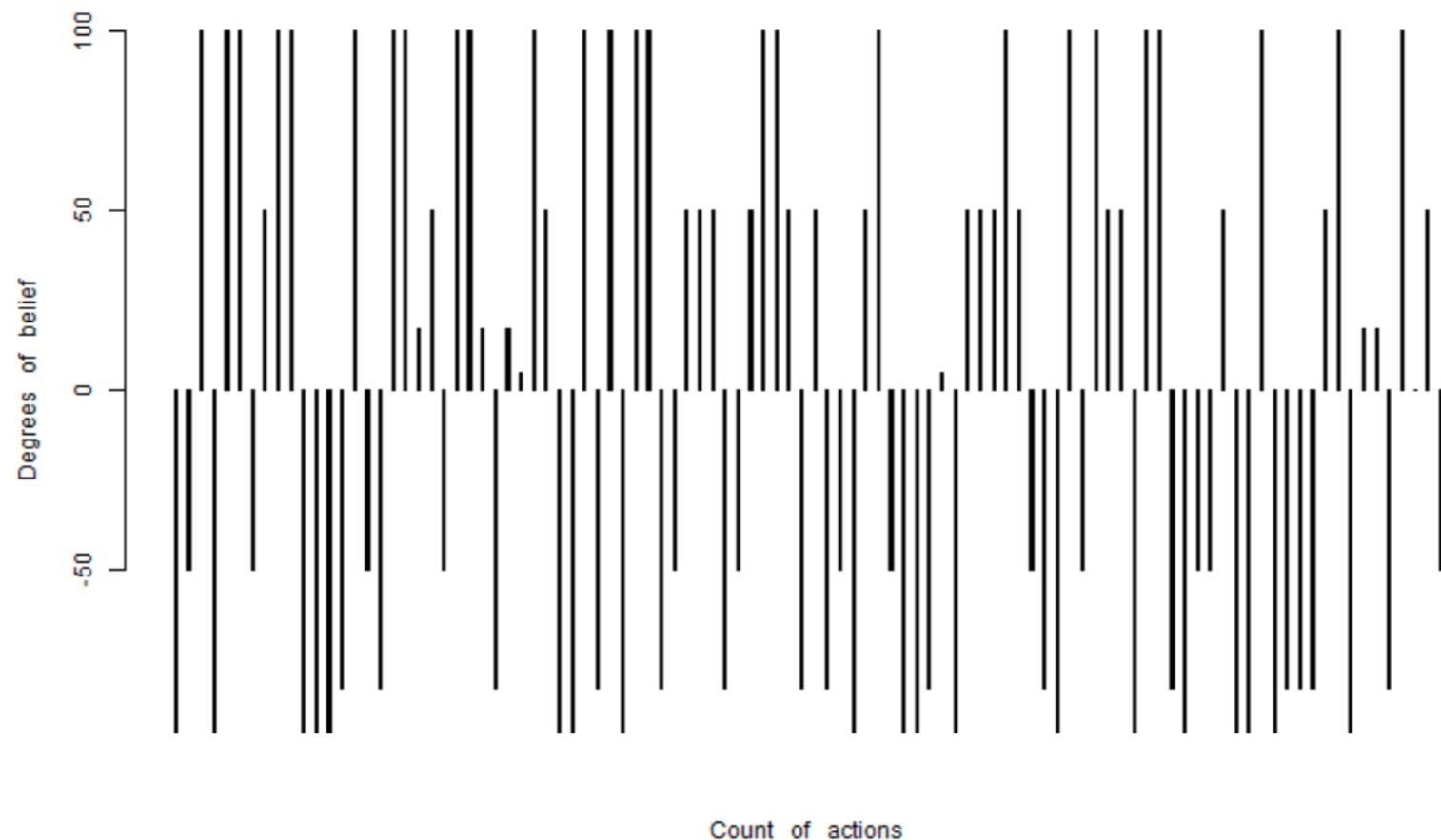
(5% belief cat is alive, 95% belief cat is dead)

$$S_4 = ((Z + (D * T)) * X) \rightarrow \hat{V} = 0.17 |a_1\rangle \langle a_1| + 0.83 |a_2\rangle \langle a_2|$$

(17% belief cat is alive, 83% belief cat is dead)

At any given moment the observer's degrees of belief are unknown, the qDT_{33} which simulates observer's degrees of belief can be interpreted as a mixed strategy with four different strategies S_1, S_2, S_3, S_4 for the observer, and the final decision is made by “quantum projection measurement” which the observer's brain selects an action $a_i \in a_1, a_2$ with degrees of belief from one of four available strategies $S_i \in S_1, S_2, S_3, S_4$. (Believe cat is alive denote by a_1 and believe cat is dead denote by a_2). For this observer, either S_1 strategy was selected with 50/50 even probability (no valued information was obtained under total uncertainty), or S_2, S_3, S_4 strategies were selected with almost 100% sure (maximum information was obtained). The strategies selected by qDT_{33} are more likely happened in real world, if we know for sure something will happen, we will take an action; if we have no idea what will happen, best we can do is to toss a coin to “hit” the luck.

The subjective degrees of belief of the first 100 actions the observer took are shown in the figure below.



A Observer's degrees of belief (qDT_{33} : positive: believe cat is alive; negative: believe cat is dead).

Detailed information of the first ten actions the observer took is shown in the table below.

