The Standard Model of Particle Physics

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1. INTRODUCTION AND HISTORY

The Standard Model is one of the greatest scientific achievements of all time.

It consistently describes all known fundamental particles and their interactions with the exception of gravity that is still properly described at low energies.

In this sense we can now explain any fundamental physical phenomenon at the smallest distances that can be probed experimentally with spectacular success.

We may therefore claim that, so far, the Standard Model is the fundamental theory of Nature.

The Standard Model is the most successful application of quantum field theory when it comes to experimental verification.

It is the final conclusion of many decades of intense research both on the theoretical and experimental side.

Its structure was finally completed after the celebrated discovery of the Higgs particle in 2012.

Over the decades since its ingredients were combined, thousands of measurements have been made at energies $E \leq 1\text{TeV}$, all consistent with the Standard Model.

The Standard Model describes the physics of the building blocks of all visible matter: spin 1/2 quarks and leptons interacting via three fundamental forces, each mediated by spin 1 particles known as gauge bosons.

Electrically charged particles feel the electromagnetic force by exchanging photons as described by Quantum Electrodynamics (QED).

The electromagnetic interactions are of long range due to the fact that photons are massless.

In contrast, the short range weak force is responsible for certain radioactive decays such as the neutron β-decay and plays a crucial role in the thermonuclear interactions within stars.

The mediators of this interaction are the massive W and Z bosons.

Their large mass is responsible for the short range of the interaction.

The strong force binds quarks into nucleons (protons and neutrons) and indirectly nucleons into nuclei; the carriers of the strong force are appropriately called the gluons.

Leptons, such as electrons and neutrinos do not feel the strong force.

Particles made out of quarks are called hadrons which can be either baryons (made up of 3 quarks) and mesons (a quark-antiquark pair).

The aim of these lectures is threefold.

First, to let the students appreciate all the twists and turns that drove scientists in the past century to discover and develop the Standard Model.

A historical perspective is important to appreciate the magnitude of the achievements, but also the surprises and human drama that came with the development of new ideas in particle physics.

Most importantly, it demonstrates that research is not a straight line of well developed arguments as usually presented in textbooks and lecture notes, but rather a windy road with occasionally unforeseen twists and turns before a proper understanding emerges.

The second goal is for students to internalize that, despite the Standard Model being only one in an infinite number of possible field theories, its structure is extraordinary rigid and compelling.

Following general principles within our basic theories, Special Relativity and Quantum Mechanics, we take a constructive approach arguing that it is essentially unavoidable that elementary particles are determined by unitary representations of the Poincaré group limiting their nature to only a handful of possibilities, namely spins 0, 1/2, 1, 3/2, 2 out of an infinite number of possible spins.

Further, we will see why gauge invariance defining the three non-gravitational interactions is only a redundancy needed to properly describe the interactions.

Lastly, even though Field Theory is the basic formalism to describe interactions, the fundamental objects are actually the particles themselves, whereas fields are a necessary tool to describe local interactions among particles.

The third target of these lectures is for students to get familiar with the physical details of the Standard Model and be able to reproduce some of the key calculations and results that led to its successful completion.

Throughout the lectures emphasis will be given to basic principles and potential loopholes that may be important to guide us towards the unknown physics beyond the Standard Model.

That is, we provide crucial methods to build new theories (or models) of nature – a skill that is vital for any theoretical physicist.

We follow the guideline as in the quote of Steven Weinberg

> *Our purpose in theoretical physics is not to describe the world as we find it, but to explain – in terms of a few fundamental principles – why the world is the way it is.*

with the key word being to explain rather than describe.

We emphasize the explanatory power of the Standard Model towards all the experiments, but also towards some of the approximate or accidental symmetries such as isospin, flavor, baryon number, etc.

Just like the Standard Model provides a UV description of Effective Field Theories (EFTs) such as the Fermi theory of weak interactions and the pion dynamics of Yukawa, it should itself only be regarded as an EFT once gravity or other UV physics is included.

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1.1 Brief History of the Standard Model

You have already seen some of the ideas we will be discussing in this part of the class as part of the general QFT slides.

Before we now begin with a very careful treatment of the Standard Model itself, let me present here a brief sketch of the historical developments that led to the Standard Model.

1600's		Classical **Gravity**. First unification of interactions (Newton).
1800's	1861	Classical **Electromagnetism**. Second unification (Maxwell
	1869	Periodic table (Mendeleyev). Discrete nature of matter not established.
	1896	Radioactivity (Becquerel, P. & M. Curie, Rutherford).
		α-, β- & γ-decay as hint for instabilities in nature &
		hint for new **Weak Interactions** & **Strong Interactions**.
	1897	**Electron** discovered (J. J. Thomson first computation of e/m_e.
		Beginning of particle physics!
1900's	1900-1930	Quantum Mechanics developed & established (e.g. **Photons** as particles).
	1905	Special Relativity (Einstein e.g. $c =$ const. & spacetime structure).

$\Rightarrow$ *The two basic theories of nature.*

1910's	1911	Rutherford formulated a model for atoms (**Proton** nucleus of H).
		First cloud chamber is constructed (Wilson).
	1912	Cosmic rays discovered (Hess
	1915	Einstein General Relativity
	1919	F. Aston postulates the "whole numbers rule" $\to$ **proton**.
1920's		Bose & Fermi statistics.
		Beginning of Quantum Field Theory: Dirac, Jordan, Heisenberg, ...
		Dirac equation with solutions of charge ± 1.
1930's	1930	Pauli predicts **Neutrino** (energy & momentum conservation in β-decay).
	1931	Dirac predicts **Positron** e^+ as anti-particle of e^-
	1932	Anderson discovered positron e^+.
		Chadwick discovered **Neutron**.
		Heisenberg introduces isospin as symmetry between n & p.
		Fermi theory of weak interactions e.g. β-decay: $n \to p + e^- + \bar{\nu}$.
	1935	Yukawa theory of strong interactions Scalar mediators **Pions** π predicted.
		Short range potential $V \sim \mathrm{e}^{-m_\pi r}/r$ with $m_\pi \sim 100\mathrm{MeV}$.
	1936	Anderson, Neddermeyer include isospin as d.o.f. in wave function.
	1939	Wigner's description of particles as representations of Poincaré group

1940's 1947 Lamb shift (vacuum polarisation becomes relevant).