Cat's state	Observer's action selected	Degrees of belief	Strategy	Value
q ₁ : alive	a ₂ : believe cat is dead	95%	S ₃	-1
q ₂ : dead	a ₂ : believe cat is dead	50%	S ₁	1
q ₂ : dead	a ₁ : believe cat is alive	100%	S ₂	-1
q ₂ : dead	a ₂ : believe cat is dead	95%	S ₃	1
q ₂ : dead	a ₁ : believe cat is alive	100%	S ₂	-1
q ₁ : alive	a ₁ : believe cat is alive	100%	S ₂	1
q ₂ : dead	a ₂ : believe cat is dead	50%	S ₁	1
q ₂ : dead	a ₁ : believe cat is alive	50%	S ₁	-1
q ₁ : alive	a ₁ : believe cat is alive	100%	S ₂	1
q ₁ : alive	a ₁ : believe cat is alive	100%	S ₂	1

The first ten actions by qDT33

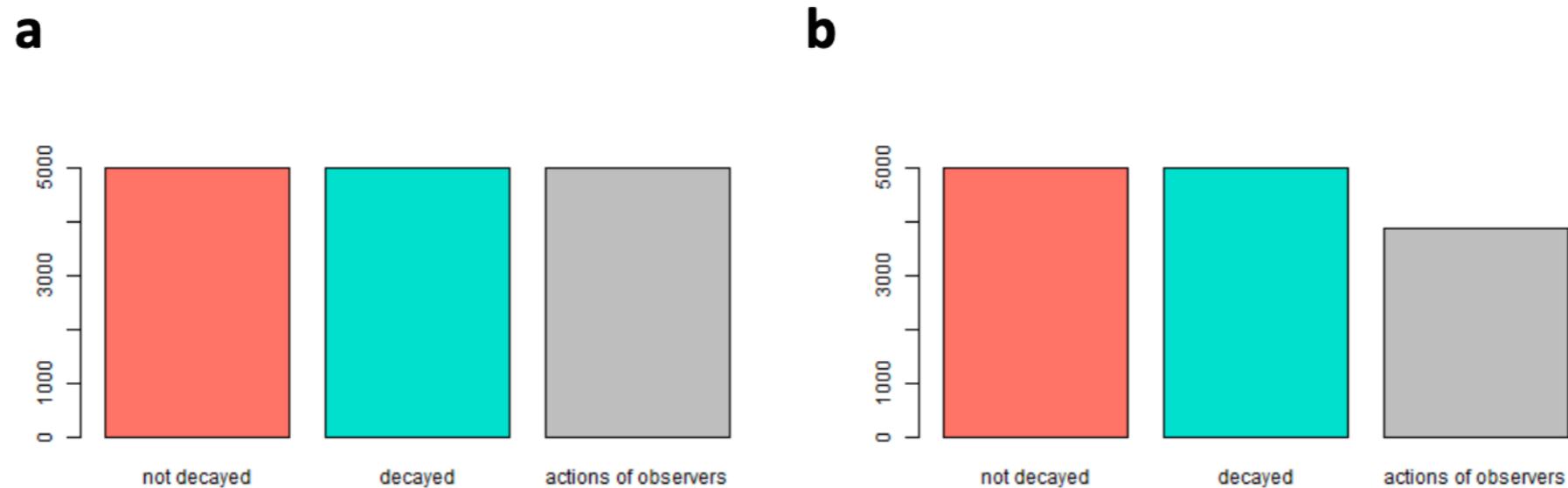
For the first action, strategy S₃ was applied by the observer who believes that the cat is dead with 95% degrees of belief, the observer got it wrong because the cat is alive when the box is opened; for the fourth action, strategy S₃ was applied by the observer who believes that the cat is dead with 95% degrees of belief, this time the observer got it right because the cat is dead when the box is opened; for the eighth action, strategy S₁ was applied by the observer who believe that the cat is alive with 50% degrees of belief, the observer got it wrong because the cat is dead when the box is opened; for the tenth action, strategy S₂ was applied by the observer who believe that the cat is alive with 100% degrees of belief, the observer got it right this time because the cat is alive when the box is opened.

According to an earlier table (see right), we can apply the majority rule to let the observer determine the cat's state based on the experience obtained from 100 qDTs; the rules are as follows:

number	state (q _k)	value (x _k)
1	0	1
2	1	0
3	1	-1
⋮	⋮	⋮
⋮	⋮	⋮
5000	0	56
⋮	⋮	⋮
⋮	⋮	⋮
9998	0	20
9999	0	21
N=10000	1	20

1. If more than 50 qDTs decide cats are alive, the observer believes that the cat is alive;
2. If more than 50 qDTs decide cats are dead,

The figure below (a) shows that the winning rate of the observer jointly determined by the 100 qDTs (the experiences of the observers) optimized based on maximum value is 50%, which is almost completely conforms to the objective frequency of the cat's state (Born rule); however, the winning rate of the observer jointly determined by 100 qDTs (generated by simple coin toss) that are not optimized based on the maximum value is only 39% (Figure 10 (b)), not conforms to the Born rule. The reason is simple: the useful information obtained from the decision of completely random coin toss (50/ 50) is 0, while the jointed decision of 100 qDTs optimized based on maximum value almost obtains the maximum information of the nature, so it conforms to the Born rule; in other words, the observer “finds” the Born rule.



(a) winning rate of 100 optimized qDTs (b) winning rate of 100 qDTs without optimized.

Discussion

$$\text{entity} = \{\text{state}_i, \text{value}_i, i = 1, 2, \dots, N\} \quad (20)$$

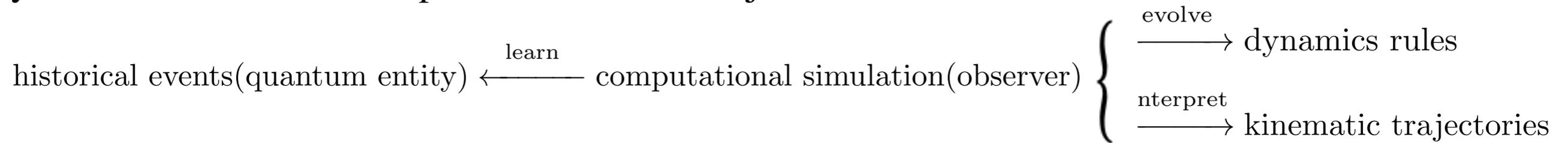
Both classical particles and quantum entities can be represented by (20); the current dynamic state of an entity is denoted by state_i , while the corresponding kinematic trajectory is denoted by $\{\text{value}_i\}$. Classical mechanics describes the laws of particles based on the dynamics equations (differential) and their kinematic trajectories (integral). A dynamic equation is essentially a set of logical rules, and the trajectory (historical data) of an entity describes the "historical events" that happened in the past of the entity. By using calculus, classical mechanics fully describe the past and future of entities as successive "events" (certain). The evolution of the dynamic state of a quantum entity is described by the Schrödinger differential equation, but the kinematic trajectories of quantum entities cannot be obtained by integration, and the "position" of a quantum entity can only be obtained by an irreversible "quantum jump" caused by quantum measurements. Quantum mechanics describes past "events" (discontinued) of quantum entities through differential equation and quantum measurements, while future "events" of quantum entities are statistically determined by Born's rule (uncertain).

We can look at quantum measurement in this way: nature asks questions and observers answer natural questions; basically it is a game between nature and observers: there is a sequence of "choices" made by nature, and observers select a sequence of actions guided by optimized strategies to decode the nature.

$$\{d_i, i = 1, 2, \dots, N\} : \text{question posed by Nature.} \quad (21)$$

$$\{qDT_i, i = 1, 2, \dots, N\} : \text{observers answer natural questions using yes/no logic in qDTs.} \quad (22)$$

Let go of the traditional differential equations dynamic variable approach; by learning the historical "events" given by nature, the yes/no logic of qDT (nest of hierarchy) is applied to evolve the natural dynamics rules, and to interpret the kinematic trajectories of nature as follows:



We divide $N=10000$ data points ("cat" states) into subset $M=100$ groups; each subset data group M_i includes $L=100$ data points, and satisfy:

1. subset $M \ll N$: The 100 data points in subset M are small enough compared to all the data points in N .
2. $M_i = L_1, L_2, \dots, L_{100}$: The subset M group data should be large enough to ensure that the objective frequency of the subset M group data is approximately equal to the objective frequency of all the data, so the subset M can approximately represent the overall sample $N = 10000$.

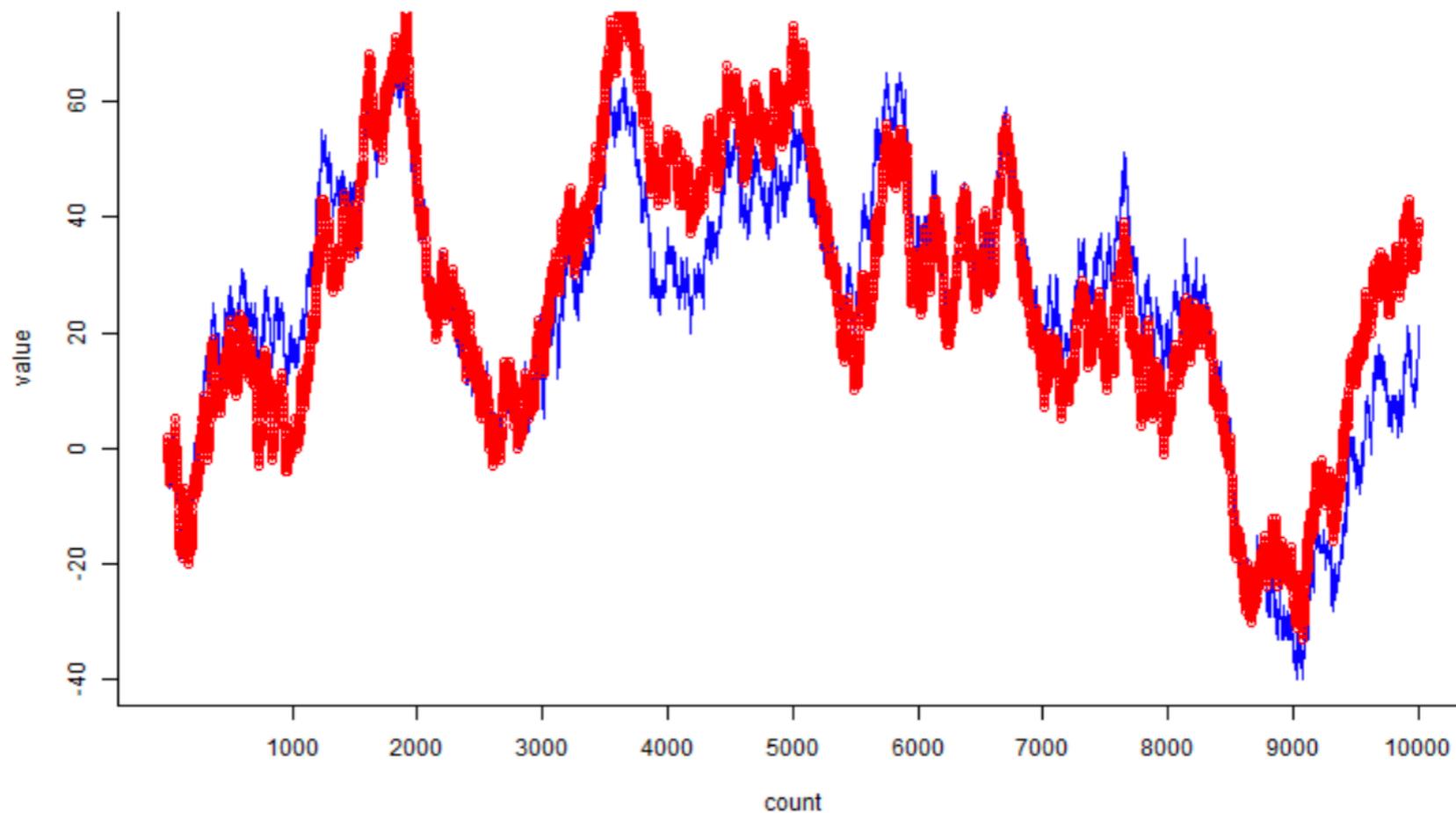
Now the question is: whether we can accurately "reconstruct" the "trajectory" of a quantum entity by learning from all subsets $M_i, i = 1, 2, \dots, 100$?