QED (Schwinger Feynman Tomonaga Dyson

Pions π are discovered (charged 1947 neutral 1950

1950's Particle accelerators and bubble chambers ($E \geq$MeV).

dozens of new particles discovered (mostly strongly interacting).

Hadrons: Kaons, hyperons, ...

2 classes: **Mesons** (bosonic) & **Baryons** (fermionic).

Classification: charge (Q), baryon number (B), lepton number (L),...,

strangeness (S) (Gell-Mann 1956 as new charge.

1954 Yang-Mills (& Shaw) theory generalising QED

1956 Parity violation theoretically conjectured (Lee & Yang Salam

implies that parity is *not* a fundamental symmetry of nature!

Discovery of **(Anti-)Neutrino** (Cowan, Reines

1957 Wu discovered parity violation experimentally

Neutrino oscillations proposed (Pontecorvo

V-A structure of weak interactions (Marshak & Sudarshan

also Feynman, Gell-Mann

1960's 1961 Eightfold Way (Gell-Mann & Ne'eman

Symmetry breaking (Nambu Goldstone, Weinberg, Salam

1962 Cabibbo mixing

Muon Neutrino ν_μ discovered (Steinberger et al.

Preliminary Electroweak unification (Glashow Salam-Ward

1964 **Quarks** u**,** d**,** s proposed (Gell-Mann & Zweig

Higgs Mechanism (Higgs Brout, Englert Guralnik et al. Kibble

Ω^- discovery CP violation Kaon decay

Colour introduced (Greenberg Han-Nambu

Charm quark predicted (Glashow, Bjorken

1967 **Electroweak Unification** (Weinberg Salam

1968 Deep inelastic scattering (Friedman, Kendall, Taylor et al.

Parton (quarks, gluons) composition of hadrons (Bjorken, Feynman).

1968 Solar neutrino puzzle (Bahcall & Davis

1969 Anomalies (Bell, Jackiw Adler

This was the period (1961-67(that I was in graduate school at Cornell University.

No one at Cornell was directly involved in the Standard Model development.

1970's	1970	Glashow-Iliopoulos-Maiani (GIM) mechanism existence of charm quarks explains suppression of flavour-changing neutral currents.
	1971	Renormalisability of Weinberg-Salam model (t'Hooft
	1973	**Asymptotic Freedom** (Gross-Wilczek Politzer
		QCD (Fritzsch, Leutwyler, Gell-Mann
		Weak Neutral Currents measured at CERN
		Kobayashi-Maskawa mixing (3-families, CP-violation)
	1974	J/Ψ discovery (Richter et al. proves existence of charm quark.
		Effective Field Theory (Wilson, Weinberg).
	1975	**Tau Lepton** τ (M. Perl et al.
		Quark Jets (hadronisation) & 2-jet events ($e^+e^- \to q\bar{q} \to 2\,\text{jets}$).
	1977	Upsilon discovered alongside the **Bottom Quark** (Fermilab
	1979	**Gluon** evidence jets $e^+e^- \to q\bar{q}q \to 3\,\text{jets}$
1980's	1983	Discovery of $\mathbf{Z^0}$, $\mathbf{W^\pm}$ (Rubbia et al. at CERN with 170GeV collisions).
1990's	1995	**Top Quark** discovery (Fermilab
		LEP precision tests of SM (< 3 light neutrinos).
	1998	Neutrino Oscillations
2000's	2001	**Tau Neutrino** ν_τ discovery
2010's	2012	**Higgs** discovery
	2016	First detection of **Gravitational Waves** by LIGO