Just as classical mechanics uses the principle of least action to obtain the trajectories of particles, we use the principle of maximum expected value to approximately obtain the "trajectories" of quantum entities. By maximizing the expected value of the observer, qGP iteratively evolves to obtain a sequence of qDTs each with different strategies in (23).

$$qGP \xrightarrow{\text{evolve}} \{qDT_i(s_1, \dots, s_m), i = 1, 2, \dots, 100\} \quad (23a)$$

$$qDT_i = \max_{\text{fitness}} \left(\sum_i qEV_i / (y - x_i) \right), \text{feedback} = (y - x_i) \quad (23b)$$

Where y_i denotes observer's subjective value (computed value), x_i denotes nature's objective value (measured value); feedback is the difference between the value computed by the observer and the value measured of nature. The observer adjusts the current value based on past experience while maximizing qEV; qEV is an assessment of the future and feedback is an assessment of the past.



the “trajectories” of quantum entity reconstructed by $\{qDT_i(s_1, \dots, s_m), i = 1, 2, \dots, 100\}$

By answering natural questions, qDTs can “reconstruct” the “trajectories” of quantum entities very well. As shown in the figure above, we were able to “reconstruct” the “trajectories” of quantum entity with 70% accuracy (blue: nature; red: observer); because we cannot get the “prior” information of quantum entity, the information of quantum entity is incomplete, that is, quantum entity may have an infinite number of "trajectories", so we cannot accurately predict the future "trajectory" of the quantum entity; It seems that nature is indeed playing games with us, and it is impossible to accurately predict the future "trajectory" of quantum entities unless we can "dance" with nature, can we?

A last summation.....

4 - Can Decoherence Solve the Measurement Problem?

Introduction

Shortly after the formulation of quantum mechanics nearly a century ago, a rather strange aspect of the theory became apparent.

It is variously known as the *quantum measurement problem*, *wave function collapse* or *wave function reduction*.

This is because the non-relativistic quantum mechanics appears to consist of two essentially different processes.

After a unitary evolution of the wave function of a microscopic quantum state following the linear Schrodinger equation, one must resort to a sudden non-unitary stochastic collapse of the wave function to obtain its classical measurement outcome.

The genesis of the problem started with the quantum pioneers headed by Niels Bohr.

They insisted that we only ever observe any physical phenomena at the macroscopic level.

We never directly deal with the quantum objects of the microscopic realm and therefore need not worry about them or their physical reality.

Accordingly, they argued that both the observer and the measurement apparatus must be kept outside the system to which quantum mechanics is applied.

This is known as the Copenhagen interpretation, which simply pronounced the issue of microscopic quantum states is out of bounds, stating that physicists just had to accept a fundamental distinction between the quantum and the classical domains.

Without being disrespectful to the esteemed founding fathers of quantum mechanics, we may inquire how can such thoughts be ever consistent with a scientific outlook?

Nonetheless, this was epitomized by the mathematical mastermind John Von Neumann in his classic axiomatic formulation of non-relativistic quantum mechanics using the linear Hilbert vector space.

Even after many decades, his skillful formulation is still taught in almost all advanced quantum mechanics classes despite the two obvious incongruities arising from the Copenhagen interpretation.

These comprise of the essential *ad hoc* role of consciousness and the postulated assumption of an abrupt collapse of the wave function.

Quantum mechanics itself does not predict the collapse, which must be manually added to the calculations.

Einstein famously likened it to God playing dice to decide what becomes “real” – what we actually observe in our classical world.

However, despite the quantum pioneers’ assertions, enormous efforts by the physics community have been made leading to many alternate postulates to explain away the irrational proposal of the inventors.

Significant progress has been made by these efforts but without leading to any consensus, although some substantial fissures have ensued in the original interpretation.

The collapse postulate

John von Neumann, way back in his formulation, postulated his non-unitary “process 1”, to emphasize the role of *consciousness* for the collapse of the wave function in the measurement process.

It was concluded by von Neumann and most of the physicists of the time that there is no physical reason for the collapse in measurement transition.

Thus evolved the rather instinctive resort to the “consciousness of an external observer,” which appears to be fading in time.

Even the stalwarts like Eugene Wigner fell for it but eventually repudiated it later.

It is quite remarkable to note that so much effort by so many eminent scientists in the early years of quantum mechanics was devoted to the role of consciousness in quantum measurement.

But this directly contradicts the obvious fact that in the early years of the universe, the conditions were not suitable for appearance of any manifest conscious agents.

Yet the universe developed to a mature state obeying quantum rules long before the possibility of emergence of conscious beings.

This has been characterized by John Bell in jest

Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a PhD?

To be fair, the conditions of the early universe were not known to the pioneers of quantum physics.

It would be reasonable to speculate that very likely they would not have put such essential emphasis on consciousness if they knew the early universe conspicuously ascertains that consciousness is not essential for the workings of quantum rules.

The other enigmatic postulate that von Neumann institutionalized is the collapse of the wave function initially alluded by Werner Heisenberg.

By then, it was well established that a quantum state in the macroscopic domain is usually a superposition of two or more states.

But in measurement using classical devices, one observes only a single state and no superposition.

Von Neumann conjectured that in the measurement process, the quantum states would collapse to one of the superposed states following his improvised projection postulate.

Again, in John Bell's words

If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time, more or less everywhere?

Gerhart Luders rejected vonNeumann's collapse postulate (except for degenerate states).

Its confirmation came in a recent experiment performed by Pokorny *et al.* who called it the ideal measurement process.

In this well-planned experiment, the authors created a microscopic superposition of three quantum states.

They were able to measure just one of the superposed states without collapsing the entire wave function also observing that the collapse happened over time and not instantaneously.

Serge Haroche and his group also demonstrated that reduction of the wave function happens gradually.

An example from the cosmic history is worth examining in this regard.

The universe about 380 000 years after the big bang consisted primarily of hydrogen ions (protons) and electrons, along with neutral helium atoms.

An electron would naturally be attracted to the proton, starting to emit electromagnetic radiation due to its motion.

But a much more rapid process would take place when the electron, while aligned in the direction of the proton, spontaneously emits a virtual photon with an amount of energy that exactly matches the potential energy of the electron in an orbital of the hydrogen atom.

In this process, the wave function of the electron can directly wind up as the wave function of a specific orbital of the hydrogen atom without having to undergo a typical collapse to any particular point.

Such episodes would reveal that the wave function does not necessarily always need to go through a traditional collapse for detection.

But the mystery of the occurrence of the quantum to classical transition continues to persist.

Consequently, substantial attempts have been made to find an acceptable solution by modifying the Schrodinger equation but without any success so far.

Despite its outstanding success, some experts like Vittorio Gorini, Andrzej Kossakowski, George Sudarshan and Goran Lindblad have attempted to modify the Schrodinger equation to solve the measurement problem.

Steven Weinberg using the Lindblad equation pointed out using data from atomic clocks that any proposed modification would need to produce an accuracy of at least one part in 10^{17} in the difference between the energy states employed in the clock.

The accuracy of the atomic clocks continues to improve requiring possibly even better improvement of the modification of the theory.

So far, such approaches do not seem to be fruitful.

An attractive scheme generally known as the Ghirardi–Rimini–Weber theory has been studied extensively over the last four decades by arbitrarily attaching a Gaussian function to the Schrodinger equation.

The modification acts as a Markovian process that has negligible effect during the unitary evolution but becomes active afterwards during the final measurement when a very large number of particles become available due to some unspecified diffusion process.

The efficacy of the Ghirardi–Rimini–Weber modified Schrodinger equation remains to be demonstrated.

It may be prudent to consider Steven Weinberg's contention

Unfortunately, these ideas about modification of quantum mechanics are not only speculative but also vague, and we have no idea how big we should expect the corrections to quantum mechanics to be.

Roger Penrose has proposed a novel scheme of a gravitational process to bring about the reduction of the wave function but without any successful experimental demonstration yet.

The most fruitful approach now seems to be the one based on quantum decoherence.

Furthermore, the decoherence time being relatively short, also seem to rule out the Ghirardi–Rimini–Weber modification and the gravitational reduction proposal since both will take some time to be built up for their effectiveness.

Further progress

It is rather amazing that not until about half a century after the advent of quantum mechanics, Heinz-Dieter Zeh was the first to emphasize that the microscopic quantum state wave function evolves unitarily obeying the Schrodinger equation in isolation from the environment.

However, for measurement, the wave function must be exposed to the ambient atmosphere as well as to the plethora of quantum systems of the measuring device.

Under this open circumstance, the various components of the superposed wave function become affected with the elements of the environment as well as the measuring device.

This led to the initiation of a more systematic study of the effect of the environment and the measuring apparatus on quantum system resulting in the loss of quantum coherence, which is now known as *decoherence*.

Use of density matrix was also initiated for decoherence by Zeh in 1970s.

Zeh continued his work on decoherence, sometimes with Erich Joos, for decades.

The next big step forward came when the idea of quantum entanglement was conjoined with decoherence for exploration of quantum measurement.

It is fascinating to appreciate how this historic conjunction came to be recognized.

The award of the 2022 Nobel Prize in physics has brought significant attention to quantum entanglement.