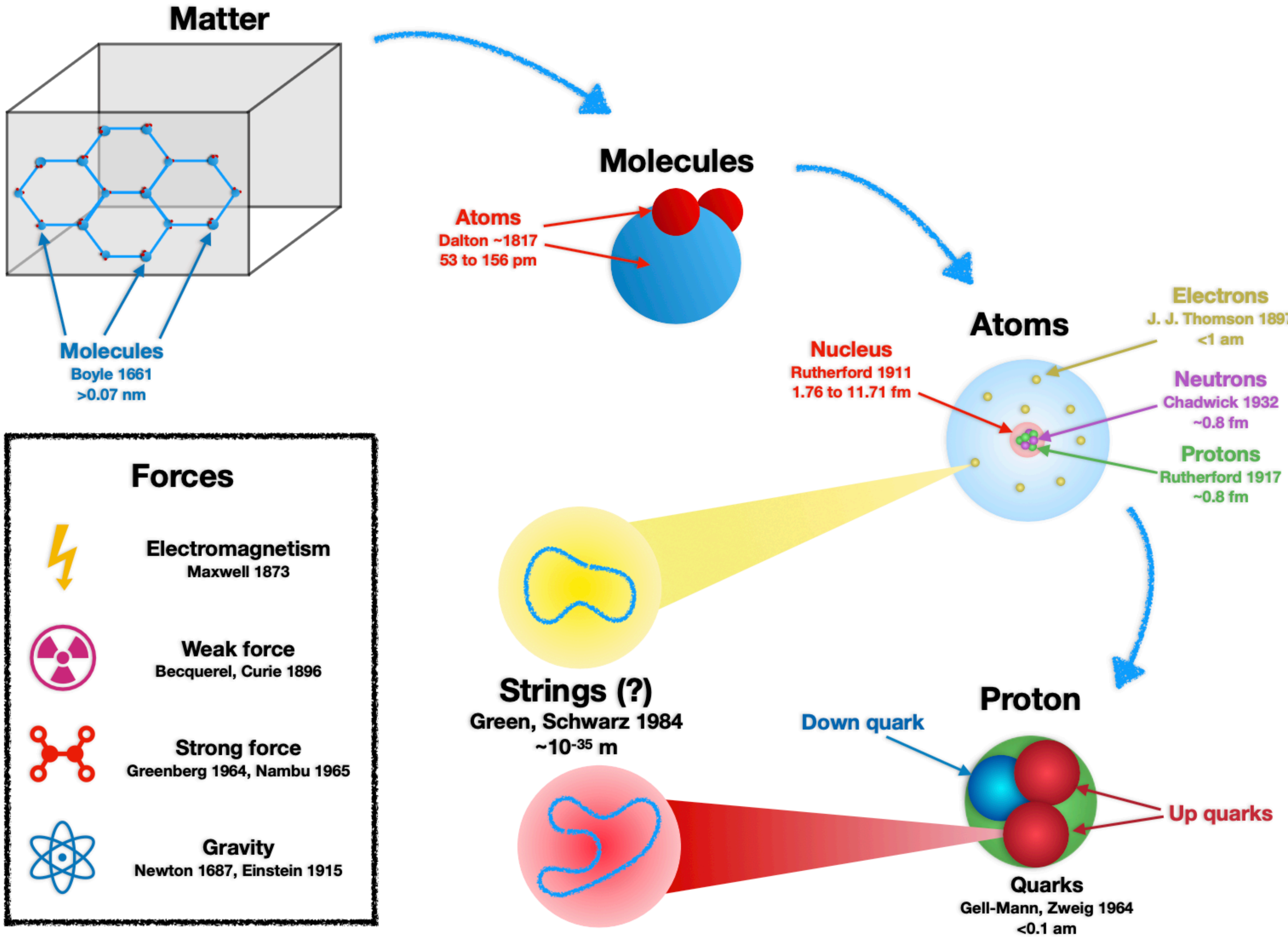

Figure 1.1: An historical account of our understanding of matter and its constituents. The complementary forces are summarized in the box on the left. The dates either refer to the earliest major contribution towards a theoretical understanding or to the first experimental evidence.

From the above, we find that the history of the Standard Model involves many interesting scientific developments, great creativity, very hard work with ups and downs with incredible achievements.

As usual, the human factor plays an important role as the following anecdotes establish:

- How a combination of great theoretical ideas with ingenious and brave experimental initiatives managed to unlock the deepest mysteries of the elementary particles.

- In particular, how the Cavendish laboratory played such a crucial role in the early part of the 20th century with some of the major discoveries that helped identify the structure of the nucleus and different elementary particles. It is interesting to notice that the discoverer of the neutron (Chadwick) was a student of the discoverer of the proton (Rutherford) who in turn was a student of the discoverer of the electron (Thomson), completing the composition of all atoms. All of them have followed the steps of the previous Cambridge giants: Newton and Maxwell who had performed the great unifications of the past, namely the gravitational forces on Earth and space, and electricity, magnetism (and optics), which can be considered the start of the Standard Model. This can make us proud and humble to address these questions in this very same place where so many developments were made.

- The fact that a few years after Thomson discovered the first elementary particle (the electron) his son managed to prove that the electron was also a wave identifying the electron diffraction patterns.

- The original reluctance to explain experimental results by introducing new particles(it delayed the identification of the neutron as an independent new particle instead of an electron-proton composite).

- Pauli's bold proposal of the existence of a totally new class of particles, neutrinos, based on arguments of conservation laws.

- Dirac's contrived prediction of the positron and anti-particles, while being again reluctant for some time to accept the positron as a new particle.

- The several independent discoveries of the positron, but most failed to appreciate it or report on time.

- Yukawa's prediction of pions as mediators of the (strong) interactions among protons and neutrons to keep them together within the nucleus dominating the electromagnetic repulsion among protons. Contrary to the electromagnetic interactions which are long range, these strong interactions had to be only at the nuclear scale and Yukawa concluded that the mediators of the interactions were massive particles. Almost immediately the muon was discovered at very similar mass as predicted by Yukawa creating confusion since the muons only interact by weak and electromagnetic interactions. The confusion finished when the pions were later discovered with a mass similar to the muons but with the properties Yukawa had predicted. It took many years for people to understand that Yukawa's theory was only an approximation of the fundamental strong interactions mediated by gluons. In current terminology Yukawa's theory is an Effective Field Theory (EFT) that is completed in the UV by QCD.

- Fermi's theory to describe weak interactions such as β decay in terms of four-particle interactions being very accurate at low energies but failing at larger energies. This is today also understood as an EFT that is completed in the UV by the exchange of the massive Z and W particles of the Standard Model.

- The role of a relatively simple issue in atomic physics such as the Lamb-shift leading to the full theoretical development of QED.

- The introduction of approximate symmetries such as strangeness and eightfold way (see Fig. 1.2) leading to the prediction of quarks.