It is now well known that the essence of quantum entanglement arose from the famous Einstein–Podolsky–Rosen paper published way back in 1935.

But it remained effectively ignored by most physicists until John Bell’s epochal article on Bell’s inequality proposed in 1964.

Again it remained on the side line for quite a while due to the lack of a suitable experimental arrangement to verify Bell’s proposal.

Eventually a feasible experimental arrangement was devised five years later by John Clauser, Michael Horne, Abner Shimony and Richard Holt.

The first experimental verification of the Bell–Clauser–Horne–Shimony–Holt theory, now popularly known as quantum entanglement, was carried out by Friedman and Clauser in 1972 with later substantiation by Clauser and Shimony.

A lack of interest of the mainstream scientists to the subject still continued perhaps because of possible loopholes in the substantiation of Bell's theorem.

The checkered history of the development of this period included an underground journal to avoid the apathy of *Physical Review* editors to quantum entanglement.

This is well documented in a book amusingly entitled, *How the Hippies Saved Physics* and penned by David Kaiser.

Eventually, a better authentication of Bell's theorem came from Alain Aspect and his group.

Further experiments to provide loophole free confirmation of Bell's theorem led to the acceptance of quantum entanglement.

An analysis of how does nature possibly accomplish nonlocal action are presented by Bhaumik.

The essential role of entanglement in quantum decoherence was soon realized by Kubler and Zeh.

However, Zeh's emphasis of entanglement in further studies of Everett's theory of quantum measurement apparently distracted him from advancing the appropriate roles of entanglement in decoherence.

Nevertheless, he continued his work with others, Erich Joos being one of them.

Details of decoherence

As soon as the closed quantum system is exposed to the environment including the detector, the unitary Schrodinger evolution in a very short order generates quantum entanglement between the system and the detector including the environment making it possible to combine the system and the detector into a single bigger system.

Since both the system and the detector comprising atoms and molecules abide quantum rules, one can build up a composite tensor product space using two sets of orthonormal basis vectors of Hilbert spaces.

The combined system evolves in a unitary fashion.

Outcome of the measurement of the selected quantum system is determined by the quantum correlations encoded in the globally entangled quantum states of the composite system.

Thus, the conspicuous feature of the decoherence program is that the laws of quantum mechanics are not suspended during measurement, contrary to the popular assumption of most of the pioneers of quantum mechanics for a long time.

Since 1980, decoherence involving quantum entanglement has been extensively studied by Wojciech Zurek with his group at the Los Alamos National laboratory for almost four decades making a very substantial improvement in our understanding of the process.

In summary, Zurek's investigations show that only the eigenstates or the pointer states survive in the complex environmental decoherence process and the number of entanglement states increases very substantially due to what Zurek calls quantum Darwinism.

Consequently, the plethora of entanglement states consisting of the robust pointer states show up in the process of measurements.

Detailed mathematical analyses of the decoherence process have been presented in a substantial number of publications by Zurek and others.

More recently, an excellent entire book on decoherence has been presented by Maximilian Schlosshauer.

A brief synopsis of the essential results of all these investigations will be presented next.

Finding the expectation values

Let us consider that the quantum system and the detector including the environment are each represented by a finite dimensional Hilbert space \mathcal{H}_S and \mathcal{H}_E , leading to a pure composite state $|\psi_{SE}\rangle$ that can be represented by a density matrix $\hat{\rho}_{SE}$ corresponding to the pure state as

$$\hat{\rho}_{SE} = |\psi_{SE}\rangle\langle\psi_{SE}|. \quad (1)$$

The expectation value $\langle\hat{A}\rangle$ of any observable \hat{A} acting on $\mathcal{H}_S \otimes \mathcal{H}_E$ is

$$\langle\hat{A}\rangle = \text{Tr}(\hat{\rho}_{SE} \hat{A}), \quad (2)$$

which is completely determined for the composite state.

Despite that the composite $\hat{\rho}_{SE}$ is pure, in general, both the $\hat{\rho}_S$ and $\hat{\rho}_E$ individually are ensemble of states.

Each of their reduced density matrices contains an incoherent mixture of N quantum state vectors $|\psi_{n,i}\rangle$

$$\hat{\rho}_i = \sum_{n=1}^N p_{n,i} |\psi_{n,i}\rangle\langle\psi_{n,i}| \quad (3)$$

where $i \in [S, E]$, $|\psi_{n,i}\rangle\langle\psi_{n,i}|$ are projection operators with probability $p_{n,i}$ and the sum of the probabilities is normalized $\sum_n p_{n,i} = 1$

Thus, there can be various ensembles of states with each one having its own probability distribution that will produce the same density matrix.

Therefore, for a single copy of unknown state $\hat{\rho}_{SE}$, it is the case that $\hat{\rho}_i$ are unknowable to any meaningful extent for either of the components.

However, if we are given multiple copies of the same composite state $\hat{\rho}_{SE}$, then $\hat{\rho}_{SE}$ and $\hat{\rho}_i$ can be reconstructed using quantum state tomography and $\langle \hat{A} \rangle$ can be obtained as the average of measurement outcomes of \hat{A} , where each measurement is performed on a new copy of $\hat{\rho}_{SE}$.

It is rather amusing to note that we may know everything about the composite entangled pure state, while we may not know anything specific for either one of the component mixed states.

Also, we may know exactly the expectation value of an observable $\langle \hat{A} \rangle$, while we may not know what the measurement outcome for each measurement run will be.