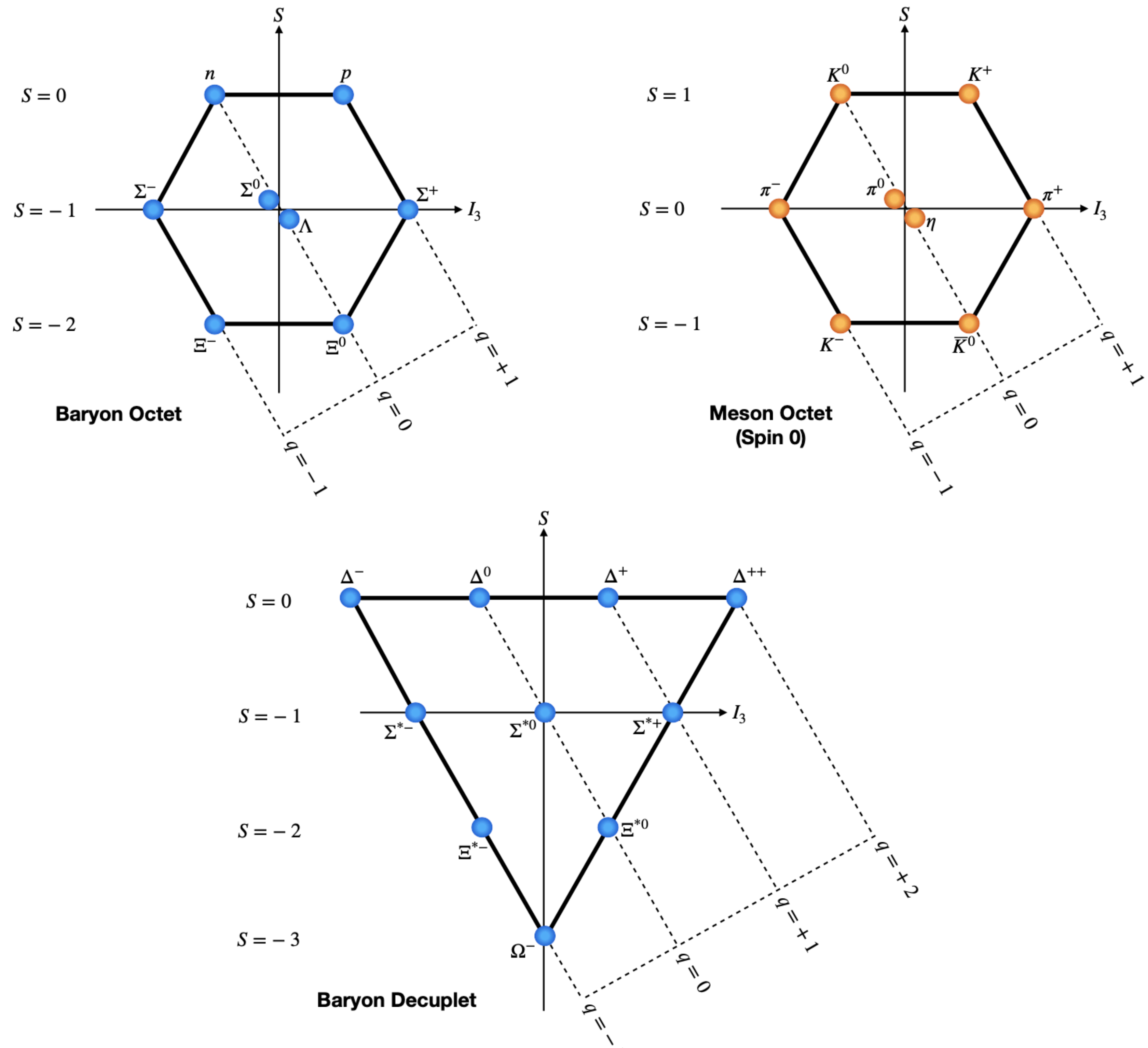

Figure 1.2: The eightfold way. The weight lattice of hadrons in their respective $SU(3)_f$ representations with the baryon octet (top left) corresponding to 8 of SU(3)f, pseudo-scalar meson octet (top right) again corresponding to 8 of SU(3)f and the baryon decuplet (bottom) corresponding to 10 of $SU(3)_f$. Here, S denotes 'strangeness' and I3 isospin.The prediction of the Ω^- particle and its subsequent discovery lead strong credibility to this approximate symmetry.

- The original reluctance to question the validity of parity violation and the great inspiration of Lee and Yang to prove it and Wu to confirm it experimentally in such a short time.

- How physicists became involved in world's politics. For instance, the discoverer of the neutron, Chadwick, was a first world war prisoner for 4 years where he still managed to perform some experiments (similar to Karl Schwarzschild who found the black hole solution of Einstein's equations while fighting in the war in 1915). Although the second world war somehow delayed fundamental scientific progress since some of the leading scientists were involved in projects such as the Manhattan project, after the war, scientists quickly returned to ask and answer fundamental questionsandevenovercamepoliticaldifferencesbyhavingcollaborationsfromboth sides during the cold war. Although cases, like Pontecorvo, took sides and escaped from the West (after a mysterious disappearance) and others moved in the other direction. Like Harald Fritzsch, one of the pioneers of QCD, escaping in a folding boat from East Germany via Bulgaria to Turkey after organizing a major protest.

- The disappearance of one of the greatest minds of the 20th century (Majorana) that still remains a mystery.

- Creating international institutions like CERN allowed scientists to show how a united effort among different countries can lead to great achievements, much earlier than politicians realized the same and proposed the European Union. CERN is still the best example of international scientific collaborations and defines the recently coined term of science diplomacy.

- Pauli's dismissal of Salam's ideas on parity violation (that were later confirmed by Lee and Yang).

- Pauli's also criticism of Yang and Mills since, as he correctly pointed out, their theory predicted massless particles that should have been observed and were not. It so happens that Pauli had obtained the same theory as Yang and Mills starting from a six-dimensional theory but decided not to publish his results because of the massless particles problem. He gave such a difficult time to Yang in a seminar at the Institute for Advanced Studies in Princeton that Yang decided to stop the seminar and sat down until Oppenheimer convinced him to continue.

- How, in turn, Salam discouraged his PhD student Ronald Shaw to publish the results of his thesis which were identical to those of Yang and Mills based on the same argument of Pauli. When years later people realised that Yang-Mills theories were the basis to describe the electroweak and strong interactions in the Standard Model (after the massless problem was solved by the Higgs mechanism and gluon confinement), Salam called the theory Yang-Mills-Shaw. But it was too late.

- How theV-A (vector minus axial vector) theory of Marshak and Sudarshan (followed by Gell-Mann and Feynman) played a key role in deciphering the weak interactions but originally contradicted four different experiments that ended up being wrong.

- How Weinberg (and the independent work by Salam) trying to understand strong interactions led him towards understanding the weak interactions and unifying them with the electromagnetic interactions in a 3-page paper where he predicted neutral currents, the $W^\pm$ and $Z0$ particles as well as the Standard Model Higgs. All these predictions were later confirmed experimentally. It is hard to find any written material in history carrying such an amount of information and successful predictions in such a few number of words.

- How the intuition of Bjorken and Feynman combined to extract the relevant information of the deep inelastic scattering experiments that determined the composite nature of protons and neutrons and finally are the best evidence for the existence of quarks.