In the situation when the system S and the environment E are quantum correlated by entanglement, an observer having access only to the system S can compute the expectation values for any local observable using only the system's reduced density matrix

$$\hat{\rho}_S = \text{Tr}_E (\hat{\rho}_{SE}) \quad (4)$$

where the reduced density matrix $\hat{\rho}_S$ is obtained by tracing out the degrees of freedom of the environment in the joint system–environment density matrix $\hat{\rho}_{SE}$.

The statistics of all possible local measurements on the system S is comprehensibly encoded in the reduced density matrix.

Thus, for any local observable $\hat{A}_S \otimes \hat{I}_E$ that relates only to the Hilbert space \mathcal{H}_S of the quantum system S , the reduced density matrix $\hat{\rho}_S$ will be sufficient to calculate the expectation value of the observable

$$\langle \hat{A}_S \rangle = \langle \hat{A}_S \otimes \hat{I}_E \rangle = \text{Tr} (\hat{\rho}_S \hat{A}_S) \quad (5)$$

Although the concept of the reduced density matrix was introduced by Paul Dirac in 1930, oddly its significance does not appear to have been appreciated for almost half a century until the advent of quantum entanglement.

An essential element of Zurek's milestone contributions to the decoherence program turned out to be the utilization of entanglement and consequently the reduced density matrix for dealing with expectation values among others.

The problem with decoherence

Although Zurek and his colleagues have advanced the decoherence program in leaps and bounds over the last four decades, there are still some conspicuous complexities in resolving the measurement problem.

To begin with, although the trace rule provides a convenient way to obtain the reduced density matrix and hence the expectation value of an observable, the trace operation is a non-unitary process stroking a whiff of the collapse theory.

More importantly, their work does not seem to provide a satisfactory explanation of where does the probability in measurement come from.

Zurek's derivation has been criticized, among others, by Steven Weinberg. In his classic textbook *Lectures on Quantum Mechanics*, Weinberg states

There seems to be a wide spread impression that decoherence solves all obstacles to the class of interpretations of quantum mechanics, which take seriously the dynamical assumptions of quantum mechanics as applied to everything, including measurement.

Weinberg goes on to characterize his objection by asserting that the derivation of Born's rule by Zurek is

clearly circular, because it relies on the formula for expectation values as matrix elements of operators, which is itself derived from the Born rule.

Maximilian Schlosshauer has become a champion advocate of the application of decoherence toward the resolution of the measurement problem among others.

In a paper on Zurek's derivation of the Born rule, he and Arthur Fine comment

Certainly Zurek's approach improves our understanding of the probabilistic character of quantum theory over that sort of proposal by at least one quantum leap.

However, they also criticize Zurek's derivation of the Born rule of circularity, stating

we cannot derive probabilities from a theory that does not already contain some probabilistic concept; at some stage, we need to "put probabilities in to get probabilities out."

In a recent paper Bhaumik presented a plausible solution that supplements decoherence with some basic aspects of the well-established Quantum Field Theory of the Standard Model of Particle Physics.

Our argument relies on some characteristics of the universal quantum fields that predetermine the values of the complex coefficients involved in the inherent superposition of eigenstates before measurement.

This has been also briefly hinted by Leonard Susskind by stating that the probability of a quantum state does not change during unitary evolution, which is its attribute.

Thus, one of the major obstacles in using decoherence for quantum measurement could be considered resolved.

The other significant problem is that although the reduced density matrix gives a convenient way to find the expectation value of an observable, unfortunately, decoherence does not provide the pointer states separately.

We only get those states still entangled with the environment including the detector states and that is not what an experimenter will measure.

For that purpose, we need separable or product states such as

$$|\psi_S\rangle \otimes |\psi_E\rangle \quad (6)$$

We now present some plausible ways to accomplish this.

Product states using quantum rules

From the available facts so far, it appears fruitful to bring about the innate existence of the ubiquitous *vacuum quantum fluctuations* for this objective.

During the unitary evolution of some superposed quantum states, no substantial effect of the fluctuations has been observed other than their essential participation in *spontaneous emission*.

The vital part played by the quantum fluctuations in facilitating spontaneous emission, which is a unitary process according to quantum electrodynamics, has been known from the early days of quantum mechanics.

It was conveyed in a recent article, how some additional properties of matter like the well-known Lamb shift, anomalous g-factor, etc., would not exist without the ubiquitous fluctuations of the electromagnetic quantum fields.

These quantum fluctuations, essential for spontaneous emission, could very likely separate the pointer states from the entanglement with the environment.

The quantum fluctuations are known to be represented by a Gaussian function.

The effect of the Gaussian quantum fluctuations has never been witnessed to affect the unitary Schrodinger evolution to any appreciable degree.

But its significant effect could be cumulatively operative during the measurement process when a substantial number of the entangled states have been produced.

Thus, it seems reasonable to explore if the quantum fluctuations could make the disentanglement effective in aiding quantum measurement in somewhat of a manner envisioned by the Ghirardi–Rimini–Weber proponents but without any modification of the Schrodinger equation as well as not requiring a very large number of quantum states during the final measurement.

In our goal to understand the effect of quantum fluctuations to produce disentanglement in recovering the product states, it seems prudent to explore some of the relevant new activities being pursued by the quantum computation community.

After Peter Shor's publication of his celebrated algorithm for quantum computing in 1994, extensive studies have been carried out in both decoherence and disentanglement, which is of critical importance to quantum technology for avoiding loss of quantum coherence.

Hence the activities on these topics have exploded exponentially in the last two decades.