- The fact that quarks for several years were only considered mathematical objects (even by Gell-Mann himself) since they did not exist in liberty, but theorists and experimentalists were clever enough to find evidence for them and also for gluons despite the fact that they are confined within hadrons.

- The origin of color (one of the most fundamental properties of the SM) just to address a problem with Pauli exclusion principle in the $\Omega-$ (and other) particles.

- The importance of path integral techniques that allowed ’t Hooft to prove the renormalisability of spontaneously broken gauge theories and how it suddenly gave credibility to the Weinberg-Salam model that had been totally ignored for 4 years.

- The relevance of a simple minus sign that allowed Gross, Wilczek and Politzer to explain and understand the strong interactions (asymptotic freedom). How several groups failed to identify it or recognize the importance and how the two groups finally agreed with this important sign.

- The prediction of the charm quark based first on general symmetry arguments by Glashow and Marshak and later on a way to understand the suppression of flavour changing neutral currents (GIM Mechanism of Sheldon Glashow, John Iliopoulos and Luciano Maiani) and subsequent discovery of charm in terms of the J/Ψ particle.

- The fact that several groups missed the J/Ψ discovery (one of the most surprising and exciting discoveries that helped to confirm the quark theory and particularly the existence of charm) but one of them (Lederman) was lucky enough to later find the upsilon particle leading to the unexpected discovery of the bottom quark.

- The different stages of trust in quantum field theory to describe elementary particles from the early attempts in the 1930s to excitement after QED in 1948 to almost rejection before asymptotic freedom and back to life again after that.

- How a very simple SU(3) extension of the mixing ideas of Cabibbo for SU(2) by Kobayashi and Maskawa had the important information about CP violation in the Standard Model and how the whole scientific community was surprised and disappointed for Cabibbo to be left out of the Nobel prize.

- How several groups simultaneously came up with the idea of the Higgs mechanism but failed to identify the importance for the weak interactions and did not even mention the Higgs particle (except only for Higgs but only apparently after the suggestion of the referee).

- The persistence for decades of one experimentalist (Ray Davis) and one theorist (John Bahcall) to insist that the solar neutrino problem (the fact that a much smaller number of neutrinos are detected than predicted) was a fundamental rather than astrophysical issue. This can be solved by noticing that the different types of neutrinos can oscillate converting from one type to another (as we will see in this course) and the ones that are produced in the Sun are changed to another kind of neutrino while leaving the core of the Sun explaining why less are detected when they arrive on Earth.

- The impressive international efforts towards the search and final discovery of the top quark, the W,Z and Higgs particles, etc

- Before the Higgs discovery, how the CERN international efforts gave rise, essentially by accident, to the world-wide-web (WWW) that made internet accessible to the general public, one of the most influential developments of the past decades, confirming the importance to study basic science, even for its impact on spin-off applications.

These are just a few of the highlights for this beautiful set of events that illustrate, as well as anything else, how science, as a human endeavor, is made and how there is usually a large amount of confusion before things get properly understood and then written in a logical way in textbooks and lecture notes like these.

This history is also a lesson for current times in which there are many open questions and no clear guidance for the future rather than the knowledge acquired that led to the Standard Model and some open questions.

Knowing how scientists have addressed and solved questions of the past is a good guidance for how to address new challenges.

Even though this historical tour was brief and many of the concepts are unknown to the students, we would like to encourage the students to come back to some of these events after we have discussed the fundamental aspects of the Standard Model in a systematic rather than historical way to appreciate the greatness of the achievements and the combination of consistency, rigor, creativity, imagination and luck that is needed to do fundamental research.

As Weinberg emphasized, this is one of his four gold rules for scientists: you can get great satisfaction by recognizing that your work in science is a part of history.

1.2 Summary and Motivation

Let us start with a first overall glimpse at the Standard Model just to introduce the concepts that will be developed during the subsequent lectures.

1.2.1 A Brief Introduction to the Standard Model

Definition: The Standard Model (SM) is a construction (or model or theory) that describes all the known elementary particles and their interactions in terms of relativistic quantum field theories.

Ingredients:

(1) *Spacetime*. The spacetime is (3 + 1)-dimensional Minkowski spacetime with (global) symmetry group

$$G_{\mathrm{gl}} = \mathbb{R}^{3,1} \rtimes \mathrm{O}(3,1) \tag{1.2.1}$$

that is the semi-direct product of spacetime translations and the Lorentz group, corresponding to the Poincaré group of special relativity.

Matter. The particle content can be classified by the spin s (or helicity), i.e., there are the *Higgs H* with s = 0 as well as 3 families of quarks and leptons with s = 1/2, see also table 1.1.

Name	Label	$\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$	Spin/Helicity
Quarks	$Q_L^i = \left(u_L^i, d_L^i\right)$	$\left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right)$	$\frac{1}{2}$
	u_R^i	$\left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}\right)$	$\frac{1}{2}$
	d_R^i	$\left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}\right)$	$\frac{1}{2}$
Leptons	$L_L^i = \left(\nu_L^i, e_L^i\right)$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$	$\frac{1}{2}$
	e_R^i	$(\mathbf{1}, \mathbf{1}, -1)$	$\frac{1}{2}$
	ν_R^{i*}	$(\mathbf{1}, \mathbf{1}, 0)$	$\frac{1}{2}$
Higgs	H	$\left(\mathbf{1}, \mathbf{2}, +\frac{1}{2}\right)$	0
Gluons	g_α	$\left(\mathbf{8}, \mathbf{1}, 0\right)$	1
W/Z-Bosons	$W^\pm, Z^0$	$\left(\mathbf{1}, \mathbf{3}, 0\right)$	1
Photon	γ	$\left(\mathbf{1}, \mathbf{1}, 0\right)$	1
Graviton*	$h_{\mu\nu}$	$\left(\mathbf{1}, \mathbf{1}, 0\right)$	2

Table 1.1: Particle content of the Standard Model and the corresponding group representations. The right-handed neutrino and the graviton are included here for completeness with the understanding that the couplings of the graviton to all other particles can be studied as long as the energies are small enough in terms of an effective QFT.

(3) *Interactions*. The interactions are given by 3 gauge interactions with associated gauge bosons of spin s= 1 and, in general, also gravitational interactions transmitted by a particle of spin s= 2 known as the graviton. The gauge forces are encoded in the gauge (or local) symmetry group

$$G_{\mathrm{SM}} = \underbrace{\underbrace{\mathrm{SU}(3)_C}_{\text{color}}}_{\text{strong}} \times \underbrace{\underbrace{\mathrm{SU}(2)_L}_{\text{left}} \times \underbrace{\mathrm{U}(1)_Y}_{\text{hypercharge}}}_{\text{electroweak}} \tag{1.2.2}$$

where the subindex C refers to *color* with $SU(3)_C$ determining the strong interactions. The strong force binds quarks into nucleons and nucleons into nuclei; the carrier of the strong force is appropriately called the *gluon*. The L in $SU(2)_L$ refers to *left-handed* in the sense that weak interactions only act on left-handed particles. Finally Y refers to *hypercharge*. The gauge group in (1.2.2) is broken by the Higgs boson through a non-zero vacuum expectation value, to a subgroup, namely

$$G_{\mathrm{SM}} \xrightarrow{\mathrm{SSB}} \underbrace{\mathrm{SU}(3)_C}_{\mathrm{QCD}} \times \underbrace{\mathrm{U}(1)_{EM}}_{\mathrm{QED}} . \tag{1.2.3}$$

In the process referred to as spontaneous symmetry breaking (SSB), the corresponding gauge bosons of the broken group ($W^\pm$ and Z^0) receive a mass, but there is also a remaining massless boson corresponding to the unbroken $U(1)_{EM}$ with EM standing for electromagnetic. This is the familiar photon of the electromagnetic interactions. To reiterate, after the breaking of the symmetry, only the gluons and photons remain massless. Photons are free to move but gluons together with quarks are confined within the particles of strong interactions such as protons and neutrons. The representations of the particles involved are summarized in table 1.1.

(4) *Three families*. For the quarks and leptons, there are 3 distinct families coming with the same copies of the representation:

$$\text{Leptons:} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \qquad \text{Quarks:} \quad \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} .$$

Only the first family (with electron, its neutrino and up and down quarks) are enough to make the matter we know. The second (muon, its neutrino, charm and strange quarks) and third (tau-lepton, its neutrino, top and bottom quarks) are more massive and the corresponding particles are unstable having the particles of the first family as end results of their decay.

It is remarkable that these simple ingredients are enough to account for the structure of the Universe as we know it including every single experience and measurement we make. There are some comments in place:

- *Chirality*. Since the right-handed quarks u_R and d_R transform under the trivial representation of $SU(2)_L$ in table 1.1, that is to say they are $SU(2)_L$-singlets, they do not feel the $SU(2)_L$-interactions. This is why we call the Standard Model a *chiral* gauge theory. Physically, this implies that left- and right-handed fermions feel certain gauge interactions differently, i.e., they couple non-democratically to the mediators (gauge fields) of a given force. Mathematically speaking, left- and right-handed fermions transform in different $SU(2)_L$-representations. Thus, the weak interaction is not parity invariant under exchange of left- and right-handed particles.

- *Charge quantization*. The electric charge is defined as

$$Q = T_3 + Y \tag{1.2.4}$$

where T_3 is the third generator of $SU(2)_L$ which is the diagonal matrix with entries $(1/2, -1/2)$ and Y the hypercharge under $U(1)_Y$. For instance to compute the electric charge of the left-handed electron we observe that its hypercharge is $Y = -1/2$ and its value of T_3 is $-1/2$ giving $Q = -1$. For the right-handed electron the

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corresponding T_3 value is zero because it is a singlet and then charge and hypercharge are the same (here +1 for the positron). Computing the electric charges of quarks give multiples of 1/3 instead of integers as we are familiar for electrons and protons.

- *Consistency conditions*. We observe that the assignment of these numbers such as hypercharge and the different representations of the Standard Model particles is not arbitrary. For instance it is easy to verify the following conditions for the hypercharges

$$\sum_{Left} Y - \sum_{Right} Y = \sum_{Left} Y^3 - \sum_{Right} Y^3 = 0\,. \tag{1.2.5}$$

Also the total number of particles transforming as a **3** of $SU(3)_C$ equals the number of $\bar{\mathbf{3}}$ (#**3** = # $\bar{\mathbf{3}}$) and the total number of $SU(2)_L$ doublets (#**2**) is even. Any modification of these numbers would render the theory mathematically inconsistent. This will be crucial in ensuring anomaly cancellation within the Standard Model. If the Standard Model was not chiral, these conditions would be trivially satisfied. It is the chiral structure of the Standard Model that makes it subject to potential inconsistencies and therefore makes it more interesting when they are satisfied.

- *Coleman-Mandula theorem*. The total symmetry of the Standard Model is given by a direct product between a spacetime and an internal symmetry

$$\text{spacetime}\otimes\text{internal gauge} = (\mathbb{R}^{3,1} \rtimes \mathrm{SO}(3,1))\otimes(\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y)\,. \tag{1.2.6}$$

The Coleman-Mandula theorem states that this structure is the most general for the full symmetry group, i.e., a direct product of the Poincaré group and an internal (gauge) group.

- *Gravity as an effective field theory*. We can treat gravity only as what is called an effective QFT (EFT) by working with energies E well below the Planck scale:

$$E \ll M_P = \sqrt{\frac{\hbar c}{G}} \sim 10^{18}\text{GeV} \,. \tag{1.2.7}$$

  At energies $E \sim M_P$, quantum effects of gravity become important and the EFT has to be replaced by a more fundamental theory that is ultra-violet complete. But for energies well below $E \ll M_P$ working with quantum aspects of gravity as an EFT are predictable and reliable. We will have to say more about the role of gravitational interactions within the Standard Model later in the course.