As a resource, quantum entanglement has now been measured, increased, decreased or even distilled and teleported.

The necessity for investigating decoherence as well as disentanglement for quantum technologies is opposite to our requirement in quantum measurement but there could be a commonality.

The quantum fluctuations, so essential for spontaneous emission, could very well be involved in terminating the entanglement with the detector states.

Particularly, it could be fruitful to pursue the surprising experimental observation called ESD, which stands for *early stage disentanglement* or *entanglement sudden death*, that has been observed by several groups.

In these experiments, astonishingly a very swift disappearance of entanglement altogether has been reported.

Other works report entanglement breaking channels.

In the simplest experimental setup, two entangled atoms in their excited states are placed one each in two widely separated cavities without any direct interaction.

When the two atoms reach their ground states by spontaneous emission, surprisingly the entanglement suddenly disappears completely and the two atoms in their ground state constitute product states.

Although not yet fully understood, the sudden disappearance of the entanglement is an experimental fact that could possibly be caused by a process like what occurs in the unitary quantum electro-dynamical depiction of spontaneous emission.

If that turns out to be true, since unitary process preserve the probability, the final reduced quantum state would have the same probability all the way from superposition to reduction.

In contrast, the von Neumann collapse postulate assumes a non-unitary process following Born's rule.

The act of spontaneous emission appears to be a sudden non-unitary jump, however, if one were to keep track of all the vacuum modes, as per quantum electrodynamics the combined atom–vacuum system in fact undergoes a unitary time evolution.

Thus, there could be a plausible chance that the ESD process might be unitary although the details are not yet fully understood.

Further studies are planned to explore this propitious possibility.

Another feasible process resembling some aspect of the Ghirardi–Rimini–Weber procedure appears promising.

This is the experimentally observed disentanglement caused by quantum fluctuations.

Like a Markovian process, the quantum fluctuations does not affect the normal Schrodinger evolution indicating a limitation of the strength of the relevant interaction.

It appears to take place only after enough states are made available by quantum Darwinism, when the cumulative strength of interaction would cause the disappearance of entanglement leading to the desired separable states.

However, since the same information about the pointer observable is stored independently in many fragments of the environment, suitable detectors can measure the observable in different fragments even without any observer involved.

From the experimentally observed results of the consistent effect of quantum fluctuations in diminishing quantum entanglement, this approach appears to be assured for accomplishing the desired separable states.

Our goal is to capture the disentangled observables in the detector.

So we need to find out what could cause the disentanglement.

Several experiments clearly confirm that the vacuum quantum fluctuations cause the disentanglement.

Thus, the essential agent has been clearly identified and we could leave at that.

But the work would be more complete if we can provide the rate and consequently the disentanglement time that could be reasonably short.

We know that the quantum fluctuations can be represented by a Gaussian.

So we need to find the rates and value of the constants for the Gaussian and then possibly making some calculations like in the Ghirardi–Rimini–Weber model to predict the time taken for disentanglement.

It is not essential but would complete the program.

However, because studies of the details of the disentanglement process is still continuing vigorously and many of the results does not clearly identify whether it was done in a cavity where quantum electrodynamics can give more than a number of rates say for spontaneous emission.

Again, the most important part is to experimentally identify the mechanism that causes disentanglement and that we already have accomplished with reasonable confidence.

So the principal objective can be considered reasonably accomplished.

Concluding remarks

It is evident by now that the advent of quantum entanglement has led to a quantum leap for a resolution of the enduring measurement problem through the decoherence procedure.

Additionally, the distinctive appeal of this program revealing that the quantum rules are not suspended during the measurement process is unique.

Although many others have contributed, the decades of concerted efforts by Zurek and his group have advanced the progress of the decoherence program to a fairly mature stage.

The primary deprecation of their advancement concerns, however, is the lack of a satisfactory answer to the origin of probability and the occurrence of separable product states in the measurement process.

A cogent perspective is presented here that appears to alleviate the deficiency of achieving the expected observables and their probabilities in measurement.

Therefore, along with the prior article by the author, a solution of the century old quantum measurement problem could be on hand.

Significantly, much of the process of the reduction of the wave function or quantum to classical transition occur following quantum rules in contrast to the visions of the pioneers of quantum physics.

The universe is quantum at the core and so are we.

About seven octillions of electrons and a plethora of other elementary particles inhabit our body.

Our existence in the familiar classical world is made possible by continuing transition from the quantum to classical domains additionally, of course, with the irreversible metabolic processes.

The quantum origin of objects in the classical arena is patently supported by the recent observation of a sliver of residual quantum activity in a man size 40-kilogram mirror in the Laser Interferometer Gravitational-Wave Observatory (LIGO).

In fact, in a variety of experiments, quantum effects have been observed from mesoscopic to macroscopic entities clearly indicating a transition from quantum to classical is the abiding rule when a quantum system is exposed to a huge number of quantum particles.

Most significantly, deriving the wave function of a non-relativistic quantum mechanics from the fundamental reality accessible to us so far by the standard model of particle physics and utilized by the author in a series of publications, illustrate quantum mechanics could be considered weird no more.

We must recognize that there are two distinct parts of reality, the quantum and the classical with their characteristic rules, but one transitioning to the other.

The perception of weirdness arise when we try to understand our daily classical world through the lens of quantum rules.

The quantum theory could be as splendid a theory based on fundamental reality as has been both the non-relativistic Newton's laws as well as Maxwell's theory of electrodynamics.