- *Accidental symmetries*. There are accidental symmetries known as *Baryon number* B and *Lepton number* L. That is, the total number of baryons, such as the neutron and proton, and the total number of leptons such as the electron and neutrino are conserved in every interaction.

- *Approximate symmetries*. The three families of quarks and leptons in which the members of each family behave the same as the other families except that the particles are heavier for each generation (e.g., the muon is like a heavier copy of the electron, the top quark of the up quark, etc.) implies that there are approximate symmetries known as f*lavor symmetries* such as $SU(3)_f$ known as the *eightfold way*, see Fig. 1.2. This flavor $SU(3)_f$ should not be confused with the color $SU(3)_C$ which is the symmetry describing the strong interactions.

- *Phases of the Standard Model*. The Standard Model is relatively simple, although not the simplest model we can imagine. Actually, it is rich enough to illustrate the 3 main phases of gauge theories: The *Coulomb phase* for U(1)EM meaning that the corresponding gauge boson, the photon, moves freely; the *confining phase* for $SU(3)_C$ meaning that the interactions are so strong that the corresponding gauge bosons, the gluons, and the quarks are confined within hadrons; and the *Higgs phase* for the weak interactions in $SU(2)_L \times U(1)_Y$ meaning that the corresponding force is short range since the gauge bosons W, Z are heavy after symmetry breaking.

1.2.2 Motivation for the Standard Model

Why do we need to learn about the Standard Model?

- *It is fundamental*. This is currently the most fundamental theory in science describing the nature and interactions of the building blocks of nature.

- *Robustness*. It is based on self-consistent elegant and robust mathematical principles based on the two fundamental theories of physics, namely quantum mechanics and relativity, that can be used to explain the observable world from basic principles. Symmetries and their whole mathematical structure play a crucial role in the implementation of these basic principles.

- *It is true!* The SM is one of the greatest achievements in science history. It is mathematically consistent and agrees with all experimental tests so far. In fact, many experiments have spectacularly confirmed predictions such as the existence of the $W^{\pm}$, Z^0 bosons, the top quarks, the Higgs particle, etc. On top of that, the observable physical quantities have been measured with unprecedented precision. For instance, the anomalous magnetic dipole moment of the electron is

$$a = \frac{g-2}{2} = (1159.65218091 \pm 0.00000026) \times 10^{-6} \tag{1.2.8}$$

The agreement between theory and experiment is within one part in a trillion, which is probably the best precision test of any scientific theory. Also, the fine structure constant (measured at small energies $E \ll 10^2 \text{GeV}$)

$$\alpha^{-1} = \frac{\hbar c}{e^2} = 137.035999084\,(21) \tag{1.2.9}$$

has been tested to one part in a billion. These are only a few of the many experimental tests that the Standard Model has passed successfully over many years and the experimental results have been reproduced by independent experiments many times adding to the robustness of the theory.

- It is the best test of validity of QFT. Relativistic QFT is a very general framework, but with only a handful of general experimental predictions such as the existence of anti-particles, the CPT theorem, the spin-statistics connection and the running of couplings. The main reason that QFT is trusted is through its big success in describing the Standard Model. Only one in an infinite number of possible QFTs that happens to describe our world.

- *Cosmology*. The Standard Model is the main theoretical framework to successfully describe the early history of the universe known so far. The great success of the past decades to test cosmological theories with strong precision uses the properties of the Standard Model and provides a further experimental way to test the Standard Model. For instance, the excellent agreement between theory and observation on the abundance of the different elements coming from the big-bang for light elements and from the core of stars for the heaviest can all be traced to properties of the Standard Model. Furthermore, the fact that no more than three light neutrinos are expected fits extraordinarily well with precision tests of the Standard Model as well as cosmological observations of the cosmic microwave background that put stringent bounds on what is known as dark radiation, essentially ruling out further neutrino species.

- It is incomplete! We know that the Standard Model cannot be the final theory. For example, there is no description of some key observational facts such as the explanation of baryogenesis (why we are made up of baryons and not anti-baryons), the identity of dark matter for which there is overwhelming evidence, the explanation of the nature and origin of dark energy, responsible for the current acceleration of the universe and most importantly a fully quantum description of gravity. But whatever physics will replace it, the SM will remain as the valid description of the world at low energies and understanding the basic principles behind the Standard Model is a prerequisite to look for alternative theories to modify it and/or generalize it. We summarize some of these directions in chapter 9.

1.3 Outline for the lectures

As is common in many textbooks, the structure of these lecture notes diverges from the historical development of the Standard Model summarized earlier. The primary reason for this is pedagogical clarity, which at times may obscure some of the unexpected discoveries and existential challenges the theory faced. Ultimately, the Standard Model's final form is the result of numerous breakthroughs and innovative ideas. While these lectures often present the definitive answers to foundational questions in particle physics upfront, it is important to remember that many physicists wrestled with these same issues for years – this is the nature of research. Therefore, students should not be discouraged by the seemingly ad-hoc choices made in presenting some of the material.

In chapter 2, we revisit Wigner's classification of elementary one-particle states through unitary irreducible representations of the Poincaré group. We begin with an overview of the Poincaré algebra and its representations, giving special attention to spinor representations, where the concepts of left- and right-handedness – crucial for chiral gauge theories – are introduced. Finally, we explore in detail how both massless and massive elementary particles are described through Poincaré group representations.

After this classification of all "relevant" unitary representations of the Poincaré group that potentially play a role in nature in form of elementary one-particle states, chapter 3 is concerned with the question: what do we do with them? Traditionally, we pick one of two options by either building an on-shell formalism for amplitudes or we introduce off-shell objects that transform covariantly under Lorentz transformations.

In these lectures we pick the second option. To describe interactions among many particle states, we formulate axioms for a proper quantum theory in section 3.1 such as unitarity and locality.

We define fields as the superposition of one-particle states of fixed mass and spin/helicity. Subsequently, we can formulate an action principle for these fields which allows to systematically introduce interactions among particles in terms of operators in the Lagrangian, while producing the correct on-shell conditions for corresponding particles in the absence of interactions. Next, we revisit standard arguments for organizing physics by energy scales in section 3.3 which will be crucial in describing physical phenomena at low energies. The organization of interactions and their relevance at low energies are both heavily determined by the notion of symmetries. We summarize the various types of such symmetries in section 3.2 most of which will play important roles in the Standard Model.

The advent of fields comes however also at a cost. Among others, it introduces unphysical auxiliary degrees of freedom for particles with spin/helicity ≥1. This ultimately demands gauge redundancies as will be explained in chapter 4. The story is quite simple: to describe the interactions of many species of massless fields of helicity 1, the operators in the action have to follow certain rules dictated by an algebra underlying certain Lie groups. This can in fact be derived from only requiring Lorentz invariance and unitarity of scattering amplitudes (in form of Ward identities) as detailed in App. B. These so-called gauge theories play an outstanding role in the Standard Model since they describe the dynamics of the mediators of elementary forces like the photon in QED. We collect their properties relevant for these lectures and also review soft theorems making statements about e.g. charge conservation in Sect. 4.2.

The next chapter is concerned with the breaking of symmetries. We distinguish mainly two ways in which symmetries get broken, namely explicitly or spontaneously. In the former case, a symmetry that exists under certain assumptions gets spoiled once other interactions are allowed. In contrast, the spontaneous breaking of a symmetry just means that the ground state of a theory does not respect the full symmetry group, but merely some sub-group.

In chapter 5, we explain how this idea solves unitary problems in EFTs with massive spin-1 bosons. In particular, the spontaneous breakdown of gauge symmetries will be key in understanding why the weak interactions are short ranged as well as how matter fields like the electron receive their masses in the Standard Model.

Yet another important aspect of symmetries is their manifestation in quantum theories. The notion of anomalies introduced in section 5.6 explains which classical symmetries can be promoted to quantum theories. In the presence of non-vanishing anomalies, global symmetries are explicitly broken in the quantum theory, whereas for gauge (= local) symmetries the theory is rendered inconsistent. The discussion of anomalies in the Standard Model will be deferred to Sect. 8.3 once the full content of the Standard Model has been described.

A first step towards building up the Standard Model is taken in the subsequent chapter. There, we introduce the Electroweak Theory as the unification of the weak interactions mediated by massive spin-1 bosons $W^{\pm}$, Z^0 and the electromagnetic force due to the photon. We will see the Higgs mechanism at play giving masses to W±,Z0 as well as the matter fields. Chapter 6 will also analyze in great detail how the Higgs particle cancels the dangerous contributions in the scattering of massive vector bosons, thereby preventing the otherwise expected loss of perturbative unitarity. We explicitly write down the interactions of matter fields with the Higgs and vector bosons. We show that in a basis of mass eigenstates some of the interactions are not flavour diagonal which leads us to the CKM mixing matrix. A significant part of chapter 6 is devoted to neutrino physics which remains a very active field of research on the Standard Model. For example, we illustrate how the decay of the Z-boson into lepton-antilepton pairs constrains the number of light species, i.e., neutrinos in Sect. 6.4.3. Lastly, we show how the electroweak interactions can be approximated by the 4-Fermi theory at low enough energies. As an application of these results, we compute the decay $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ and compare our theoretical results with experimental measurements.

Chapter 7 concerns the strong interactions which is theoretically described by Quantum Chromodynamics (QCD). At its heart, QCD is an SU(3) gauge theory providing additional charges, so-called colors, for the quarks. Ultimately, this was key in explaining the classification of hadrons through the eightfold way in Fig. 1.2. The arguably most important property of QCD is however asymptotic freedom which ensures that the interactions becomes strong at low energies, while at the same time being well behaved in the high energy limit. Ultimately, this result explains e.g. why we cannot see quarks and gluons in isolation – the farther away a quark from a hadron, the stronger the interactions. The last part of chapter 7 discusses chiral Lagrangians obtained when treating the light quarks as effectively massless. In this way, the resulting approximate symmetry helps us classifying hadrons, while its breakdown quantifies the mass hierarchies observed in e.g. the baryon octet in Fig. 1.2.

In chapter 8, we provide a short summary of parameters in the Standard Model with a particular focus on the Θ-term and its relevance for quark masses. As it turns out, this term combines in an interesting way the weak and strong interactions descriptions within the Standard Model. Furthermore, we study potential anomalies of local and global symmetries in the Standard Model which also combine the strong and electroweak sectors of the model. We show explicitly that, despite being a chiral theory, quite remarkably the Standard Model is indeed free of gauge anomalies and therefore quantum mechanically consistent in a highly non-trivial way, while there are harmless anomalous global symmetries like lepton and baryon number.

The subsequent chapter addresses some of the open questions in the Standard Model and possible extensions. First, we list the major open problems within the Standard Model and classify them according to their nature. In section 9.2, we provide a short account of physics Beyond the Standard Model such as supersymmetry, grand unification or axions. We emphasize also bottom-up approaches. In this sense the power of effective field theories again play a major role in order to learn from a model independent way what may lie beyond the Standard Model. The final chapter 10 summarizes the most important concepts derived over the course of these lectures and provides some concluding remarks.

CHAPTER 2 SPACETIME SYMMETRIES

Importance: *If it were not for these symmetries, the work of science would have to be redone in every new laboratory and in every passing moment.*

In this chapter, we review basic techniques for constructing suitable representations of the Poincaré group – the symmetry group of (Minkowski) spacetime – which defines Special Relativity and is relevant for constructing the Standard Model. We begin with a general discussion of the Poincaré algebra and its properties before introducing spinor representations of the Lorentz group SO(3,1). Subsequently, we will detail Wigner's classification of irreducible representations of the Poincaré group which define for us elementary particles. We discuss in detail how these particles have to transform under discrete spacetime transformations which allows us to count all of the relevant polarization states. Among others, we will find that massless particles with helicity ≥ 0 have always just two degrees of freedom which will severely constrain the field theories to be studied in the reminder of these lectures.

2.1 Poincaré symmetry and spinors

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